Comparative Analysis of Multiposition Strapdown Azimuth Determining Schemes

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Abstract—**Navigation is a method of determining or planning a vehicle's position and path/trajectory; employing sensors, geometry, astronomy and radio signals etc. Navigation is a combination of science, engineering and art. The determination of the position, velocity and attitude of a moving body with respect to a known reference-frame is the science or engineering of navigation. The planning and maintaining the intended path from one point to another, evading hindrances and crashes, is the art of navigation and has the forms as guidance, pilotage, or routing depending on the type of applications. Inertial Navigation System (INS) is a completely self-contained form of navigation and regarded as the most robust and complete navigation-system, as it renders navigation information including position, velocity, acceleration, attitude and attitude rates of the host body. INS always requires information of initial position, velocity and attitude-angles. The azimuth (angle of forward axis of vehicle with respect to the True North) is the most important and critical initial information required for the INS. Initial azimuth angle can be provided to INS by optical means or gyro-compassing. Conventional Gyrocompasses (using suspended gyroscopes) have been in use for high precision azimuth determination for last several decades. Conventional gyrocompass gives azimuth information by sensing the combined effect of Earth spin-rate and local-gravity. But gyrocompasses are highly delicate and expensive equipment requiring careful handling/transportation, periodic calibration and challenging storage conditions. To overcome these issues, extensive research has been going-on for development of rugged north finding systems based on Strapdown inertial sensors for last few decades. Herein one or more strapdown gyroscopes are used to measure the component of Earth Spin Vector in the local horizontal plane. Azimuth is mathematically computed from the output of gyroscopes. This research paper is focused on the comparative analysis of different schemes used in strapdown Azimuth Determining Systems (ADS), including, single-position, 2-position, 4-position and multi-position azimuth finding schemes. We have elaborated the methodology and analyzed the pros & cons of all schemes vis-à-vis accuracy/precision, sampling-time, effect of different types of errors of gyroscopes and total process time. We have concluded that 4-position azimuth finding scheme renders the optimal performance in terms of accuracy and time for azimuth finding.**

Keywords—**MEMS gyroscope, fiber optic gyroscope, accelerometers, inertial sensors, azimuth determining, Global Navigation Satellite System (GNSS), inertial navigation, strapdown navigation systems**

I. Introduction

Determination of high precision azimuth angle is a critical requirement for accurate surveying, alignment and various navigation systems, especially INS. Azimuth is the angle between the forward direction of the object under consideration and the true North, measured from North to East, in horizontal plane.

Azimuth determination before and during navigation ensures that the navigating bodies including ships, land-vehicles and aircraft etc. will maintain their intended courses and reach the destination efficiently, avoiding hazards [1−4]. In military applications, errors in azimuth can lead to significant deviations from intended targets. In astronomy, precise azimuth measurements are necessary for telescope alignment, star tracking, and the study of certain celestial phenomena. High precision azimuth angles are used in search and rescue operations to navigate and locate missing persons or objects. Moreover, in geophysical surveying, precise azimuth measurements are essential for exploration and extraction activities. Golfers use azimuth data to analyze and improve their swings and ball trajectories.

The concept of azimuth determination has roots in ancient astronomy and navigation. Early navigators relied on celestial bodies to determine their heading, using tools such as the astrolabe, star-trackers and sextant [5]. With the advancement of technologies, more sophisticated methods and instruments have been developed. Magnetic compasses have also been a method of determining azimuth although they are highly vulnerable to the environment and magnetic interference requiring periodic calibration. A scheme using dual-antenna GNSS receiver can calculate the azimuth angle by measuring the positions of the two antennas installed sufficiently apart from each other [6]. Emergence of the high precision Inertial Sensors technology has given new dimensions to the azimuth finding techniques and methodologies [5]. Conventional Gyrocompass (GC) tracks the true north by attempting to align the gyro-axis with the horizontal component of the Earth's spin vector by processing or swinging around the meridian and horizontal line. GCs are immune to magnetic interferences, but their mechanical complexity and maintenance requirements call for more robust solutions.

Strapdown Azimuth Finding systems are gaining acceptance due to their robustness, ruggedness and accuracy [7]. Presently certain strapdown ADS based on Fiber Optic Gyroscope (FOG), Ring Laser Gyroscope (RLG) and Hemispherical Resonator Gyroscope (HRG) outperform the conventional gyrocompasses [8, 9]. High precision MEMS Gyroscopes are also being used in modern ADS [10]. We have conducted our research on different schemes of strapdown azimuth finding using simulated and real data of Fiber Optic Gyroscope and high precision MEMS Gyroscopes. We have analyzed the effect of different types of errors on the accuracy of azimuth finding. We have elaborated the schemes and comparative analysis vis-à-vis effects of different types of errors, processing time and other effects. Explanation of schemes and respective issues are presented in subsequent paragraphs of this paper.

