An Accurate 3-D Netted Radar Model for Stealth Target Detection Based on Legendre Orthogonal Polynomials and TDOA Technique

Mohamed Barbary and Peng Zong

***Abstract*—Normally, it is very difficult to statistically model a real stealth target by theoretical models which have analytical probability density function (pdf) expressions, because there are very few parameters which can be used to approximate the pdf of stealth target radar cross section (RCS) in conventional target models. A novel non-parametric detection technique for stealth target model F-117A based on time difference of arrival (TDOA) and legendreorthogonal polynomials methods is proposed. TDOA is applied for an accurate localization of stealth target based on the real stealth RCS data which predicted by Physical Optics (PO) approximation method to improve the performance of netted radar, while the Legendre orthogonal polynomials are used to reconstruct the pdf of stealth target RCS data. The proposed scheme improves RCS measurement accuracy and computes the stealth target position based on maximum – likelihood (ML) estimation. Simulations demonstrate that the new detection method gives much higher estimation accuracy of stealth target model and reduces location errors comparing to the traditional TDOA that using theoretical model which have analytical expressions.**

***Index Terms*—Stealth RCS, TDOA, netted radar, legendre orthogonal polynomials, PO.**

I. Introduction

The threat of electronic jamming to military radar is well known. But in the event of future wars, there are two serious threats to radar: stealth target and antiradar missiles (ARM).The goal of stealth technology is to make an airplane invisible to radar. In other words, whenever the aircrafts Radar Cross Section area (RCS) isvery small, the returned signals received by the radar cannot be differentiated from the clutter/interference and noise; the refore, it will be undetectable by a normal radar system reliably .The overall result is that a stealth aircraft like anF-117A can have the radar signature of a small bird rather than an airplane. The anti-stealth radar can be divided into two types. The first one is raising the capability of radar detection to stealth target with RCS reduced by increasing the power-aperture product of radar (PA), this is not a good way that will pay investment which is almost directly proportional to the PA. The second kind of counteracting stealth is making RCS of aircraft hardly reduce to expected level by selecting lower

The second kind of counteracting stealth is making RCS of aircraft hardly reduce to expected level by selecting lower radar carrier frequencies and using the biostatic (multistatic) or netted radar system. These two kinds of measures are alternative or mixed to be realized [1].

Netted radar employs several spatially distributed transmitters and receivers for information retrieval. This system topology offers many advantages over traditional monostatic and bistatic systems which use a single transmitter and a single receiver. For example, it provides better utilization of reflected energy, more flexible system arrangement and enhanced information retrieval capability. Therefore, the netted radar system is of emerging interests among radar researchers [2].Several researches deal with improving the Radar detection and tracking by using the netted radar systems based on the localization techniques such as time difference of arrival (TDOA), frequency difference of arrival (FDOA), angle of arrival (AOA) [3]-[5] etc. These researches didn't study an important evaluation criterion of aircraft's stealth performance, only using the conventional theoretical target fluctuation models (i.e., Swerling's case I—IV, chi-square, log-normal and Rice model etc.,) to the statistical analysis and modeling of areal target RCS based on the simple consideration of using flat RCS ($0.025\ m^2$) [6]-[11]. However, in practice, the scattering of electromagnetic energy from a stealth target is a rather complicated phenomenon, which depends on a number of factors (stealth target geometry, size, shape, orientation (aspect), altitude with respect to radar antenna etc.).The stealth target parameters are often practically unknown and even time-varying. In this case, the probability density function (pdf) of the stealth target RCS cannot be approximated well by any existing theoretical models which have analytical expressions. Therefore, the parametric method is not suitable for the application of stealth target modeling [12], [13].

Generally, the localization techniques need to be combined with a non-parametric method for statistical modeling of stealth target detection, which using an approximation methods to predict the real stealth target RC Sdata, such as Method of Moments (MoM),Finite Element Method (FEM), Geometrical optics (GO) and Physical Optics (PO) [14].This paper proposes a new non-parametric technique for stealth target detection using a novel combination of statistical Legendre orthogonal polynomials

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M. A. Barbary is with College of electronic and information engineering, Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China (e-mail: mbarbary300@gmail.com).

Peng Zong is with College of Astronautics, Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China (e-mail: pengzong@nuaa.edu.cn).

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model to reconstruct the pdf of the stealth target RCS and TDOA localization technique. This combination based on a real RCS data predicted by PO approximation method to achieve high location accuracy of the stealth target.

The rest of this paper is organized as follows. In Section II, we present the proposed detection scheme through a review PO, TDOA methods and illustrate the Least Square (LS) position estimation technique adopted for proposed scheme. In this section we also discuss the non-parametric method for statistical modeling of stealth target detection based on Legendre orthogonal polynomials to reconstruct the pdf of the stealth target RCS. The performance of the proposed scheme is evaluated via computer simulation in Section III, followed by the conclusion in Section IV.

II. Proposed Scheme

This section reviews the physical optics (PO) method to calculate RCS of stealth model based on F-117A, the time difference of arrival (TDOA) localization technique and presents the proposed netted radar scheme to estimate the accurate position for stealth model.

A. The Physical Optics (PO) Formulation to Predict RCS of Stealth F-117A Mode

The physical optics (PO) approximation is a well-known technique used to analyze very large conducting structures. In scattering problems and radiation of large reflectors, the PO technique provides acceptable accuracy, for some applications. This technique allows avoiding the hard solution of the MoM linear system by approximating this solution by the explicit PO current [15]. In the presence of a perfectly conducting surface, the total electromagnetic field of a source may be expressed as superposition of the incident fields (E_i, H_i) and the fields (E_s, H_s) which are scattered by the surface. The scattered fields can be expressed in terms of the radiation integrals over actual currents induced on the surface of the scatterer.The PO assumes that the induced surface currents on the scattered surface are given by the geometrical optics (GO) currents over those portions of the surface directly illuminated by the incident magnetic field, $\vec{H_i}$, and zero over the shadowed sections of the surface:

$$\vec{J_S} = \begin{cases} 2\hat{n} \times \vec{H_i} \quad , & \text{illuminated region} \\ 0 \quad , & \text{shadow region} \end{cases} \tag{1}$$

where $\hat{n}$ denotes the out ward unit normal vector on a surface.

The authors in this paper use the PO method to predict the RCS of a geometry model of stealth target based onF-117A, which are modeled with the use of triangular facets. To calculate the PO-scattered field, the surface of the scatterer is approximated using planar facets. The geometry model of stealth target based on F-117A is approximated by a model consisting of many triangular facets is described in terms of the Cartesian coordinates of a large number of points on the surface. This surface is then approximated by planar triangular facets connecting these points. An arbitrary midpoint $(p)$of the triangle surface is assigned the coordinates (r_p, θ_p, ϕ_p), the observation point is assigned the coordinates $(r_s,\ \theta_s,\ \phi_s)$ and the unit vectors $(\hat{r}_s,\ \hat{\theta}_s,\ \hat{\phi}_s)$. Normal vector $\hat{n}$ is a unit vector with its tip at the midpoint of the triangle. Then $\hat{n}$ can be expressed as the cross product of the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$. Once these vectors are found,$\hat{n}$can directly be foundby, $\hat{n} = \overrightarrow{AB} \times \overrightarrow{AC} \ / \ |\overrightarrow{AB}||\overrightarrow{AC}|$ .These parameters are depicted in Fig. 1.

