Low-Level Modeling for Routing and Scheduling Trains through Busy Railway Stations with Expandable Coupling/Decoupling Mechanism

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***Abstract*—This paper studies train routing and scheduling problem for busy railway stations. The train routing problem is to assign each train to a route through the railway station and to a platform in the station. The train scheduling problem is to determine timing and ordering plans for all trains on the assigned train routes. Our objective is to allow trains to be routed in dense areas that are reaching saturation. Unlike traditional methods that allocate all resources to setup a route for a train until the route is freed, our work focuses on the use of resources as trains progress through the railway node. This technique allows a larger number of trains to be routed simultaneously in a railway node and thus reduces their current saturation. In this paper, we consider that trains can be coupled or decoupled and trains can pass through the railway station without stopping at any platform. To deal with this problem, this study proposes an abstract model and a mixed-integer linear programming formulation to solve it. The method is illustrated on a didactic example.**

***Index Terms*—Busy railway stations, mixed-integer linear programming, offline railway station management, train coupling, train decoupling, train platforming, train routing, train scheduling.**

I. Introduction

Nowadays, the railway network in Europe and most areas in the world have a great demand for transport. It is necessary to make the best use of railway resources while satisfying commercial objectives without conflicts between trains and resources. In order to fully explore the capacity of railway infrastructure, searching for optimal platform stops and passing through busy railway stations is important. In most researches, two main problems are investigated: train routing and train scheduling [1].

The train routing problem is to assign each train to a route through the railway station and to a platform in the station. The number of routings available to each train strongly affects the size of the problem and the time required to optimally solve it.

The train scheduling problem is to determine timing and ordering plans for all trains on the assigned train routes. The number of possible solutions can be very large depending on the network structure, the number and type of trains.

A train routing and scheduling problem in railway stations consists of assigning trains to platforms, so as to satisfy several constraints such as headway, dwell time and platform occupation. The schedule must satisfy some commercial objectives such as desired train arrival and departure times, platform stops, etc.

Some works dealing with train routing and scheduling problem focus mainly on low traffic densities within a reasonable computation time. In such case of simple railway structures with few lines, the problem is easy since there are few numbers of routes for each train. Reference [2] proposes a mixed-integer program to find train routing concerning with assigning trains and train times for rail links, stations stop..., so as to avoid train conflicts while minimizing costs and satisfying travel demands. The numerical example in this paper has 10 nodes, 28 links, 10 trains and requires less than one minute to be solved. The strategy of scheduling is to find the route of trains one at a time until all trains are routed and if necessary, the route of trains can be rescheduled until a feasible solution is found. References [3], [4] investigate computational complexity of the problem of routing trains through railway station. They consider the reservation of a complete route which guarantees that each train can travel without interruption along the reserved route. They also include shunting decisions, which are the move of a train to a depot track from a platform in the station (and inversely), and small deviations for preferred arrival time and departure time of trains. They prove that if each train has at most two routing possibilities, a solution can be computed in polynomial time.

The routing and scheduling problem becomes difficult in busy railway stations, having busy lines and several alternative platforms. Some research focus on complex railway stations. Reference [5] proposes a linear model. Heuristic methods are developed according to train planners' objectives. The algorithm schedules each train one by one. For each train, they check feasible platforms and for each of these platforms, they check if there are any conflicts with other trains that are already scheduled. If there are conflicts, the arrival time and departure time of train are changed to resolve conflicts. The experiment example has 12 main platforms (with 34 sub-platforms) and 491 trains with 900 arrivals and departures. The computation times can be from a few seconds to several hours depending on the heuristic method and the train planners' objectives. Reference [6] proposes a model dealing with the routing and scheduling problem for busy complex railway stations by applying a hybrid algorithm combining branch-and-bound and heuristic algorithms. In this model, they consider the reservation of a

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complete path and the deviation of departure time in a similar way to [3], [4]. The experiment example has 250 trains divided in sub-groups, the biggest group has about 60 trains. The computation time is a few minutes with 182 minutes deviation of departure times of 37 trains that contains 3 trains postponed by more than 10 minutes, 8 trains by more than 6 minutes and 29 trains by less than 5 minutes. Reference [7] improve the model of [3]. The problem is formulated as a weighted node packing model by making some assumptions about shunting decisions, preferences of trains for platforms and routes. Reference [7] also includes preprocessing and reduction techniques in the solution process. Reference [8] proposes a track-circuit based model dealing with perturbations. In this paper, all track-circuits belonging to a block must be reserved for trains. Reference [9] proposes a set packing model to deal with the problem of routing trains through railway junctions. The route locking and sectional release system is used in this model, a sequence of track sections must be reserved before the arrival of trains.