II. ERTH SPIN AND DIRECTION OF TRUE NORTH

The Earth spins about its polar axis at an angular rate of 360°/Sidereal Day given by following expression:

$$\Omega_{ie} = \frac{360^{\circ}}{23h\ 56\,min\ \ 4.1Sec} = \frac{15.04107^{\circ}}{h}$$
$$= 7.2921159 \times 10^{-5}\,\frac{rad}{s}$$

It may be noted that the Earth spins about its axis through an angle of approximately 361° in 24 hours (Solar Day). The phenomenon is depicted in Fig. 1. A solar day is longer than a sidereal day by about 4 minutes because Earth' s orbital motion around the Sun requires an additional rotation of 0.9856° to realign with the Sun.

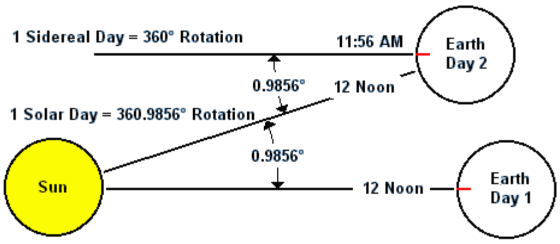

Fig. 1. Solar day and sidereal day.

Earth spin vector defined in Earth Centered Earth Fixed Frame is given by Eq. (1).

$$\omega_{ie}^{e} = \begin{bmatrix} 0 & 0 & \Omega_{ie} \end{bmatrix}^{T} \tag{1}$$

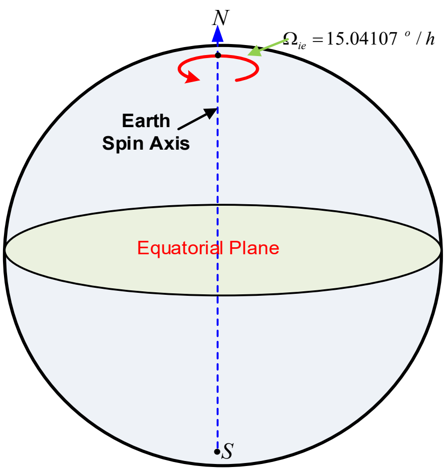

Fig. 2. Spinning of earth.

Transformation Matrix from Earth Fixed Frame to Local Level North Point Frame (ENU) is give as follows:

$$C_{e}^{n} = \begin{bmatrix} -Sin\lambda & Cos\lambda & 0 \\ -Sin\phi Cos\lambda & -Sin\phi Sin\lambda & Cos\phi \\ Cos\phi Cos\lambda & Cos\phi Sin\lambda & Sin\phi \end{bmatrix} \tag{2}$$

where $\lambda = Longitude$ and $\varphi = Latitude$

ENU stands for East, North and Up. It means that x, y and z axes of the navigation frames are pointing towards East, North and Up respectively.

Therefore, Earth Spin Vector transformed in the ENU Frame is given as follows:

$$\omega_{ie}^{n} = C_{e}^{n}\omega_{ie}^{e} = \begin{bmatrix} -Sin\lambda & Cos\lambda & 0 \\ -Sin\phi Cos\lambda & -Sin\phi Sin\lambda & Cos\phi \\ Cos\phi Cos\lambda & Cos\phi Sin\lambda & Sin\phi \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \Omega_{ie} \end{bmatrix} \tag{3}$$

$$\omega_{ie}^{n} = \begin{bmatrix} 0 \\ \Omega_{ie} Cos\phi \\ \Omega_{ie} Sin\phi \end{bmatrix} \tag{4}$$

Decomposition of Earth Spin Rate Vector into Local Level North Pointing (ENU) Frame is depicted in Fig. 3.

From Fig. 3 and Eq. (4), it can be seen that the Earth Rate component about North and Vertical axis are as follows respectively:

$$\omega_{ieN} = \Omega_{ie} Cos\phi \tag{5}$$
$$\omega_{ieU} = \Omega_{ie} Sin\phi \tag{6}$$

Component about East Axis is zero ($\omega_{i_{eE}} = 0$). True north is the direction represented by a horizontal line in the plane of the meridian or the intersection of the local horizontal plane (local level) and the local meridian as shown in Fig. 4.

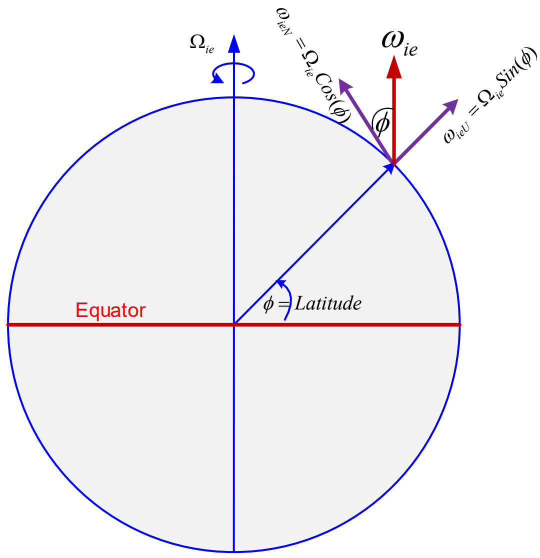

Fig. 3. Components of earth spin rate vector.