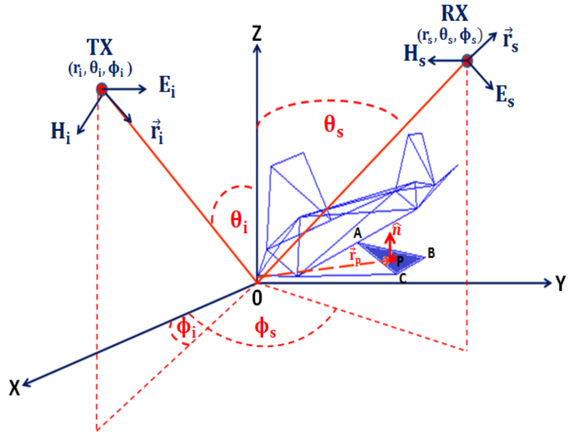


Fig.1. Vector definitions of an approximation of a stealth F-117A model using triangular facets on the surface.

Thus far, the discussion has involved the calculation of the scattered field from a single facet. Superposition is used to calculate the scattered field from the stealth target. First, the scattered field is computed for each facet. Then, the scattered field from each facet is vector summed to produce the total field in the observation direction.

If the source is at a great distance from the target, it will illuminate the target with an incident field which is essentially a plane wave. The incident electric field intensity is given by, $\overline{E_i} = (E_{i\theta}\hat{\theta}_i + E_{i\phi}\hat{\phi}_i)e^{-j\vec{k}_i\hat{r}_i \cdot \vec{r}_p}$, where $E_{i\theta}$,$E_{i\phi}$ are the orthogonalcomponents in terms of the variables θ and ϕ, $(r_i, \theta_i, \phi_i)$are the spherical coordinates of the source and $(\hat{r}_i, \hat{\theta}_i, \hat{\phi}_i)$are the unit vectors, so the magnetic field intensity of the incident field is given by:

$$\vec{H}_i = \frac{\vec{k}_i \times \vec{E}_i}{Z_0} = \frac{1}{Z_0}(E_{i\phi}\hat{\theta}_i - E_{i\theta}\hat{\phi}_i)e^{j\vec{k}_i h} \tag{2}$$

where($k = \frac{2\pi}{\lambda}$), $\vec{k}_i$ is the propagation vector is defined as$\vec{k}_i = -k(\hat{x}\sin\theta_i\cos\phi_i + \hat{y}\sin\theta_i\sin\phi_i + \hat{z}\cos\theta_i)$, Z_0 is the intrinsic impedance of free spaceand $h = \hat{r}_i \cdot \vec{r}_p = x_p \sin\theta_i\cos\phi_i + y_p\sin\theta_i\sin\phi_i + z_p\cos\theta_i$. Since radiation integral for the scattered field is calculated by employing a GO approximation for the currents induced on the surface, it can be concluded that PO is a high frequency method, which implies that target is assumed to be electrically large. For the scattered field, the vector potential is given by [16]:

$$\vec{A} = \frac{\mu}{4\pi r_s}e^{-jkr_s}\iint_S \vec{J}_s e^{jk\hat{r}_s \cdot \vec{r}_p}\, ds \tag{3}$$

where μ is the permeability of a specific medium. For a far-field observation point, the following approximation holds

$$\vec{E}_s(r,\theta,\phi) = -jw\vec{A}$$
$$= -\frac{jw\mu}{2\pi r_s}e^{-jkr_s}\iint_S \hat{n} \times \vec{H}_i e^{jk\hat{r}_s \cdot \vec{r}_p}\, ds$$

$$= \frac{e^{-jkr_s}}{r_s}\left(E_{i\emptyset}\hat{\theta}_i - E_{i\theta}\hat{\phi}_i\right) \times \underbrace{\left(\frac{j}{\lambda}\right)\iint_s \hat{n} e^{jk(h+g)}\, ds}_{\vec{S}} \quad (4)$$

where

$$g = \hat{r}_s \cdot \vec{r}_p = x_p \sin\theta_s \cos\phi_s + y_p \sin\theta_s \sin\phi_s + z_p \cos\theta_s$$

However, it is not possible to obtain an exact closed form solution for $\vec{S}$ with this integral. Given that the incident wave front is assumed plane and that the incident field is known at the facet vertices, the amplitude and phase at the interior integration points can be found by interpolation. Then, the integrand can be expanded using Taylor series, and each term integrated to give a closed form result. Usually, a small number of terms in the Taylor series (on the order of 5) will give a sufficiently accurate approximation with unit amplitude plane wave (|Ei|=1) [17].

$$\vec{S} = \left(\frac{j}{\lambda}\right)\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| e^{jD_0}\left\{\left[\frac{e^{jD_p}}{D_p\left(D_q - D_p\right)}\right] - \left[\frac{e^{jD_q}}{D_q\left(D_q - D_p\right)}\right] - \frac{1}{D_q D_p}\right\} \quad (5)$$

where

$$D_p = k[(x_B - x_A) \times \sin\theta_s \cos\phi_s + (y_B - y_A) \times \sin\theta_s \sin\phi_s + (z_B - z_A) \times \cos\theta_s\,] \quad (6a)$$

$$D_q = k[(x_C - x_A) \times \sin\theta_s \cos\phi_s + (y_C - y_A) \times \sin\theta_s \sin\phi_s + (z_C - z_A) \times \cos\theta_s] \quad (6b)$$

$$D_0 = k[x_A \sin\theta_s \cos\phi_s + y_A \sin\theta_s \sin\phi_s + z_A \cos\theta_s] \quad (6c)$$

It is now possible to write the formula of PO current as, $\vec{J}_s = \left(J_{sx}\,\hat{x} + J_{sy}\,\hat{y} + J_{sz}\hat{z}\right)e^{jkh}$. In the general case, the local facet coordinate system will not be aligned with the global coordinate system. In the local facet coordinate system ($x^{"}, y^{"}, z^{"}$), the facet lies on the $x^{"}y^{"}$ plane, with $\hat{z}^{"}$ being the normal to the facet surface, hence $\hat{n} = \hat{z}^{"}$.For any arbitrary oriented facet with known global coordinates, its local coordinates can be obtained by a series of two rotations. First, the angles α and β, are calculated from $\alpha = \tan^{-1}\left[n_y/n_x\right]$ and $\beta = \cos^{-1}(\hat{z} \cdot \hat{n})$, where $\hat{n} = n_x\,\hat{x} + n_y\,\hat{y} + n_z\hat{z}$.The local coordinates can be transformed to global coordinates [18]:

$$\begin{bmatrix} x^{"} \\ y^{"} \\ z^{"} \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (7)$$

