Consideration of the Optimal Form of Rib by Observing the Stress and the Tooth Deflection of a Small and High-Strength Spiral Bevel Gear Fabricated Using Five-Axis Controlled Machining Center

Masahiro Saito*, Shinya Toyota, Toshiki Hirogaki, and Eiichi Aoyama

***Abstract*—A small and high-strength spiral bevel gear fabricated using a five-axis controlled machining center has been developed. In previous paper, we proposed a novel spiral bevel gear with ribs at the end of the tooth width to reinforce the tooth bending strength, which is machined with a ball-nose end-mill tool. However, it was shown that the stress could be concentrated around the ribs depending on the shape although the tooth root stress was reduced. Therefore, in this paper, we investigated the influence of the rib shape and size on the maximum stress at the tooth and rib root fillet and the tooth deflection with FEM analysis. The results revealed a novel guideline for designing the shape and size of rib at the end of the teeth width for the small and high-strength spiral bevel gear.**

***Index Terms*—Novel spiral bevel gear design, rib, FEM, five-axis controlled machining center**

I. Introduction

Many automobile companies speed up developing Electric Vehicles (EV) [1]. Since the government offices of many developed countries are encouraging the use of Electric Vehicles for realizing Zero Emissions Research and Initiatives. A spiral bevel gear is commonly applied as a final reduction gear in an automobile. It is applied even in Electric Vehicles [2], [3]. Therefore, the spiral bevel gear is needed reducing gear loss and downsizing without reducing torque capacity. The issue of a downsizing spiral bevel gear is insufficient strength. The spiral bevel gear loses its strength when the actual tooth contact moves toward the heel side during high-load operation and slips out of the heel side [4]. In addition, since the spiral bevel gear's sliding friction is small, the bending stress at the tooth root is more important in strength design than the surface pressure.

With reference to machining, many studies have focused on the machining of a spiral bevel gear with a five-axis controlled machining center or with multi-functional machine tools [5]-[9]. However, the shape of the work piece is the same as the shape machined by the uni-purpose machine. Furthermore, none of the studies have investigated the machining of a bevel gear with these multi-functional machine tools from a design perspective. Therefore, we propose a novel spiral bevel gear supported by a rib at the end of the teeth, fabricated using a five-axis machining center [10, 11]. We also discuss a design for estimating the tooth bending strength by FEM model based on the 1DCAE method [12]. However, we have not yet focused on both stress and tooth deflection at the same time. Therefore, in this report, on the premise that triangular ribs are added at the end of the tooth width, we discuss the relationship between the stress and tooth deflection generated around the tooth root and ribs by the rack-shaped FEM model based on the concept of 1DCAE.

II. Proposed a Small and High-Strength Spiral Bevel Gear

We propose a new concept pinion-gear, shown in Fig. 1. It has ribs on its tooth width end. The ribs support the tooth and strengthen the gear stronger. The gear is made of carburizing chromium molybdenum alloy steel. It has eight teeth, a module of 4 mm, a pressure angle of 19.5°, and a tooth width of 30 mm. Here, we consider a pinion gear module is less than 4mm. The gear is machined by a medium five-axis controlled machining center to generate teeth, 261 min are required per piece before the heat treatment. For machining, the R1.5 and R0.5 ball-end-mills were applied. After the heat treatment, the gear teeth are finished by the same machining center and R0.5 ball-end-mills.

III. The Concept of The Rib

A. The Types of the Tooth Damages in Spiral Bevel Gear

A large stress occurs when a force is applied to a cross point in a structure. Ribs are placed at the cross points to prevent the concentrated stress or to increase the rigidity of the entire structure. Regarding gears, when the gear is damaged by the bending load, the cracks occur from the tooth root fillet. Therefore, the purpose of the rib in the proposed of the tooth width far from the rib, the concentrated stress gear is that the concentrated stress at the tooth root fillet decrease. In general structures, it is common way to place ribs in the areas where stress is most concentrated. However, in the case of the proposed gear, Ribs are placed only at the end of the tooth width. Therefore, when the load is at center does not decrease at the tooth root fillet. The load on a gear is distributed in a

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certain amount of the tooth face. When we focus only on the stress at the tooth root fillet and place ribs to increase tooth rigidity at the end of the tooth width, the distributed load concentrates in the center of the tooth width, and furthermore, the stress at the tooth root fillet is increased. Therefore, in the gear supported by ribs, instead of simply avoiding the stress concentration at the tooth root filet, the load should be distributed evenly over the entire tooth width by bending the tooth. The rib shape can do this is the most suitable.