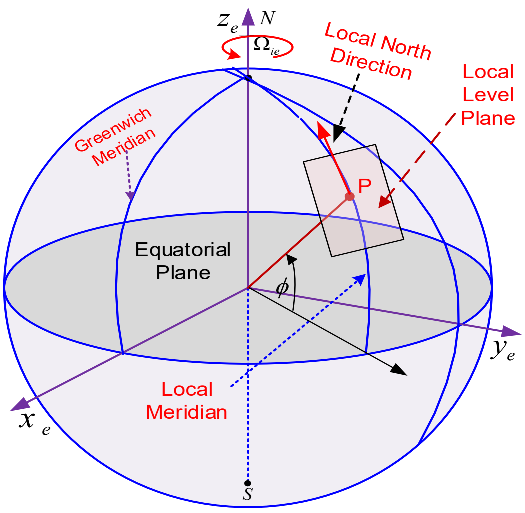

Fig. 4. Local north direction.

III. Strapdown Azimuth Determination System

Schematic diagram of the strapdown ADS investigated in this research is depicted in Fig. 5. The system is composed of following parts and sub-systems:

- 1 × Gyroscope
- 2 × Accelerometers or inclinometers for levelling
- Rotation Table with Encoder
- Supporting hardware and Electronics
- Data-acquisition/processing system
- Azimuth Computation Algorithm
- User Interface

The gyroscope in the strapdown Azimuth Determining System does not precess or swing like that in gyrocompass. Herein gyro remains fixed and stationary at the time of measurement of the component of the Earth Spin Vector's along its sensing axis. Azimuth angle is mathematically computed from the output of gyroscope.

IV. Methodology

In the subsequent sections of this paper, we will describe the detail of Single-Positions, 3-Position, 4-Position and multi-position schemes. We use a singular setup of hardware in all of these schemes wherein we use a high precision gyroscope mounted on a rotary table in such a way that the sensing axis of gyroscope is in horizontal plane and perpendicular to the rotation axis of a leveled rotary table. Levelling can be achieved using high precision inclinometers or accelerometers. The schematic of the methodology is depicted in Figs. 5–6.

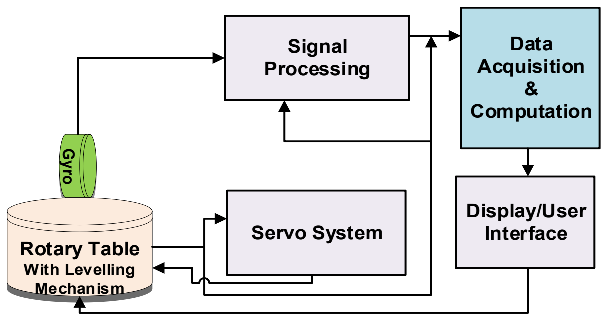

Fig. 5. Azimuth determining system.

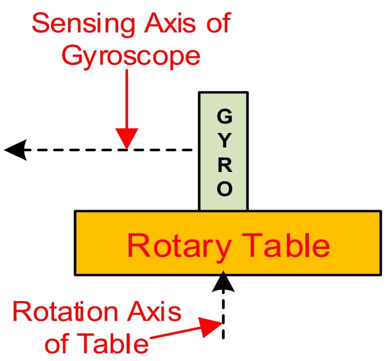

Fig. 6. Strapdown-ADS.

A. Single Position Azimuth Finding Scheme

If the sensing axis of FOG is directed exactly northward. The output of the Gyro, as shown in Fig. 7, will be [11, 12].

$$\omega = \omega_o + \Omega_{ie} Cos\phi + \varepsilon \tag{7}$$

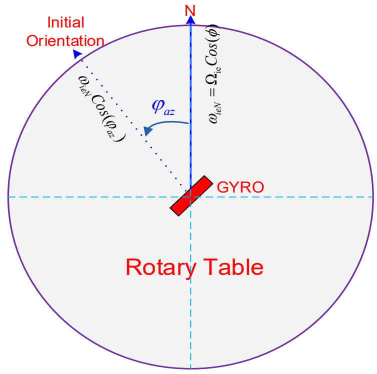

Fig. 7. Position azimuth finding scheme.

If the sensing axis is pointing at an unknown angle φ_{az} from true north in horizontal plane, the output of the gyro will be:

$$\omega = \omega_o + \Omega_{ie} Cos\phi Cos\varphi_{az} + \varepsilon \tag{8}$$

whereas:

ϕ: Local Latitude
φ_{az}: Azimuth Angle to be determined
ε: Random Drift of Gyro
ω: Gyro's output
ω_o: Fixed bias of Gyro
Ω_{ie}: Earth Spin Rate

Random bias can be eliminated by taking several readings and taking average [12, 13]. Fixed bias can also be estimated and compensated for every gyro by calibration methods.