However, in facet local coordinates, the surface current does not have a $\hat{z}^{"}$component, since the facet lies on the $x^{"}y^{"}$ plane. Hence the local surface current is given by, $\vec{J}_s = (J''_{sx}\hat{x}^{"} + J''_{sy}\hat{y}^{"})e^{jkh}$, the surface current components are[18]:

$$J''_{sx} = \left[\frac{E^{"}_{i\theta}\cos\phi^{"}\cos\theta^{"}}{2R_s + Z_0\cos\theta^{"}} - \frac{E^{"}_{i\phi}\sin\phi^{"}}{2R_s\cos\theta^{"} + Z_0}\right]\cos\theta^{"} \quad (8a)$$

$$J''_{sy} = \left[\frac{E^{"}_{i\theta}\sin\phi^{"}\cos\theta^{"}}{2R_s + Z_0\cos\theta^{"}} + \frac{E^{"}_{i\phi}\cos\phi^{"}}{2R_s\cos\theta^{"} + Z_0}\right]\cos\theta^{"} \quad (8b)$$

where $E^{"}_{i\theta}, E^{"}_{i\phi}$ are the components of the incident field in the local facet coordinates ,$\theta^{"}, \phi^{"}$are the spherical polar angles of the local coordinates and R_s being the surface resistivity of the facet material. When $R_s = 0$, the surface is a perfect electric conductor and let's assume the surface is smooth. To obtain the total scattered field, simply replace Eq. (8a), Eq. (8b) in the radiation integral for the triangular facet, which was determined in Eq. (5), the total number of facets (*m*=20), so

$$\vec{E}_s(r,\theta,\phi) = \sum_{m=1}^{20} \frac{-jkZ_0 e^{-j(kr_s - D_{0m})}}{4\pi r_s}\left(J''_{s_m x}\hat{x}^{"} + J''_{s_m y}\hat{y}^{"}\right) \times \left|\overrightarrow{AB}_m \times \overrightarrow{AC}_m\right| \times \left\{\left[\frac{e^{jD_{P_m}}}{D_{P_m}\left(D_{q_m} - D_{P_m}\right)}\right] - \left[\frac{e^{jD_{q_m}}}{D_{q_m}\left(D_{q_m} - D_{P_m}\right)}\right] - \frac{1}{D_{q_m}D_{P_m}}\right\} \quad (9)$$

Once the scattered field is known, the RCS in that direction is computed in terms of the incident and scattered electric field intensities, The RCS is given by [19]:

$$RCS(r,\theta,\phi) = \lim_{R\to\infty} 4\pi R^2 \frac{\left|\vec{E}_s(r,\theta,\phi)\right|^2}{\left|\vec{E}_i\right|^2} \quad (10)$$

where *R* is distance between the radar transmitter and the target. For most objects, radar cross section is a three-dimensional map of the scattering contributions, which vary as a function of aspect angles (azimuth and elevation) and polarization. The scattering matrix describes the scattering behavior of a target as a function of polarization, normally contains four RCS values ($\theta\theta, \theta\phi, \phi\theta$ and $\phi\phi$), where the first letter denotes the transmission polarization, the second letter is the polarization at receive. Therefore, the RCS can be derived at any polarizations:

$$RCS(r,\theta,\phi) = \lim_{R\to\infty} 4\pi R^2 \begin{bmatrix} |S_{\theta\theta}|^2 & |S_{\theta\phi}|^2 \\ |S_{\phi\theta}|^2 & |S_{\phi\phi}|^2 \end{bmatrix} \quad (11)$$

The s_{pq}denote the scattering parameters, where the first index specifies the polarization of the receive antenna and the second refers to the polarization of the incident wave. The elements of the scattering matrix are complex quantities and in terms of the RCS [19].

$$RCS_{pq} = 4\pi R^2 S_{pq}{}^2 e^{-2j\psi_{pq}},\ \ \psi_{pq} = \tan^{-1}\left\{\frac{Im(\frac{E_{sp}}{E_{iq}})}{Re(\frac{E_{sp}}{E_{iq}})}\right\} \quad (12)$$

B. The Time Differences of Arrival (TDOA) Method

The most widely used position location technique for radar detection is the hyperbolic position location technique, also known as the time difference of arrival (TDOA) position location method. This technique utilizes cross-correlation process to calculate the difference in time of arrival (TOA) of a target signal at multiple (two or higher) pairs of stations. This delay defines a hyperbola of constant range difference from the receivers, which are located at the foci. Each TDOA measurement yields a hyperbolic curve along which the target may be positioned. When multiple receivers are used, multiple hyperbolas are formed, and the intersection of the set of hyperbolas provides the estimated location of the target. The TOA is dependent on the target-radar geometry and medium characteristics. With three stationary receivers, an intersection of hyperbolic curves corresponds to a possible target in two-dimensional (2-D) position localization, while with four or more stationary receivers, hyperbolic curves intersect at a target in (3-D) position-localization [20].

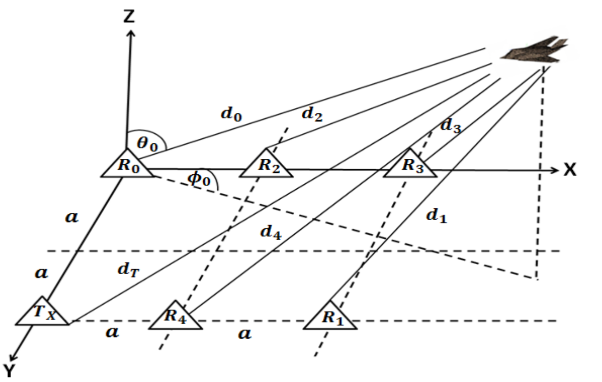


Fig. 2. The TDOA method based on netted Radar system.

The proposed model consists of one transmitter and four receivers are demonstrated in Fig. 2. Benson [21] investigates modeling method to optimize the location of receivers in order to achieve maximum coverage of aircraft moving around Cape Town International Airport. Due to this method the optimum spacing between radar stations is constant for our system geometry. The 3-D coordinate system must be converted for each radar station according to the following equations as shown in Table I.

In the case of a constant velocity medium as assumed in the following study, the TOA is function of the target-radar ranges. Assume each radar is capable of performing TOA observation, t_i , then TDOA observation is defined as $T_i = t_i - t_1\ (i = 2, \ldots, N)$ .Expressing (3-D) TDOA observation as a function of stationary receiver co-ordinates, a hyperbola has the form:

$$CT_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} \quad (13)$$

where C is the speed of light, (x_1, y_1, z_1) and (x_i, y_i, z_i) are the co-ordinates of $d_1$and d_i, respectively, and (x, y, z) is the unknown stealth target position. Consequently, the stealth target position is determined by solving the intersections of a set of $N - 1$ hyperbolas.

The least-squares estimation is a common technique to solve TDOA equations line a razed by using the first two terms of their Taylor series [22]. Denote the initial guess of the stealth target position as(x_0, y_0, z_0), the linearization of Eq. (13) is given by:

$$m_{xi}x + m_{yi}y + m_{zi}z = CT_i - f_i + m_{xi}x_{0+}m_{yi}y_{0+}m_{zi}z_0 \quad (14)$$

where

$$m_{xi} = \frac{(x_0 - x_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2}} - \frac{(x_0 - x_1)}{\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}} \quad (15a)$$

$$m_{yi} = \frac{(y_0 - y_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2}} - \frac{(y_0 - y_1)}{\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}} \quad (15b)$$

$$m_{zi} = \frac{(z_0 - z_i)}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2}} - \frac{(z_0 - z_1)}{\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}} \quad (15c)$$

$$f_i = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2)} - \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2} \quad (15d)$$

Expressing the set of linearized equations in matrix form:

$$A_t X = B_t \quad (16a)$$

where

$$A_t = \begin{bmatrix} m_{x2} & m_{y2} & m_{z2} \\ m_{x3} & m_{y3} & m_{z3} \\ m_{x4} & m_{y4} & m_{z4} \end{bmatrix} \quad (16b)$$

$$B_t = \begin{bmatrix} CT_2 - f_2 + m_{x2}x_{0+}m_{y2}y_{0+}m_{z2}z_0 \\ CT_3 - f_3 + m_{x3}x_{0+}m_{y3}y_{0+}m_{z3}z_0 \\ CT_4 - f_4 + m_{x4}x_{0+}m_{y4}y_{0+}m_{z4}z_0 \end{bmatrix} \quad (16c)$$

$X = [x\, y\, z\,]^T$ is the vector of the unknown variables x , y and z.