In view of the above, the reservation of a complete route is popularly used to solve the routing and scheduling problem in railway stations since it can guarantee that trains travel safely without interruptions. In this method, all sections in the route of trains are reserved until the trains release the complete route. One complete route can be reserved by only one train at a time. In principle, the reservation duration of each section of route can be calculated. It depends on the length and speed of train and the length of section. In this paper, we want to assess the interest and performance of a model considering the reservation of each section independently. This implies low-level modeling consideration with respect to the speed and length of train. A section can be reserved when a train arrives and it can be released after the train leaves it, so that the use of available resources can be more efficient. It allows the full exploitation of the capacity of railway infrastructures.

We proposed in [10] an abstract model and a mixed-integer linear programming formulation to solve it. We considered that every train consists of two circulations. One circulation goes from outside of railway station to a platform of railway station and the other circulation leaves the railway station. In this paper, we extend our early study by describing many types of trains. Thereafter a train can consists of a combination of one, two or three circulations. We consider that trains can be coupled or decoupled, which correspond to frequent railway operations. We consider also trains that pass through the railway station and do not stop at any platform. The paper is structured as follows. In the Section II, we propose the main concepts for describing the problem. In the Section III, we propose a mathematical model allowing a resolution by a mixed integer programming approach. Section IV is an application of the proposed model to a case study to illustrate the feasibility of our approach. In the Section V, we conclude with the lessons of this work and indicate its perspectives.

II. Description of the Problem

We propose to study a topology based on two types of generic components:" section" and" connector".

A section is a segment of railway infrastructure that can contain only one train at a time.

The set of sections in a railway infrastructure is denoted by $S = \{s_1, s_2, .., s_S\}$ where S is the cardinal number of S.

A connector is a point which connects several sections.

The set of connectors in a railway infrastructure is denoted by $C = \{c_1, c_2, .., c_C\}$ where C is the cardinal number of C.

Relations between sections and connectors: The topology we consider corresponds to a sequence of sections and connectors, see Fig. 1. Each section is bounded by only two connectors.

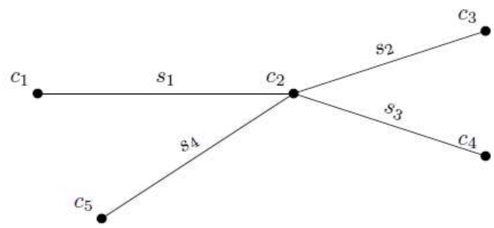

Fig. 1. Relations between sections and connectors.

For every $c \in C$ we denote the set of sections connected with connector c by S_c. In Fig. 1 , $S_{c_2} = \{s_1, s_2, s_3, s_4\}$.

Sections doublet. (s_1, s_2) is a doublet of connector c_1 when s_1, $s_2 \in S_{c_1}$, and trains can traverse from section s_1 to section s_2 by connector c_1. The set of doublets of a connector c is denoted by $K_c = \{(s_1, s'_1), (s_2, s'_2), .., (s_K, s'_K)\}$ where K is the cardinal number of K_c.

We must remark that a doublet of connectors represents only one travel direction. For example, a doublet (s_1, s_2) of connector c_1 represents the travel direction from section s_1 to section s_2 by connector c_1. The reverse exists only in case that we have another doublet (s_2, s_1) for connector c_1.

For example, in Fig. 1, if trains can traverse from left to right, then $K_{c_2} = \{(s_1, s_3), (s_1, s_2), (s_4, s_2), (s_4, s_3)\}$.

For every $s \in S$, we denote the set of reachable sections from section s by S_s. In Fig. 1, $S_{s_1} = \{s_2, s_3\}$.

For every $s \in S$, we denote the set of sections which have section s as a reachable section by $\hat{S}_s$. In Fig. 1, $\hat{S}_{s_2} = \{s_1, s_4\}$.

For every $s \in S$, for every $s' \in S_s$, it exists only one connector denoted as $c_{ss'}$ between these two reachable sections. In Fig. 1, $c_{s_1 s_3}$ is c_2.