If we know the fixed bias of the gyro, using Eq. (8), we can compute the estimated value of the unknown Azimuth Angle (φ_{az}) as follows:

$$\varphi_{az} = Cos^{-1}\left(\frac{\omega - \omega_o}{\Omega_{ie} Cos\phi}\right) \tag{9}$$

Azimuth Computation as given in Eq. (9) is termed as Single Position Azimuth Finding. It may be noted that we have used a single-axis FOG having a bias-stability of 0.005 °/h and Scale Factor Stability of ≤15ppm in this research work.

Undetermined fixed errors, errors growing with time and random errors in the bias (ω_o) will introduce significant errors in Azimuth computation in this scheme. In order to eliminate the effect of problematic fixed bias (ω_o), 2-position, 4-position and multi-position schemes can be adopted for determining the azimuth angle (ϕ_{az}) [14].

B. 2-Position Azimuth Finding Scheme

In 2-Position Scheme, we take and record output of gyroscope at two orientations 180 degrees apart from each other. The output of gyroscope at initial arbitrary orientation is:

$$\omega_1 = \omega_o + \Omega_{ie} Cos(\phi) Cos(\varphi_{az}) + \varepsilon \tag{10}$$

After taking above mentioned first reading of gyroscope, the rotary table is rotated exactly 180° with respect to the first orientation as shown in Fig. 8. The output of Gyroscope at rotated position will be as follows:

$$\omega_2 = \omega_o + \Omega_{ie} Cos(\phi) Cos(\varphi_{az} + 180) + \varepsilon \tag{11a}$$

$$\omega_2 = \omega_o - \Omega_{ie} Cos(\phi) Cos(\varphi_{az}) + \varepsilon \tag{11b}$$

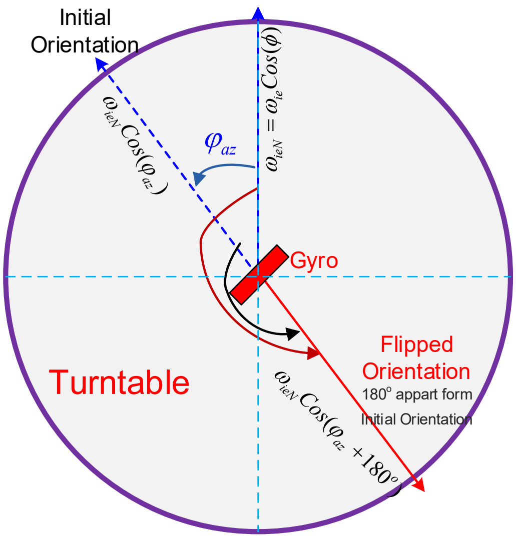

Fig. 8. Position azimuth finding scheme.

Ignoring the random errors and subtracting Eq. (11b) from Eq. (10), we have:

$$\omega_1 - \omega_2 = 2\Omega_{ie} Cos(\phi) Cos(\varphi_{az}) \tag{12}$$

By rearranging Eq. (12), the desired Azimuth angle can be computed as follows:

$$\varphi_{az} = Cos^{-1}\left(\frac{\omega_1 - \omega_2}{2\Omega_{ie} Cos(\phi)}\right) \tag{13}$$

This is called 2-Position Azimuth Computation Scheme.

C. Error Mittigation in 2-Position Scheme

The output of gyroscope at first position as given by Eq. (10) can be further improved by acquiring a third data of the gyro with the sensing axis re-aligned to the first angular orientation and acquiring the data; and obtaining an average of the first data and the third data. The average is used instead of the first data for determining the earth rate component [15]. These series of measurements improve the result when the bias of gyroscope drifts or grows at a constant rate. An average of the first measurement and the third measurement gives the estimated value of bias introduced in the averaged measurement at the same timing as that of the second measurement (taken 180° apart). The earth rate component at the first angular orientation is determined by subtracting the second data from the average of the first and the third data and dividing the difference by two. This method of measurement procedures using the 1st 2nd and 3rd data is called as bias-cancelling process by flipping [12].