TABLE I: 3-D COORDINATES OF THE SYSTEM GEOMETRY

No	X-direction	Y-direction	Z-direction
d_0	$d_0 \cos\phi_0 \sin\theta_0$	$d_0 \sin\phi_0 \sin\theta_0$	$d_0 \cos\theta_0$
d_1	$d_0 \cos\phi_0 \sin\theta_0 - 2a$	$d_0 \sin\phi_0 \sin\theta_0 - 2a$	$d_0 \cos\theta_0$
d_2	$d_0 \cos\phi_0 \sin\theta_0 - a$	$d_0 \sin\phi_0 \sin\theta_0$	$d_0 \cos\theta_0$
d_3	$d_0 \cos\phi_0 \sin\theta_0 - 2a$	$d_0 \sin\phi_0 \sin\theta_0$	$d_0 \cos\theta_0$
d_4	$d_0 \cos\phi_0 \sin\theta_0 - a$	$d_0 \sin\phi_0 \sin\theta_0 - 2a$	$d_0 \cos\theta_0$

C. The Range-Measurement Accuracy of Proposed Scheme Based on the Real RCS Stealth Data Predicted by PO Method

It is desirable to know the accuracy of the estimated stealth target position, for example, in order to properly initialize the covariance matrix in a tracking filter. Usually, the accuracy of the measurement of the bistatic parameters is known (it can be estimated from the size of the range resolution cell and the signal-to-noise ratio). Our aim is to calculate the accuracy of the position estimate based on the known accuracy of the bistatic parameters. Denote the variance of the bistatic range error corresponding to the i^{th} receiver as $\sigma_{R_{1,i}}$ .The covariance matrix W of the measurement error ofthe bistatic parameters is given by

$$W = \left[\sigma_{R1,2}\sigma_{R1,3}\sigma_{R1,4}\right]^T \quad (17a)$$

The range-measurement accuracy is characterized by the root mean square measurement error(RMSE), σ_R , computedbythree error components [23].

$$\sigma_R = ({\sigma_{RN}}^2 + {\sigma_{RF}}^2 + {\sigma_{RB}}^2)^{1/2} \qquad (17b)$$

where$\sigma_{RN}$is SNRdependent random range measurement error,$\sigma_{RF}$is range fixed error, the rss (root-sum-square) of the radar range fixed error and the range fixed error from propagation and$\sigma_{RB}$is range bias error, the rss of the radar range bias error and the range bias error from propagation. The SNR-dependent error usually dominates the radar range error. It is random, with a standard deviation given by:

$$\sigma_{RN} = \frac{\Delta R}{\sqrt{2(SNR)}} = \frac{C}{2B\sqrt{2(SNR)}} \qquad (18a)$$

where B is waveform bandwidth, C is the speed of light and ΔR is Range Resolution.By the subscripts correspond to d_i, d_1 in Eq. (13), the accuracy of the radial length measurements given by, $\sigma_{R_{1,i}} = \left({\sigma_{RN_i}}^2 + {\sigma_{RN_1}}^2\right)^{1/2}$ .From Eq. (18a), the accuracy of the radial length measurement depending on the SNR is given by:

$$\sigma_{R_{1,i}} = \frac{C}{2\sqrt{2}B}\left(\frac{SNR_1 + SNR_i}{SNR_i SNR_1}\right)^{1/2}, \quad i = 2, \dots, N \qquad (18b)$$

The netted form of radar equation is developed here to evaluate netted radar sensitivity properties. A fully coherent radar network is considered, which means that the radars comprising the whole network have a common and highly precise knowledge of time and locations. The whole radar network is composed of $m$transmitters and n receivers. It is assumed that the whole network is well synchronized to achieve the common awareness of frequency and phase and works cooperatively such that each receiver is capable of receiving echoes due to any transmitters in the network. Precise synchronization method is required in a spatially coherent netted radar system to achieve the common awareness of frequency and phase. A possible way to keep the satisfactory coherency of radar network is to use Global Positioning System (GPS) as reference signals. It is also assumed that the target is anon-isotropic (stealth) radiator, giving a fluctuation RCS in all directions. Under these assumptions, it is reasonable to calculate the overall radar sensitivity by summing up the partial signal to noise ratio is given by [24]:

$$SNR_{netted} = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{P_{ti}G_{ti}G_{ri}RCS_{ij}{\lambda_i}^2}{(4\pi)^3 KT_s B_i {R_{ti}}^2 {R_{rj}}^2 L_{ij}} \qquad (19)$$

where P_{ti}isthe i^{th} peak transmitted power, $G_{ti}$is the i^{th} transmitter gain, $G_{rj}$is the j^{th} receiver gain, $RCS_{ij}$is RCS of the target for i^{th} transmitter and j^{th} receiver, $\lambda_i$isi^{th} transmitted wavelength, $B_i$isBandwidth for the i^{th} transmitted waveform, $k$is Boltzmann's constant, T_s is receiving system noise temperature, L_{ij} is System loss for i^{th} transmitter and j^{th} receiver, $R_{ti}$is distance from i^{th} transmitter to target and $R_{ri}$is distance from target to j^{th} receiver. Most of the previous research in netted radar system only considered the simplest case of netted radar sensitivity that the radar parameters for every transmitter and receiver are the same and an isotropic radiator, giving a constant RCS in all directions except for the distance from transmitters and receivers to target, this assumption is given by [6 -10].

$$SNR_{netted}(r,\theta,\phi) = \frac{P_T G_T G_R \lambda^2 RCS}{(4\pi)^3 KT_s B_n L_R L_T}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1}{{R_{ti}}^2 {R_{rj}}^2} \qquad (20a)$$

But this is not an accurate consideration to calculate the SNR of stealth target because the RCS value varies with elevation angle and azimuth angles. Therefore the accurate Formula of netted radar sensitivity dependent on real Bistatic RCS of stealth target should be written as:

$$SNR_{netted}(r,\theta,\phi) = M\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{RCS_{ij}(r,\theta,\phi)}{{R_{ti}}^2 {R_{rj}}^2} \qquad (20b)$$

$$M = \frac{P_T G_T G_R \lambda^2}{(4\pi)^3 KT_s B_n L_R L_T} \qquad (20c)$$

where the other parameters such as $P_T, G_T, G_R, \lambda, L_R$ and L_T still constant during the detection process.