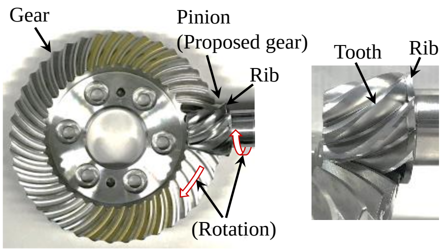


Fig. 1. Proposed spiral bevel gear.

B. Basic Guidelines for Studying Rib Shape

Many parameters are in the rib shape. However, few parameters can be adjusted when aiming for equal distribution of tooth root stress in consideration of restrictions such as the limitation of the range in which ribs can be placed. At the detailed design stage, it is necessary to consider the gear specifications, surrounding gear boxes, shaft rigidity, etc. However, in considering the effectiveness of ribs and their outline shape, there are not needed. We believe it is important to observe the rib shape, tooth root stress and tooth deflection with FEM analysis by simple model based on 1DCAE.

A lot of software of gear meshing analysis and design support are developed. They are based on the systematic study of tooth bending stress and bending deformation. In particular, when the tooth width is finite, the analysis considering the influence of the end of the tooth width has been generalized since it begun from the place where the flat plate cantilever beam was targeted [13]. However, their premise is different from the premise in the gear supported by ribs, since ribs placed to the end of a tooth width support the deformation of the teeth. In the previous research, we proceeded from the study applying flat plates [10]. In this report, we consider the rib effect by FEM analysis of the rack shape model.

In the previous report, we reported the effect of reducing the stress at a tooth root by a rib was not obtained when a load was at the center of the tooth width sufficiently far from the rib. On the other hand, when a load was at the end of the tooth width, the effect of reducing the stress at a tooth root by a rib was obtained just a small rib supporting only the small area around the tooth root. However, we focused only on the tooth root stress and we did not discuss the stress around the rib part caused by placing the rib. We will consider this in this report.

IV. FEM Analysis in the Proposed Gear

A. CAE Model

Fig. 2(a) shows a simple model for a FEM analysis, and furthermore, Fig. 2(b) shows detail definitions around ribs.

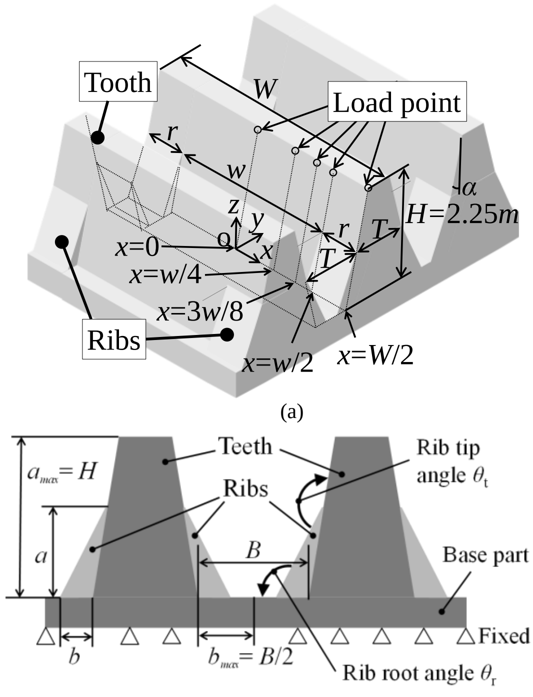


Fig. 2(a): A whole model; (b): Definition of dimensions around ribs. Proposed spiral bevel gear.