D. 4-Position Azimuth Finding Scheme

After taking average output at first orientation, if gyroscope is rotated by 90°, 180° and 270° respectively with respect to the initial orientation as shown in Fig. 9 then its output is given as follows:

$$\omega_1 = \omega_o + \Omega_{ie} Cos(\phi) Cos(\varphi_{az}) + \varepsilon(1) \tag{14}$$

$$\omega_2 = \omega_o - \Omega_{ie} Cos(\phi) Sin(\varphi_{az}) + \varepsilon(2) \tag{15}$$

$$\omega_3 = \omega_o - \Omega_{ie} Cos(\phi) Cos(\varphi_{az}) + \varepsilon(3) \tag{16}$$

$$\omega_4 = \omega_o + \Omega_{ie} Cos(\phi) Sin(\varphi_{az}) + \varepsilon(4) \tag{17}$$

Manipulating above-mentioned four equations, desired azimuth angle can be computed as follows:

$$\omega_1 - \omega_3 = 2\Omega_{ie} Cos(\phi) Cos(\varphi_{az}) \tag{18}$$

$$\omega_4 - \omega_2 = 2\Omega_{ie} Cos(\phi) Sin(\varphi_{az}) \tag{19}$$

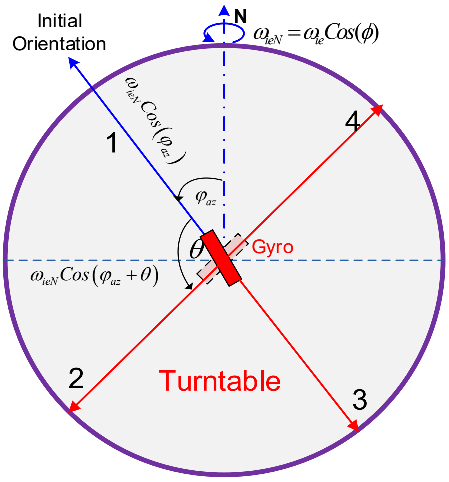

Fig. 9. Position azimuth finding scheme.

From Eqs. (18−19) we have:

$$\varphi_{az} = Tan^{-1}\left(\frac{\omega_4 - \omega_2}{\omega_1 - \omega_3}\right) \tag{20}$$

This scheme is called 4-Position Azimuth Finding Scheme.

V. Multi-Position Azimuth Finding Schemes

The multi-position Azimuth Finding Scheme is demonstrated in Fig. 10.

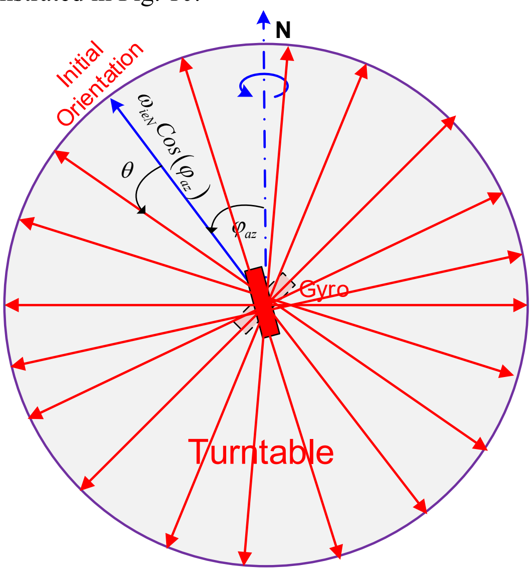

Fig. 10. Multi-position scheme.

Output of gyroscope at any arbitrary azimuth from the true North is given as follows:

$$\omega = \omega_o + k\Omega_{ie} Cos(\phi) Cos(\theta + \varphi_{az}) + \varepsilon \tag{21}$$

Using the cosine angle formula, we have:

$$\omega = \omega_o + k\Omega_{ie}Cos(\phi)\left(Cos\theta Cos\varphi_{az} - Sin\theta Sin\varphi_{az}\right) + \varepsilon \quad (22)$$

$$\omega = \omega_o + k\Omega_{ie}Cos\phi Cos\theta Cos\varphi_{az} - k\Omega_{ie}Cos\phi Sin\theta Sin\varphi_{az} + \varepsilon \quad (23)$$

We write Eq. (23) as follows:

$$A = k\Omega_{ie}Cos\phi Cos\varphi_{az} \quad , \quad B = k\Omega_{ie}Cos\phi Sin\varphi_{az} \quad (24)$$

$$\omega = \omega_o + ACos\theta - BSin\theta + \varepsilon \quad (25)$$

Here k is the scale factor of gyroscope.

In multi-position scheme we take output (ω_i) of Gyro at n equally spaced even number of position (i=1,2,…, n), [10]. For every respective angular position, we have following n Eqs. (26)−(27).

$$\omega_1 = \omega_0 + ACos\theta_1 - BSin\theta_1 + \varepsilon \quad (26)$$

$$\omega_2 = \omega_0 + ACos\theta_2 - BSin\theta_2 + \varepsilon \quad (27)$$

$$\omega_n = \omega_0 + ACos\theta_n - BSin\theta_n + \varepsilon \quad (28)$$

Writing above set of equations in Vector-Matrix Form, we have:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \\ \omega_n \end{bmatrix} = \begin{bmatrix} Cos\theta_1 & -Sin\theta_1 & 1 \\ Cos\theta_2 & -Sin\theta_2 & 1 \\ & & \\ Cos\theta_n & -Sin\theta_n & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ \omega_0 + \varepsilon \end{bmatrix} \quad (29)$$

$$W = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \\ \omega_n \end{bmatrix}, M = \begin{bmatrix} Cos\theta_1 & -Sin\theta_1 & 1 \\ Cos\theta_2 & -Sin\theta_2 & 1 \\ & & \\ Cos\theta_n & -Sin\theta_n & 1 \end{bmatrix} \quad (30)$$

$$X = \begin{bmatrix} A \\ B \\ \omega_0 + \varepsilon \end{bmatrix} \quad (31)$$