In practice, however, the range measurement accuracy is always present in the TDOA measurements,thereforetheRCS measurement accuracy is characterized by the RMSE measurement, we can express the formula of TDOA depending on the(RMSE) of RCS measurement accuracy between receiver's i and 1according to Eq. (18b) and Eq. (20)as:

$$CT_i = (d_i - d_1) + \frac{1}{\sqrt{8MB^2}} . d_t\left(\frac{RCS_i\, {d_1}^2 + RCS_1\, {d_i}^2}{RCS_1 RCS_i}\right)^{\frac{1}{2}} \qquad (21)$$

According to the physical optics (PO) method to calculate the RCS usingEq.(12),the accurate Formula ofTDOA between receiver's i and 1base onthescattering RCS can be written as:

$$CT_i = (d_i - d_1) + \frac{1}{4\sqrt{2\pi MB^2}} \times \left[\frac{{d_1}^2 {S_i}^2 e^{-2j\psi_i} + {d_i}^2 {S_1}^2 e^{-2j\psi_1}}{{S_i}^2 {S_1}^2 e^{-2j(\psi_i+\psi_1)}}\right]^{\frac{1}{2}} \qquad (22)$$

Then re-arranging Eq. (16) in a matrix form with the covariance matrix of the measurement error which consist of the range measurement accuracy between receiver's i and 1,W using Eq. (22) and (i=4) we have:

$$A_t X + W = B_t \qquad (23)$$

It is desired to estimate the target location that best fits TDOA measurements. In particular, to find the $\hat{X}$ that minimizes the sum of squares of difference between the measurements and the estimated functions by the following weighted (LS) calculation is a natural choice for a goodness-of-fit criterion as:

$$\hat{X} = arg_X min\left\|W^{1/2}(B_t - A_t X)\right\|^2 = \left({A_t}^T W A_t\right)^{-1}\left({A_t}^T W B_t\right) \qquad (24a)$$

The estimate position of stealth target $(\hat{x}, \hat{y}, \hat{z})$ can be picked out by pre-multiplying the both sides of Eq. (24) by a 3x4 matrix composed mostly of zeroes. Denote the variance of the bistatic range error corresponding to the i^{th} receiver as ${\sigma_{R_{1,i}}}^2$ which expresses the covariance matrix, W of the measurement error of the bistatic parameters, then the position estimation using a maximum – likelihood (ML) estimate can be written as [25]:

$$\begin{bmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\left({A_t}^T W^{-1} A_t\right)^{-1}\left({A_t}^T W^{-1} B_t\right) \quad (24b)$$

For radar network system that adopts plot fusing in radar intelligence processing, radar network system detection probability can be calculated with the principle of OR rule. That is:

$$P_{dnet} = 1 - \prod_{i=1}^{n}(1 - P_{di}) \quad (25)$$

where P_{di} is the probability of detection for single radar.

In [13] the authors introduce a new method of statistical modelling, where the first n central moments of the RCS data for real targets are combined through the use of Legendre orthogonal polynomials to reconstruct the pdf of the target RCS. In our paper, we use this statistical model to achieve the accurate estimation for stealth target detection. Assuming a stealth RCS random variable σ with the mean $\bar{\sigma}$, the subtraction of minimum RCS from maximum RCS is$\sigma_L = \sigma_{max} - \sigma_{min}$.The Legendre polynomials formula to reconstruct the pdf of σ

$$p_d(\sigma) = \frac{1}{\sigma_L} p_d\left(\frac{\sigma - \bar{\sigma}}{\sigma_L}\right) = \frac{1}{\sigma_L}\sum_{n=0}^{\infty} a_n L_n\left(\frac{\sigma - \bar{\sigma}}{\sigma_L}\right) \quad (26)$$

where L_n is the expression of the Legendre polynomialwhich given byand the coefficients a_n are to be determined from the kth central moments of σ, $M_\sigma^{(k)}$ as

$$L_n\left(\frac{\sigma - \bar{\sigma}}{\sigma_L}\right) = \sum_{k=0}^{[n/2]} \frac{(-1)^k (2n-2k)!}{2^n k!\,(n-k)!\,(n-2k)!}\left(\frac{\sigma - \bar{\sigma}}{\sigma_L}\right)^{n-2k} \quad (27)$$

$$a_n = \frac{2n+1}{2}\sum_{k=0}^{[n/2]} \frac{(-1)^k (2n-2k)!}{2^n k!\,(n-k)!\,(n-2k)!}\,\frac{M_\sigma^{(n-2k)}}{{\sigma_L}^{n-2k}} \quad (28)$$

where

$$M_\sigma^{(k)} = \int_{-\infty}^{+\infty} (\sigma - \bar{\sigma})^k p_\sigma(\sigma)d\sigma\,, \quad (\mathrm{k} = 0,1,2,\ldots.) \quad (29)$$

From the above expression we can calculate the accurate probability of detection for stealth target P_{di} based on the real stealth RCS data by using PO approximation method, and then calculate the netted radar probability of detection for stealth target by using Eq. (25).

Fig. 3 shows the flow chart of proposed scheme. Which describe the steps of stealth target detection based on the combination of PO, Legendre orthogonal polynomials and TDOA methods, staring from using PO method to calculate RCS of stealth model by reading the coordinates and its facets then Legendre orthogonal polynomials to reconstruct the pdf of the stealth target RCS, finally estimate the accurate position of stealth model depending on netted radar sensitivity and TDOA detection method.

TABLE II: RADAR TRANSMITTER PARAMETERS

Parameter	value
P_t(Kwatt)	250
G_t , G_r (dB)	32
f (MHz)	3000
B_n (MHz)	1
F_t, F_r	1
L_t , L_r(dB)	5
optimum radar spacing *a (km)*	50

The radar transmitter and receivers parameters are illustrated in Table II Suppose the range resolution, ΔR is $150m$, the fixed error, σ_{RF} , is $3m$ and the bias error, $\sigma_{RB}$is$10m$.

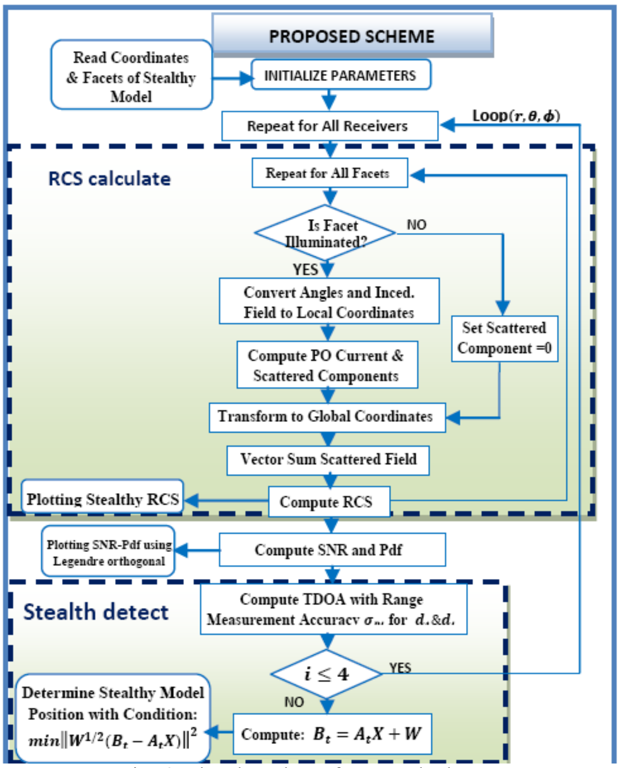
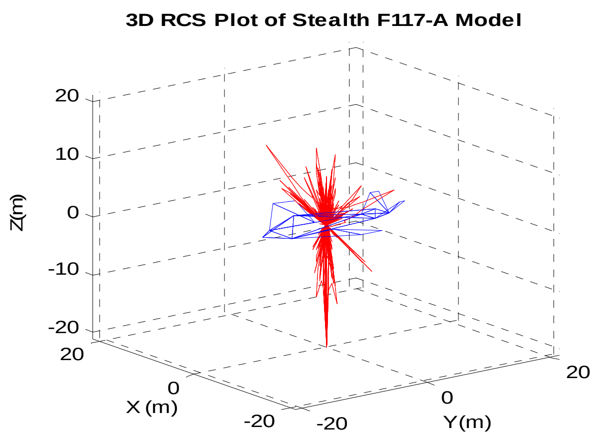
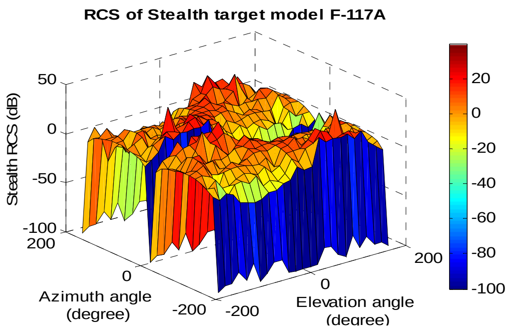