TABLE I: FEM Model Definitions

Designation	Unit	Symbol	Data type
Module	mm	m = 4	Fixed
Whole depth	mm	H = 2.25m	Fixed
Pressure angle	°	α = 14.5	Fixed
Tooth thickness	mm	T = πm/2	Fixed
Width of space	mm	T = πm/2	Fixed
Rib thickness	mm	r = πm/2	Fixed
Effective face width	mm	w = 7.5m	Fixed
Face width	mm	W = w + 2r	Fixed
Rib hight	mm	a	Variable
Rib width	mm	b	Variable
Load	N	F = 9.8	Fixed

Table I shows the model definition of dimensions and variables in detail for FEM model. We observed stress distributions by the models that had the various rib shapes. The model is a rack gear shape with a rim. The rim root is rigid. the roundness of a tooth root fillet and a rib fillet are 0.5mm. A load at the tooth top in the edge of the effective tooth width is the normal direction of a tooth face. Ribs are the end of the teeth width. The areas of upper ribs cannot be applied as a gear tooth face because ribs interfere a pair gear tooth if the pair gear tooth is in the areas of upper ribs. The mesh was automatically generated applying quadratic tetrahedral element with an element size of 0.5 mm basically. However, the element size at a fillet is smaller than 0.5mm. The element size is sufficiently small in terms of the FEM analysis accuracy by referring to the report [14], which is considering the FEM analysis accuracy with the bending stress in a gear. The material constants are uniform in the model, and the Young's modulus is 205.8GPa and the Poisson's ratio is 0.30, referring to the assumed material of the gear, CrMo steel.

We focus on the rib height (the ratio $a/a_{max.}=a/H$) and the rib width (the ratio $b/b_{max.}=b/H$) in Fig. 2(b). The rib height $a/a_{max.}$ is assigned to 3 different numerical values (25, 50,75%). The rib height $b/b_{max.}$ is assigned to 4 different numerical values (25, 50, 75, 100%). Thus, the 13 models (=3×4+1) include the no rib model applied FEM analysis. The effect of the roundness of a tooth root fillet and a rib fillet is not considered in this paper. The roundness is important factor in considering concentrated stress, however, we investigate the influence of the rib shape and size regarding to rigidity.

B. Analysis Point

Fig. 3 shows a stress distribution as an analysis result. We reached convergence in the FEA results by observing the stress and deflection. Stresses are concentrated at 3 points. First point is at tooth root. Stress is concentrated tooth root in a gear supported by ribs as same as general gear not supported by ribs. Second point is at rib root and third point at rib top. The 2 points are unique cases in the gear supported by ribs. Therefore, we observe the stress at the 3 points and consider about the relationship between a rib shape and the stress at the 3 points.

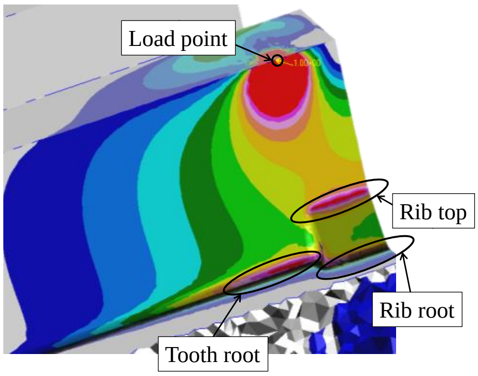


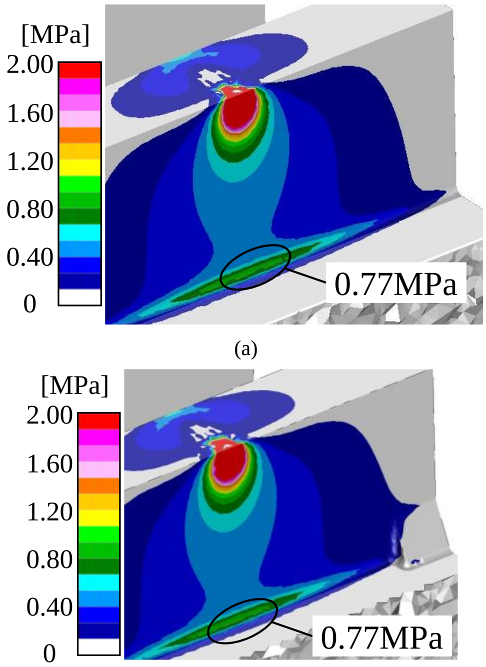
Fig. 3. This is an analysis result. There are three weak areas in bending stress distribution.