Using the Least Square Estimation method, we have:

$$\hat{X} = \begin{bmatrix} \hat{A} \\ \hat{B} \\ \omega_0 + \varepsilon \end{bmatrix} = (M^T M)^{-1} M^T W \quad (32)$$

The desired initial azimuth angle is computed from the Eq. (32) as follows:

$$\varphi_{az} = Tan^{-1}\left(\frac{\hat{B}}{\hat{A}}\right) = Tan^{-1}\left(\frac{X(2)}{X(1)}\right) \quad (33)$$

VI. Simulation and Results

We have performed number of experimentation and simulation for computation of Azimuth Angle employing Single Position, 2-Position, 4-Positin and multi-position azimuth finding techniques. Simulations have been performed by using MATLAB, analysing the effects of different grades of gyroscopes having variant levels of fixed and random errors. Following results (are for the single-Position, 2-Position and 4-Position azimuth finding techniques using FOG having in-run and run-to-run bias stability of (0.1°/h), (0.01°/h), and (0.005°/h), (see Fig. 11).

A. Simulation Results of Single Position Scheme

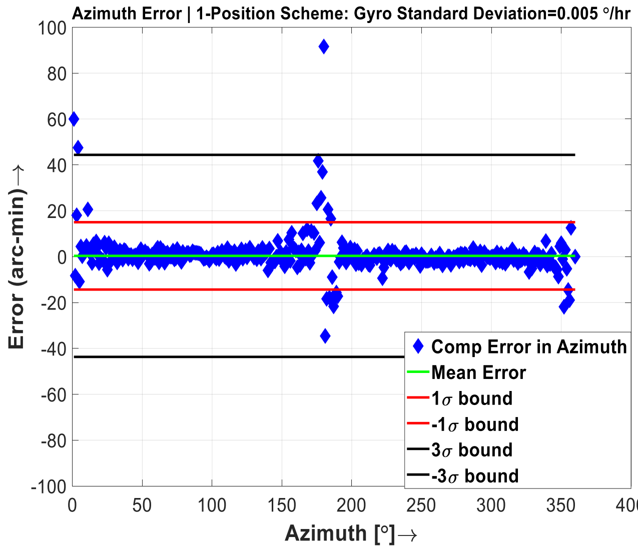

Fig. 11. Single position azimuth computation error.

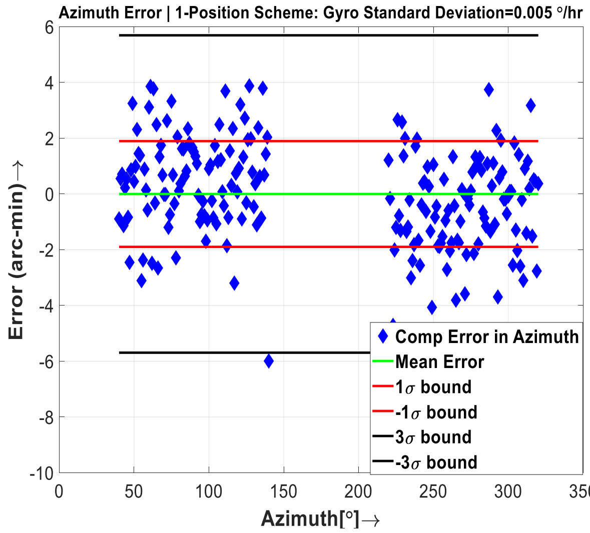

Fig. 12. Single position (azimuths near north/south omitted).

These results are obtained by employing Eq. (9). It can be seen from Eq. (9) and above figures that Azimuth Computation accuracy is effected by errors in fixed bias. Moreover the error behaviour is not symmetical in all 4 quandrants. Furthermore errors are very high if gyros initial orintation is near to North or South. Single position Azimuth finding requres information about the latitude of the test site, as well. Accuracy can be improved if initial orientation of gyro sensing axis is made to point sufficiently away from North or South direction, as shown in Fig. 12.

B. Simulation Results of 2-Position Scheme

2-Position scheme is not affected by error in fixed bias of the gyro, but accuracy is seriously affected if gyro's initial direction is near to North or South direction. Moreover, Latitude information is also required for this scheme (see Figs 13−17).