Fig. 3. The flow chart of proposed scheme.

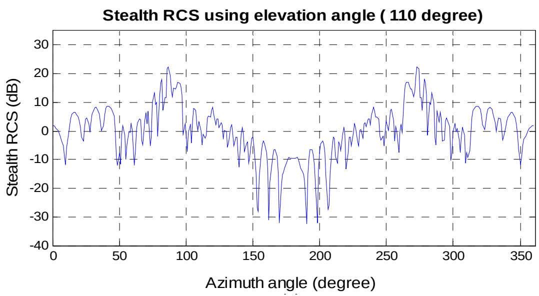


(a)

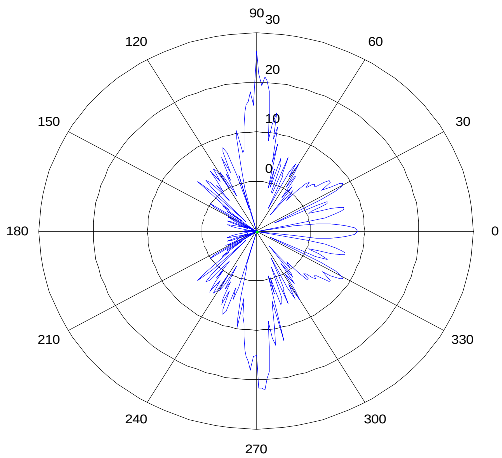


(b)

Fig. 4. The 3 –D Bistatic RCS geometry model of stealthy target based on F-117A.



(a)



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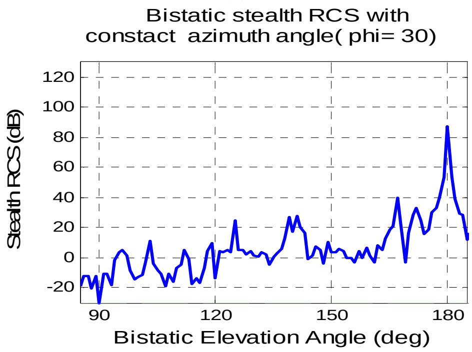


(c)

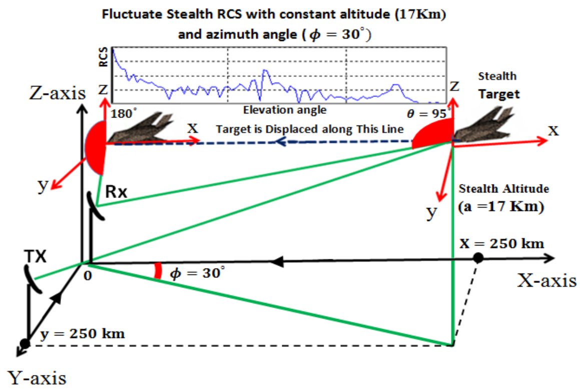
Fig. 5. Bistatic RCS of the stealth target in 2-D. (a) Bistatic RCS varying with the azimuth aspect angle with constant elevation angle ($\theta = 110^{\circ}$)inspherical coordinate system. (b)Inpolarcoordinate system.(c) Bistatic RCS varying with the elevation aspect angle with constant azimuth angle ($\phi = 30^{\circ}$).

III. Simulation Results

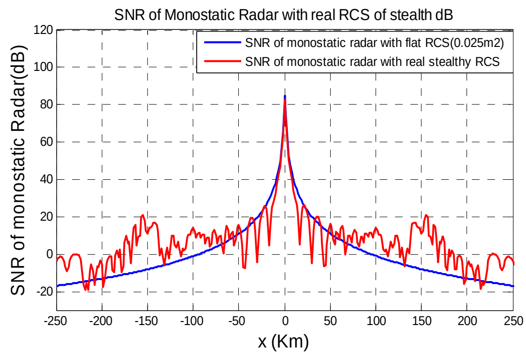
A. Establishing the Stealth Target Model and RCS Results

PO approximation is used to detect real RCS data of stealth target model, F-117A. The scientific computational features of MATLAB and GUI functions provide an efficient calculation. It provides a convenient tool for "first cut" of the RCS with a complex model outline composed of triangular facets. The simulation displays the Bistatic(3-D) RCS of stealth target based onF-117Ageometry model using the range of ($0 \leq \theta \leq 360$) and ($0 \leq \phi \leq 360$) shows in Fig.4(a).Fig.4(b)shows the Bistatic RCS of a stealth target in 3-D. Fig.5 shows the Bistatic RCS of a stealth target in 2-D, we further assume that the incident wave is (phi-polarized), the frequency is 3GHz and elevation angle ($\theta =$ 110 degree) while the azimuth angle (ϕ) between the horizon and observation direction varies from (0 to 360 degrees). (a) Using spherical coordinate plot and (b) using polar coordinate plot.(c) Bistatic RCS varying with the elevation aspect angle with constant azimuth angle ($\phi = 30^{\circ}$).

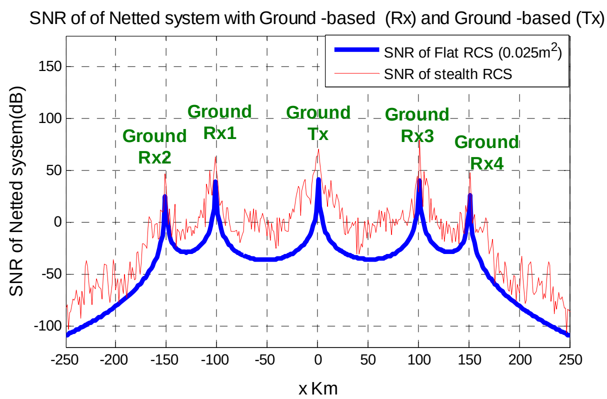
B. SNR Results for Proposed Model



(a)



(b)