In the deflection, we focus on the tip of the tooth on neutral line on normal plane of the tooth the load point. In this report, it is the maximum deflection on the intersection of the yz plane and the neutral line of the tooth at the load point.

V. Analysis Results and Discussions

A. The Standard of a Relationship between Stress and Deflection

Fig. 4 shows the analysis results when a load point is at the tip of the tooth at the center of the tooth width (x=0). It is just examples of the analysis result of the gear with no rib and the gear supported by ribs, although a slight difference is in the stress distribution comparing them, the maximum tooth root stress values are not much different, which are about 0.77MPa. Fig. 6 shows the results of analysis summarized by maximum deflection and maximum stress under the same load conditions for each rib shape (13 models including no rib). It shows all the analysis results are about 0.77MPa and 0.0575µm. Thus, when the load point at the center of the tooth width (x=0) far from the rib, even if the rib shape changes, the tooth root stress and the tooth deformation are not influenced. Therefore, when the tooth width b/m (for example, b/m = 7.5 in this report) is wide enough, the tooth root stress under the load near the center of the tooth width is not influenced by the shape of the tooth width end whichever ribs are placed or not.



(b)

Fig. 4(a) A model with no rib; (b) A model with ribs.The load point is x=0.

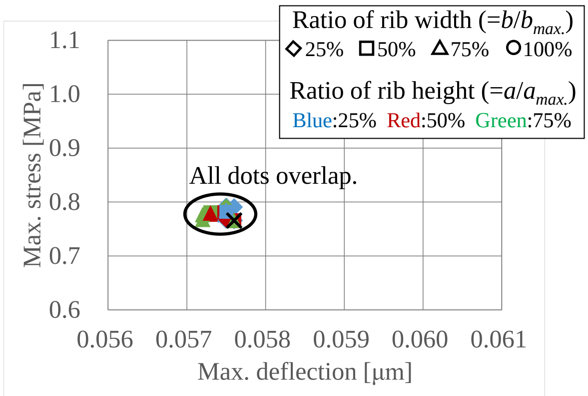


Fig. 5. This is the relationship between the max. deflection at a tooth tip and max. stress at a tooth root. The load point is x=0.

In this report, we observe the stress and the tooth deflection in ratio based on the stress of 0.77MPa and the deflection of 0.0575µm in each FEM analysis result. Furthermore, we consider the effects of changes in the rib shape observing stress and tooth deflection when the load point changes.

B. Effect of Change in Rib Shape on Tooth Root Stress and Tooth Deflection

Fig. 6 shows the maximum tooth root stress and tooth deflection when the load point is at the tooth tip of the effective tooth width end($x=w/2$) of the analysis models with each rib shape. Regarding the model with no rib, the stress is about 1.6 times, and the tooth deflection is about 1.35 times compared to the case of the load point at the tooth width center(x=0). It is the largest compared to other analysis results. Therefore, we found that any rib shape applied in this report has the effect of reducing tooth root stress and tooth deflection when a load point is at the tip of the effective tooth width end (x=w/2). In addition, a correlation is observed between the tooth root stress and the tooth deflection.

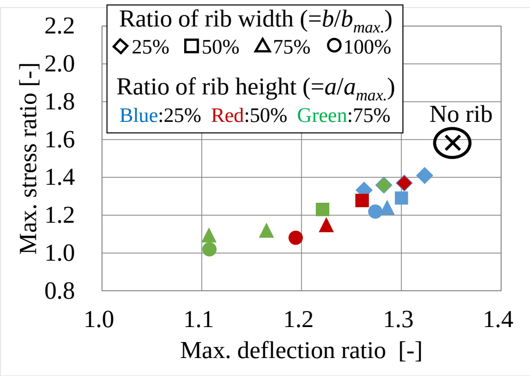


Fig. 6. This is the relationship between the max. deflection at a tooth tip and max. stress at a tooth root. The load point is x=w/2.