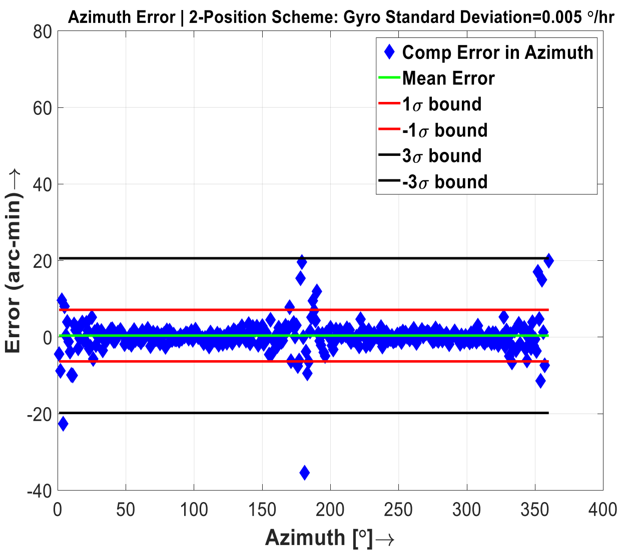

Fig. 13. Position azimuth computation error.

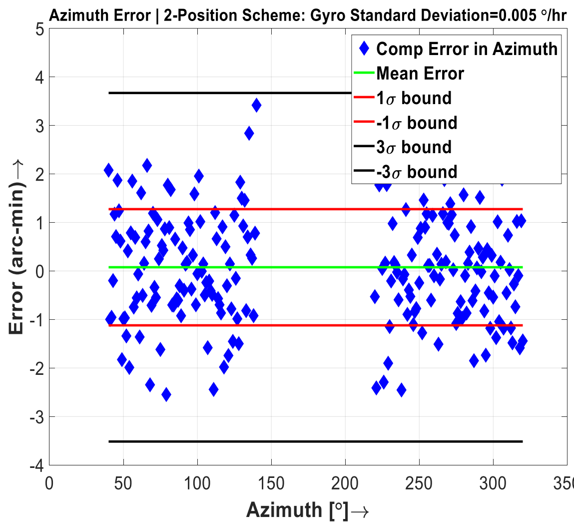

Fig. 14. Position (azimuths near north / south omitted).

Above mentioned graphical results are obtained by employing Eq. (13).

C. Simulation Results of 4-Position Scheme

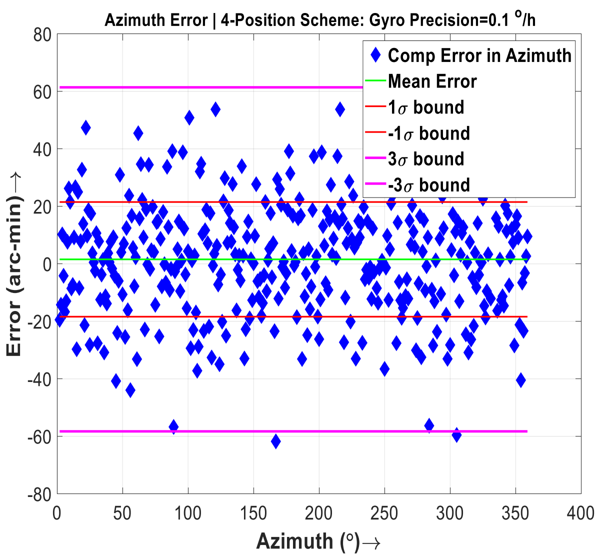

Fig. 15. Position azimuth computation error (A).

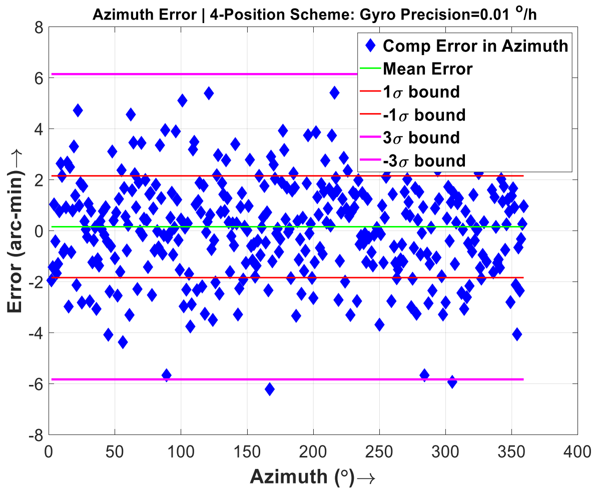

Fig. 16. Position azimuth computation error (B).