A bordering connector is a connector surrounding the railway infrastructure where trains can enter or leave railway infrastructure.

The set of bordering connectors in a railway infrastructure is denoted by $B = \{b_1, b_2, .., b_B\}$ where B is the cardinal number of B. Obviously, $B \subset C$.

An external section is a section surrounding the railway infrastructure, represented by a line which connects from a bordering connector to the outside of the infrastructure where trains can enter or leave railway infrastructure. The set of external sections in a railway infrastructure is denoted by $E = \{e_1, e_2, .., e_E\}$ where E is the cardinal number of E. Obviously, $E \subset S$.

A platform is a section which is used for passengers that can await, board or unboard from trains. Train can usually stop long-time in platforms.

The set of platforms in a railway infrastructure is denoted by $P = \{p_1, p_2, .., p_P\}$ where P is the number of platforms.

Thus, $P \subset S$ and $P \cap E = \varnothing$.

An internal section is a section inside railway infrastructure where trains can pass through. The internal sections are not platforms. The set of internal sections in a railway infrastructure is denoted by $I = \{i_1, i_2, .., i_I\}$ where I is the cardinal number of I. Thus, $I \subset S$, $I \cap E = \varnothing$, $I \cap P = \varnothing$ and $S = I \cup E \cup P$.

An example of a railway infrastructure is represented in Fig. 2 and the correspondences between sections of this figure are listed in Table I.

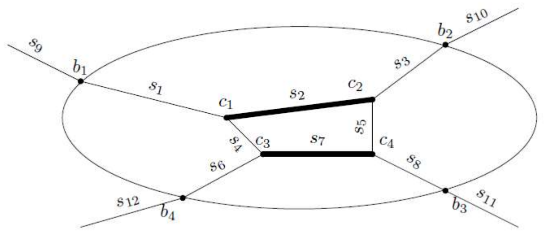

Fig. 2 An example of railway infrastructure.

TABLE I: CORRESPONDENCES BETWEEN SECTIONS OF FIG. 2

Section	External section	Internal section	Platform
s_1		i_1	
s_2			p_1
s_3		i_2	
s_4		i_3	
s_5		i_4	
s_6		i_5	
s_7			p_2
s_8		i_6	
s_9	e_1		
s_{10}	e_2		
s_{11}	e_3		
s_{12}	e_4		

Trains' Activities

Train: The traffic in the railway infrastructure is defined by a set of trains $T = \{t_1, t_2, .., t_T\}$ where T is the number of trains.

A circulation is an operation of a train which travel from one section to another.

Every train $t \in T$ consists of a set of ordered circulations $L^t = \{l_1^t, l_2^t, \dots, l_{L^t}^t\}$ where L^t is the cardinal number of L^t.

Train platform. If a train must stop at a platform, we must allocate one and only one platform to train t, denoted as $p_t \in P$. A route for the train passing through the railway station must be determined with the condition that the train arrives at and departs from the same platform p_t.

Routing of trains. A train passing through the railway station has circulations which are given external sections and need to be assigned to a route. The external sections of circulations of the train t are denoted by $e_{in}^{l_1}, e_{in}^{l_2}..$, $e_{out}^{l_3}, e_{out}^{l_4}.. \in E$ (train using coupling or decoupling mechanism must have three external sections). The circulation l of train enters the railway station from the external section e_{in}^{l}, arrives at a platform, after that another circulation l' departs from the same platform and leaves the railway station by the external section $e_{out}^{l'}$.

Three types of circulation are defined:

- **An entering circulation** is a circulation of a train which travels from an external section to a platform, see Fig. 3. The set of entering circulations is denoted by L_{ent}.

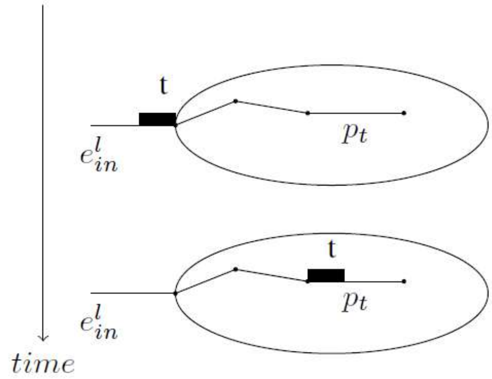

Fig. 3. Entering circulation.