C. Effect of Change in Rib Shape on Rib Root Stress and Tooth Deflection

Fig. 7 shows the maximum stress at the rib root and the tooth deflection when a load point is at the tip of the effective tooth width end (x=w/2) of the analysis model with each rib shape. We found that a proportional relationship between the deflection and the maximum stress at the rib root with respect to the change is in the rib width $b/b_{max.}$, and the slope depends on the rib height $a/a_{max.}$. The model with no rib in Fig.7 shows one's the tooth root stress and the tooth deflection as same as it in Fig.6. The values are the tooth root stress ratio is about 1.6, and the tooth deflection ratio is about 1.35. Fig. 7 shows the maximum stress at the rib root on some rib shape models are larger than the tooth root stress on the model with no rib, although the tooth deflection is smaller than the model with no rib. Therefore, it should be noted that in a gear supported by ribs, the rigidity of the tooth increase, however the maximum stress in the entire tooth may increase depending on the rib shape.

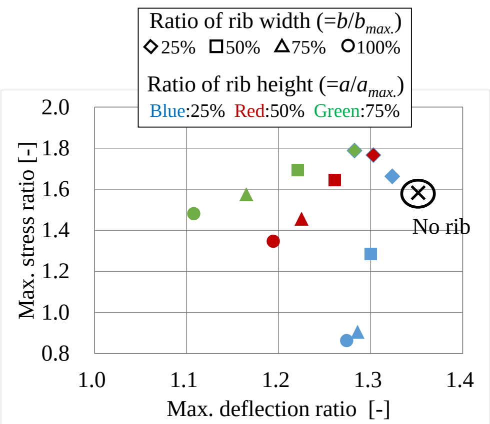


Fig. 7. This is the relationship between the max. deflection at a tooth tip and max. stress at a rib root. The load point is x=w/2.

D. Effect of Change in Rib Shape on Rib Top Stress and Tooth Deflection

Fig. 8 shows the maximum stress at the rib top and the tooth deflection when a load point is at the tip of the effective tooth width end (x=w/2) of the analysis model with each rib shape. It shows that the deflection and the stress are inversely proportional in the rib height a/a_{max}=25,50%. It is considered that the stress is concentrated on the rib top instead of increasing the rigidity of the tooth due to the increase in the rib width b/b_{max}. Regarding in the rib height a/a_{max}=75%, it is considered that since the load point and the rib are relatively close to each other, it is easy to decrease the tooth deflection, however the stress tends to concentrate on the rib top. Furthermore, it should be noted that in a gear supported by ribs, the rigidity of the tooth increase, however the maximum stress in the entire tooth may increase depending on the rib shape as same as the rib root.

E. Effect of Change in Rib Shape on Max. Stress in the Tooth and Tooth Deflection

Finally, Fig .9 shows the maximum stress in an entire tooth and the tooth deflection when a load point is at the tip of the effective tooth width end (x=w/2) of the analysis model with each rib shape. The maximum stress in an entire tooth is the maximum stress value of Fig. 6–9 shows, when a load point is at the tip of the effective tooth width end (x=w/2), the tooth deflection can be decreased with any rib shape, however the maximum stress generated in the entire tooth can be increased with some rib shapes. However, when the rib shape is a/a_{max}=25% and b/b_{max}=50%, the maximum stress when the load point is at the effective tooth width end (x=w/2) is 1.35 times as large as the maximum stress when the load point is at the effective tooth width center (x=0). Furthermore, the point occurred the maximum stress is at a rib tip. Regarding a model with no rib, the maximum stress when the load point is at the effective tooth width end (x=w/2) is 1.58 times as large as the maximum stress when the load point is at the effective tooth width center (x=0). Thus, we found that the rib can reduce the stress by about 14%. However, it cannot be reduced to 1.00 time or less, which is the maximum stress when a load point is at the effective tooth width center (x=0). It is considered that this is because the stress is concentrated on the rib root and the rib top even if the tooth root stress is reduced by placing the rib.

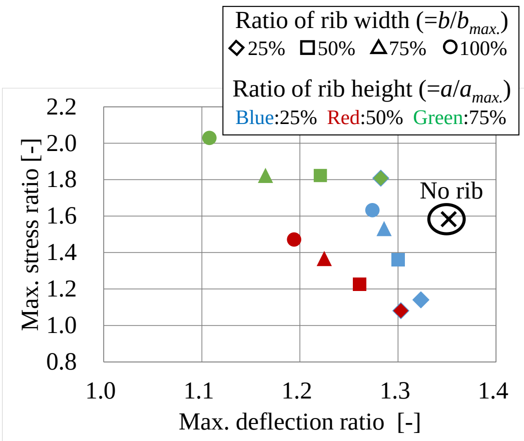


Fig. 8. This is the relationship between the max. deflection at a tooth tip and max. stress at a rib tip. The load point is x=w/2.