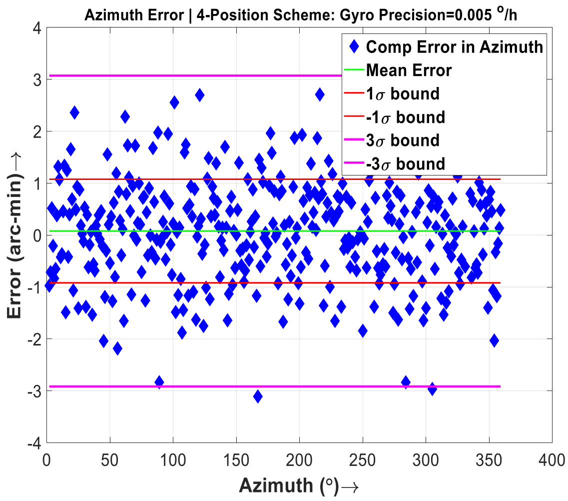

Fig. 17. Position azimuth computation error (C).

4-Position Scheme's graphical results are obtained by employing Eq. (20). 4-Position scheme is not affected by errors in fixed bias of gyros. Moreover, it is evident from the results that accuracy is not affected if gyro's initial orientation is near to the North or South. Accuracy is similar in all quadrants. Latitude of the test site is also not required in the computation of this scheme.

D. Effect of Residual Deterministic Errors

Effect of different types of errors on Azimuth computation is summarized in Table 1.

It is generally assumed that the deterministic errors of Gyros are eliminated through calibration. However, due to limitations and imperfections of calibration hardware, environmental factors and other unforeseen circumstances, some fixed errors are always there in the system.

Table 1. Effects of different errors on azimuth computation

Scheme / **Effect of Errors on Azimuth Computation**	**1-Position Scheme**	**2-Position Scheme**	**4-Position Scheme**	**Multi-Position Scheme**
Error in the Site Latitude	✓	✓	☒	☒
Fixed Bias Error	✓	☒	☒	☒
Random Bias	✓	✓	✓	☒
Scale Factor Error	✓	✓	☒	☒
Misalignment Errors	✓	✓	☒	☒

The fixed errors in Gyro include constant bias, scale factor error and misalignment error. Effects of misalignments and scale factor error elimination is guaranteed by 4-position and further multi-position schemes.

The results of simulation indicate that the residual deterministic errors in scale factor and misalignment in 1-position and 2-position scheme can seriously affect the performance of azimuth determination [16].

VII. Conclusion

It has been concluded from experimental and simulation analysis that 1-position and 2-Position Azimuth computations, although renders faster azimuth information results, has larger errors if azimuth angle is near to true north or south. Overall accuracy of 2-Position Scheme is 2 times better than that of 1-Position Scheme with gyroscope having same level of errors. This is because of elimination of bias errors in 2-Position Scheme. Furthermore, it has been concluded in the research work that improvement effect of bias cancellation in 2-Position Scheme is proportional to the slope/gradient of the growing bias. In the events of growing bias, bias-cancellation scheme renders highly improved results. However, if bias is not growing and have only random errors, the bias-cancellation scheme does not render any improvement.

It can be seen that standard deviation (1-Sigma values, excluding values near North and South orientations) are as follows:

Standard Deviation of 1-Position Scheme, using Gyroscope of 0.005 deg/h = 2 Arc Second

Standard Deviation of 2-Position Scheme, using Gyroscope of 0.005 deg/h = 1 Arc Second

Accuracy of 4-Position Azimuth Scheme remains same at any azimuth angle in all four quadrants. Moreover accuracy of 4-Position scheme is much better than those of both 1-Position and 2-Position vis-à-vis same scale of errors in gyroscopes.

It can bee seen from the Fig. 17 that standard deviation of the 4-Position scheme is 1 Arc-Minute in all quadrants including the orientations near true North and South.

Aditionally, the misalignment and scale factor error in 2-Position scheme can seriously deteriorate the performance of azimuth determination. Precision and accuracy can be enhanced by adopting 8-Position, 16-Position schemes ect, but at the cost of extra processing time and demanding requirement of rotary tables. For example 8-Position Scheme renders a bit better accuracy than that provided by 4-Position Scheme. But 8-Position Scheme required 3 times more processing time as compared to the 4-Position Scheme.

VIII. The Way Forward

Further enhanced accuray and precision in azimuth solution can be achieved by multi position techniques (8-Pos, 16-Pos, 32-Pos, 64-Pos and so on) [11]. Further research and analysis is required related with issues like total azimuth finding time, extra processing time due to additional positions, precision of encoder/motor and more demanding data acquisition systems etc. Moreover an optimization viz-a-viz data acquisition time at each position and precision required will also add values to the research. Continuous roation scheme, wherein output of gyro is taken continously, rather than at specified psotions, during the rotation of gyro, also has its own merits and demarits which needs to be investigated. Issues related with high rate of data acquistion, and fixed rotation rate requirements in Continuous Rotation Schemes are envisaged highly challenging.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Muhammad Ushaq developed the algorithms and methodology of this research; Merium Fazal Abbasi analyzed the data; M. Rasheeq Ullah Baig Mirza supervised the experiments; all three authors has approved the final draft of this paper.

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