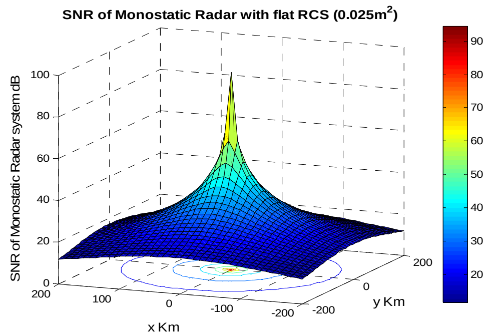


(c)

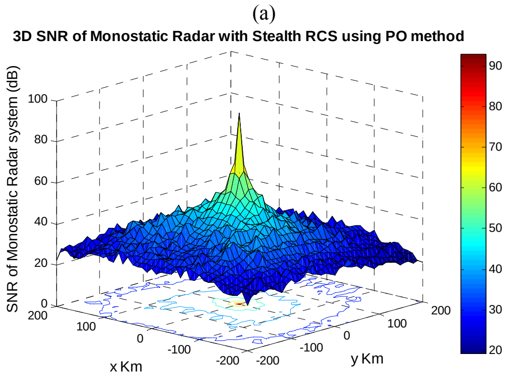
Fig. 6.The comparison between SNR of real stealth RCS data and the flat RCS ($0.025m^2$) with X-axis at constant altitude and azimuth angle where (a = 17Km) and ($\phi = 30^{\circ}$). (b) 2-D SNR for Monostatic Radar. (c) 2- D SNR for Netted Radar.

To indicate an accurate values of SNR due to the real stealth RCS data with (x, y-axis) Target-Radar range, in Fig.6 (a), we assume the stealth target moving at constant altitude and azimuth angle where ($a = 17Km$) and ($\phi = 30^{\circ}$). The relation between the varying range of stealth target and its elevation aspect angle can be obtained from Table I, therefore when the range (x-axis) varying from($250\ - 0\ km$), also the elevation aspect angle varying from ($95^{\circ} - 180^{\circ}$) similarly. In this case the fluctuation of SNR exists referring to Eq. (20b) which calculate SNR based on the real stealth RCS. A comparison between the radar sensitivity of Real RCS data based on PO method and the flat RCS ($0.025m^2$) with x-axis are demonstrated in Fig.6.

Fig. 6(b) shows a comparison between the SNR for Monostatic Radar using the two cases, which refer to the location of radar with regards to stealth target with constant altitude and azimuth angle. Fig. 6(c) showsa comparison between the SNR for Netted Radar model using the two cases. It is clear that the netted radar sensitivity of proposed scheme has been improved due an accurate estimation of the real RCS data for stealth model comparing to flat RCS ($0.025m^2$) at almost all range. The 3-D Monostatic radar sensitivity shows in Fig. 7,(a)using conventional flat RCS ($0.025m^2$). (b) Using proposed PO method with real stealth RCS. The 3-D Netted radar sensitivity shows in Fig. 8, (a) Using flat RCS ($0.025m^2$), (b)Using proposed PO method with real stealth RCS.



(a)

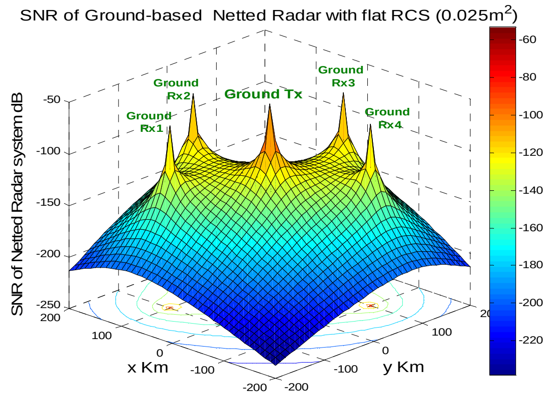


(b)

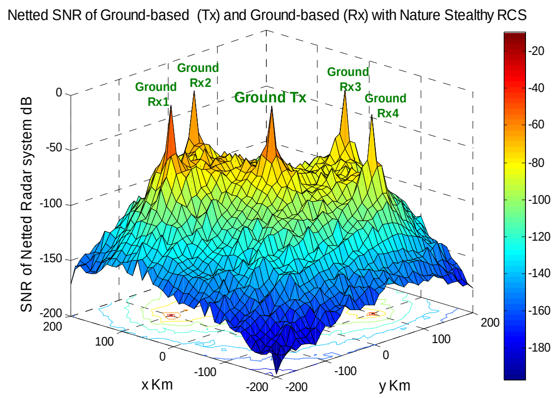
Fig. 7. The 3-D Monostatic radar sensitivity with variation of the azimuth and elevation angles (x-y axis). (a) Using flat RCS ($0.025m^2$). (b) Using proposed PO real stealth RCS data.

C. Simulation of Tracking a Stealth Target

Fig. 9 shows the comparison between the (RMSE) of stealth target detection with proposed scheme and traditional TDOA which using the conventional statistically target models. It is clear that the RMSE of proposed scheme has been improved due to the real RCS data for stealth model comparing to the conventional statistically target model sat almost all range. We can find that fluctuation of netted radar RMSE value under four cases shows a tendency around the flat(RCS $= 0.025\ \mathrm{m}^2$) value along X-axis. Comparing the RMSE plots of four cases, the RMSE curve is increasing with range changing when the conventional statistically target models is applied. While, in cases of using Legendre orthogonal polynomials, the RMSE can be even less than the other cases due to obtaining real stealth RCS. The netted RMSE have been improved, within the maximum range, the netted RMSE just equals to$50 - 100$ m.



(a)



(b)

Fig. 8. The 3-D Netted radar sensitivity with variation of the azimuth and elevation angles (x-y axis). (a) Using flat RCS ($0.025m^2$). (b) Using proposed PO real stealth RCS data.

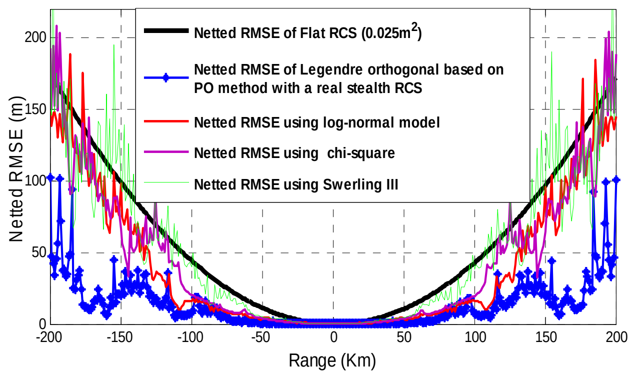
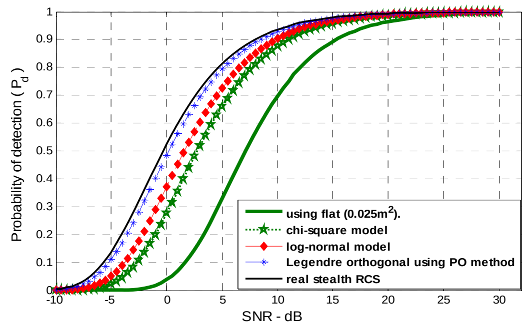
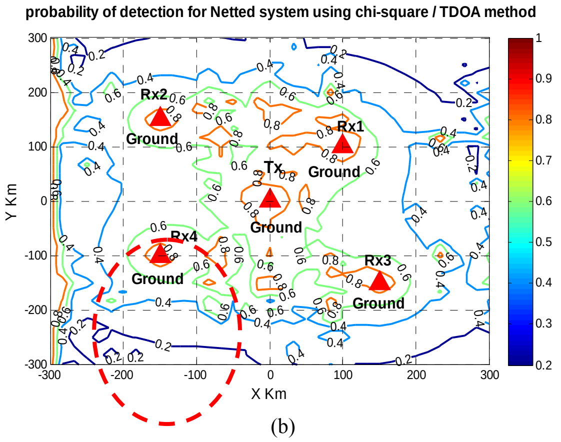