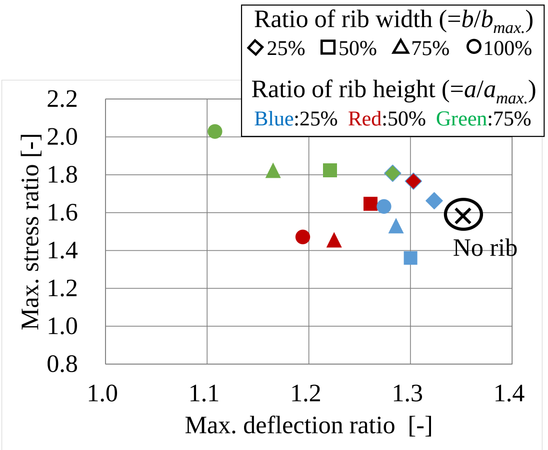


Fig. 9. This is the relationship between the max. deflection at a tooth tip and max. stress in a tooth. The load point is $x=w/2$.

In addition, where the stress is at around 1.8 times, some rib shapes differ only in tooth deflection. Thus, we found that the tooth deflection can be adjusted while maintaining the same amount of stress by adjusting the rib shape.

VI. FEM Analysis in the Proposed Gear

Therefore, we found that the gear supported by ribs can reduce the stress in a gear when a load is near the end of the tooth width by adjusting the rib shape, however the stress cannot be reduced to the same amount as the tooth root stress when a load point is near the center of the tooth width that cannot be supported by the rib.

In addition, we found that the tooth deflection can be adjusted with the same amount of stress by changing the rib shape, therefore it is possible to control the tooth deflection according to the product requirements. For example, it meets the needs of low noise in the automobile gear market.

For example, improving the actual gear contact ratio is effective in order to meet the needs under low noise [15, 16]. On the other hand, when the gear is driven in high load, the actual tooth contact move in the direction of the heel end of the tooth, and furthermore, it causes a decrease in the gear strength. In order to take care of this, the tooth surface modification to move the tooth contact to the inner end side under a light load is needed. However, the actual tooth contact ratio decreases, and furthermore, the noise increase when the gear is driven in the light load. In the gear supported by rib, the stress and the tooth deflection in a tooth is decreased when the load is at the end of the tooth width. Thus, since the tooth deflection is decreased, the actual tooth contact moving is decreased. In addition, since the stress is decreased, the actual tooth contact can be wider. Therefore, we consider that the gear supported by rib is a technology that can realize a high-strength bevel gear with low noise that is robust against load dependence.

VII. Conclusion

In this paper, we investigated an optimal rib by observing the stress and the tooth deflection regarding a proposed small and high-strength spiral bevel gear supported by ribs at the end of tooth width, fabricated using a five-axis controlled machining center. We applied FEM analysis to the simple gear-rack-shaped models placed ribs at the end of tooth width. The simple models ware considered the outline of the rib shape assuming the actual design referring to 1DCAE.

From the analysis result, we found that the larger stress can be generated around the rib fillet than at the tooth root depends on the rib shape. In this paper, one of rib shapes can reduce the max. stress in a tooth by about 14% compared to a gear with no rib when the load point is at the effective tooth width end. However, the point occurred the max. stress is at the rib tip. The maximum stress is 1.35 times as large as the maximum stress when a load point is at the effective tooth width center. Thus, the proposed gear must be cared the stress around the rib fillet not only the tooth root when we design the rib shape.

On the other hand, we found that the tooth deflection is decreased and can be adjusted with the same amount of stress by changing the rib shape. Therefore, we can control the tooth contact movement due to torque fluctuation by changing the shape of rib to some extent. Consequently, we obtained that the gear supported by rib is a technology that can realize a high-strength bevel gear with low noise that is robust against load dependence.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Masahiro Saito conducted the research, analyzed the data and wrote the paper; all authors had approved the final version.

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