- **A leaving circulation** is a circulation of a train which travels from a **platform** to an external section, see Fig. 4. The set of leaving circulations is denoted by L_{leav}.

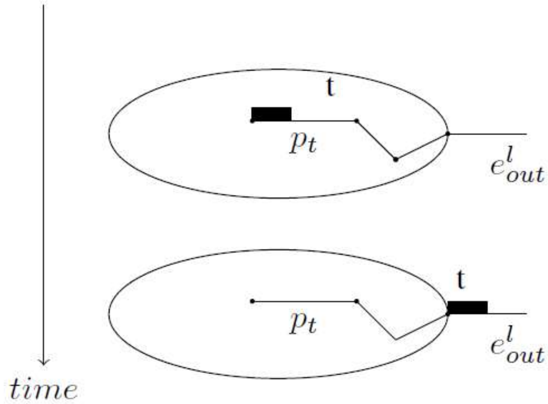

Fig. 4. Leaving circulation.

- **A crossing circulation** is a circulation of a train which passes through the railway station from an external section to another external section and does not stop at any platform, see Fig. 5. The set of crossing circulations is denoted by L_{cross}.

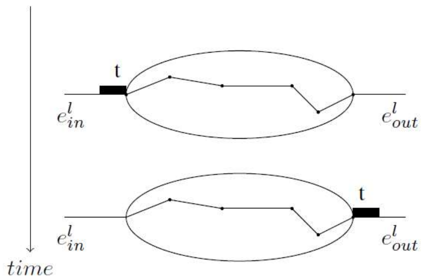

Fig. 5. Crossing circulation of a passing train.

Note: Trains can stop at only one platform but they are allowed to traverse other platforms. Crossing circulations do not stop at any platform but they can traverse platforms to go through the railway infrastructure.

Reference time. An entering circulation $l \in L_{ent}$ is associated to a reference time A^l. This reference time A^l is the preferred arrival time to the platform by the circulation l.

Circulation can arrive late to platform within a permissible deviation time. The maximum permissible deviation is denoted by L. It means that the latest arrival time of circulation at its platform is $A^l + L$.

Stopping time. The time taken for circulations remaining stopped at a platform to take passengers onboard is denoted by D^l.

Route. The route of a circulation is a sequence of reachable sections from one to another that the train uses for this circulation. One circulation can have many routes and we have to determine which one is the most appropriate.

A route of a circulation l^t of train t denoted by r consists of a set of ordered reachable sections $S^r = \{s_1^r, s_2^r, .., s_{S^r}^r\}$ where S^r is the cardinal number of S^r.

In France, nowadays the TGV (Train Ã Grande Vitesse, "high-speed train") is France's intercity high-speed rail service, operated by SNCF, French National Railway Company. TGVs have semi-permanently coupled articulated un-powered coaches (chair cars) with bogies between the coaches. At each end of the trains, Power cars, lead vehicles with machinery for supplying heat or electrical power to other parts of trains, have their own bogies. Trains can be lengthened by coupling two TGVs, using couplers hidden in the noses of the power cars.

In this study, we consider that every train t consists of a maximum of two entering circulations and one leaving circulation (or one entering circulation and two leaving circulations). Trains can stop at only one platform and they are allowed to traverse other platforms (they do not stop at these platforms). The entering circulations of a train must stop at the platform selected for the train and the leaving circulations of the train must leave the same platform. The assumption used in this model is that for all entering circulations arriving at a platform, the reference arrival time of the platform and the stopping time at platform are known.

III. Mixed-integer Linear Programming Model

In this section, we propose a mathematical model as a mixed-integer linear program with the parameters and hypotheses we presented in above.

A. Parameters

Every train $t \in T$ has some parameters corresponding, see Table II:

TABLE II: Parameters of Trains' Activities

Type	Circulation	External section	Reference time
Stopping train	$l_1, l_2 \in \mathbb{L}_{ent}$	e_{in1}^l, e_{in2}^l	A^{l_1}, D^{l_1} A^{l_2}, D^{l_2} , L
	$l_3, l_4 \in \mathbb{L}_{leav}$	e_{out1}^l, e_{out2}^l	
Crossing train	$l \in \mathbb{L}_{cross}$	e_{in}^l, e_{out}^l	

Note: Three types of stopping train in this study:

- Stopping trains that have only one entering circulation and one leaving circulation. The set of this type of train is denoted by T_{11}.
- Stopping trains that have two entering circulations and one leaving circulation (train coupling). The set of this type of train is denoted by T_{21}. We consider $A^{l_1} < A^{l_2}$, it means that circulation l_1 enters platform before circulation l_2.
- Stopping trains that have one entering circulation and two leaving circulation**s** (train decoupling). The set of this type of train is denoted by T_{12}.