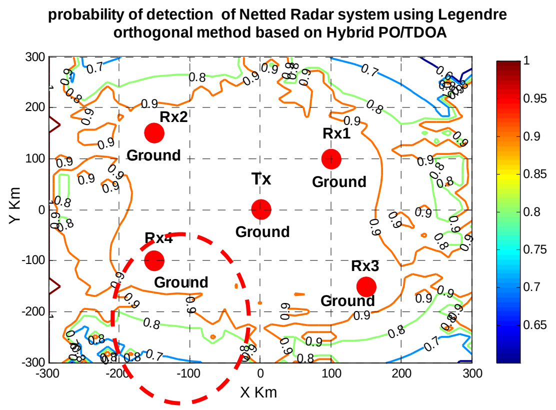
Fig. 9. The comparison between the RMSE of stealth target detection with proposed scheme using real PO stealth RCS data based on Legendre orthogonal polynomials and conventional statistically target models with flat RCS ($0.025\mathrm{m}^2$).



(a)



(b)



(c)

Fig. 10. The probability of detection for stealth target detection using the proposed scheme with real stealth RCS and other traditional models.

Fig. 10*a* showsthe comparison of detection probability between the traditional statistically target models and the proposed scheme, which used Legendre orthogonal polynomials to reconstruct the pdf of the stealth target RCS. It is clearly that the probability of detection using proposed scheme has been improved comparing to the conventional statistically target models based on flat RCS ($0.025m^2$) at almost all range.For the same detection probability 0.8, the required SNR of Legendre orthogonal polynomial is 5 dB while the others method need 8 dB and more.

TABLE III: THE DETECTION PROBABILITY OF DIFFERENTSTATISTICALLY TARGET MODELS

Detection methods	Pd of Receiver			
	Inner contour		Outer contour	
	Value	Range (Km)	Value	Range (Km)
TDOA-Chi-square	0.8	20	0.2	180
TDOA- Legendre	0.9	80	0.7	200

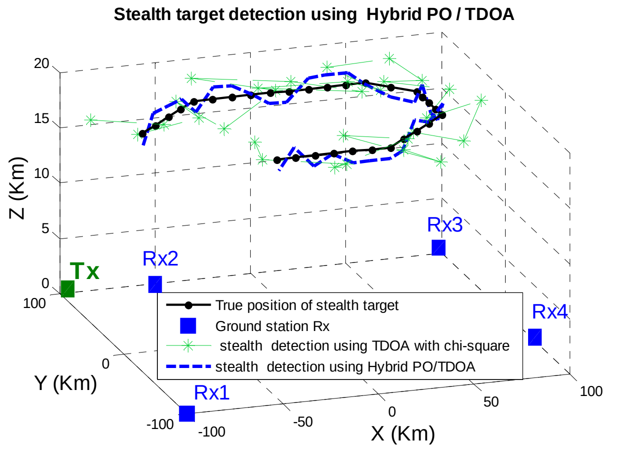


Fig. 11. The comparison between the tracking of stealth target using proposed scheme and TDOA method based on chi-square model with flat RCS.

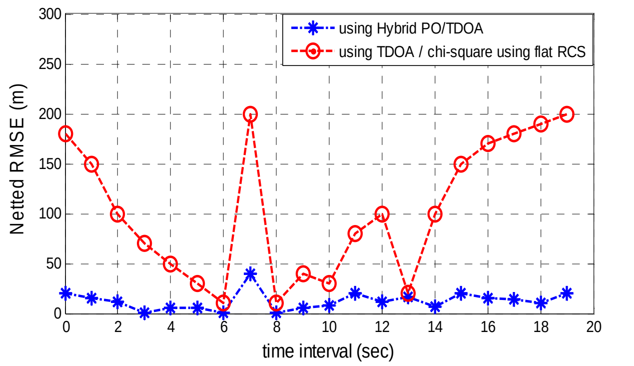


Fig. 12. The comparison between location estimation error of stealth target with proposed scheme and TDOA using chi-square (flat RCS) with time interval.

The detection probability corresponding to different statistically target models are shown in Fig. 10b-Fig. 10c. The worst case occurs when using TDOA based on chi-square statistically model, shown in Fig. 10b. In this case, the detection probability around ground-based transmitter and receivers is poor that the inner contour (0.8) is located at 20 km and the outer contour (0.2) is located at 180 km from Ground-based receiver ($R{\times}4$). The optimal case when using TDOA based on Legendre orthogonal polynomials as shown in Fig. 10c, the detection probability is optimized that the inner contour (0.9) is located at 80 km and the outer contour (0.7) is located at 200 km from the receiver ($R{\times}4$). The results of Fig.10 are also summarized in Table III.

In Fig. 11, the comparison between the tracking of stealth target using proposed scheme and TDOA based on chi-square statistically model. It is clear that the position estimate error of stealth target model was reduced using the proposed scheme. Fig. 12 shows the comparison between the RMSE of two model with time intervals.

IV. CONCLUSION

New framework of 3-D netted Radar by using a new scheme has been proposed for stealth target detection. This proposed scheme applies a nonparametric method for statistical model based on a real RCS data which predicted by PO method, it improves the performance of netted radar system against stealth technology. The Legendre orthogonal polynomials are used to reconstruct the pdf of a real stealth RCS data. The comparison of accurate tracking stealth target based on TDOA is done between proposed scheme and conventional TDOA methods that using traditional statistically target models. The results revealed that the proposed scheme gives higher location accuracy than the conventional model and demonstrate a high probability of stealth detection. It is clear that the netted radar SNR and RMSE of proposed scheme have been improved due to an accurate estimation of the real data of stealth model comparing to the conventional models. Finally the proposed scheme has better performance at almost all time intervals. The advantages of this hybrid technique are not only specific for the stealth target, but applicable for any sort of electrically tiny target with small RCSs.

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Mohamed A. Barbary received the B.Sc degree in electronics and communications from Faculty of Engineering, Alexandria University, Egypt, in 2003 and the M.Sc. in electrical engineering from Faculty of Engineering, Alexandria University, in 2012.

He is currently working towards the Ph.D. degree in College of Astronautics, Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China. He received many technical Courses in radar and electronic engineering design and Implementation.He work as a radar system engineer for more than 4 years.

Peng Zong received the B.Sc. in Nanjing University of Aeronautics and Astronautics (NUAA), China and Ph.D. degree from University of Portsmouth UK and he has been a professor in College of Astronautics NUAA since 2006. He was a research fellow in Centre for Communication Systems Research, University of Surry, England in 2001, following by employee of Hughes Network System (USA). He has been working as a software engineer and the chief designer in information and technology institute of "China Aerospace Science & Industry Corp" (CASIC) until 2006. His researcher fields are mobile communication, satellite networking mostly concerned by routing algorithm of LEO constellation, and stealthy detection of netting radar.