There are no train having two entering circulations and two leaving circulations because we can consider two trains having one entering circulation and one leaving circulation in this case.

The time taken to traverse section s by circulation l is denoted by Δ_s^l. It depends on the length of sections and the speed of trains, can be given as:

$$\Delta_s^l = \frac{\text{length of sections}}{\text{speed of circulationl}}$$

The time taken for a circulation l going through a connector is denoted by Θ^l. It depends on the length and the speed of trains, can be given as:

$$\Theta^l = \frac{\text{length of train}}{\text{speed of circulation l}}$$

The time taken for a coupling system or decoupling system at platform is denoted by Γ^t.

Note: We assume that the speed of train does not change during a circulation.

H is a sufficiently large constant.

B. Decision Variables

The function $\delta(Q)$ is an indicator such that $\delta(Q) = 1$ if the condition Q is valid, otherwise 0.

- $\boldsymbol{S_s^l}$: **boolean** variable, represents the passage of circulation l going through section s. $S_s^l = \delta$ (circulation l passes through section s).
- $\boldsymbol{C_c^l}$: boolean variable, represents the passage of circulation l going through connector c. $C_c^l = \delta$(circulation l **passes** through connector c).
- $\boldsymbol{Y_s^{ll'}}$ **:** boolean variable, represents the chronological order of two circulations l, l' using routes containing a common section s. $Y_s^{ll'} = \delta$(circulation l passes through section s before circulation l').
- $\boldsymbol{X_c^{ll'}}$ **:** boolean variable, represents the chronological order of two **circulations** l, l' using routes containing a common connector c. $X_c^{ll'} = \delta$ (circulation l passes through connector c before circulation l').
- $\boldsymbol{Z_{ss'}^l}$: boolean variable, represents the passage from section s to section s' in the route of circulation l. $Z_{ss}^l = \delta$ (circulation l travels from section s to section s').

The time interval of occupation of sections and connectors are represented in Fig. 6:

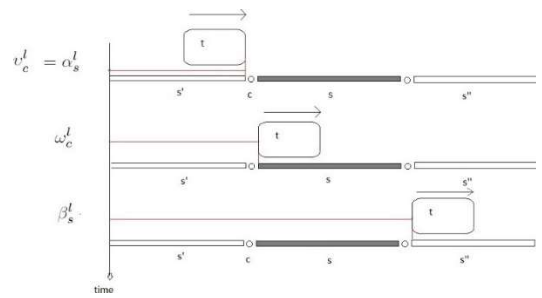

Fig. 6. Occupation time variables.

Note: A section is reserved when a train arrives at the connector connected with this section and the section is released when train leaves the other connector connected with this section.

- $[\boldsymbol{\alpha_s^l}, \boldsymbol{\beta_s^l}]$: integer **variables**, the actual time interval of occupation of section s by circulation l.
- $[\boldsymbol{v_c^l}, \boldsymbol{\omega_c^l}]$: integer variables, the actual time interval of occupation of connector c by circulation l.

- $\boldsymbol{W_s^l}$: integer **variable**, the time taken for circulation l remaining stopped at section s.
- $\boldsymbol{P_p^l}$: boolean **variables**, represents the stopping platform of circulation l. $P_p^l = \delta$ (platform p is allocated to circulation l as a stopping platform).

C. Constraints

Routing constraints. This section presents constraints which ensure that circulations can travel from their origin to their destination.

- If the **doublet** (s, s') does not exist, it means that section s is not reachable from s'. Thus, $Z_{ss'}^l$ is equal to 0:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \notin S_s \qquad Z_{ss'}^l = 0 \quad (1)$$

- If a **circulation** passes from section s to s', it cannot pass from section s' to s:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \in S_s \qquad Z_{ss}^l + Z_{s's}^l \leq 1 \; (2)$$

Route of circulation:

- If a **circulation** enters a section, this circulation must pass through this section:

$$\forall t \in T, \forall l \in L^t, \forall s \in S \qquad \sum_{s' \in \hat{S}_s} Z_{s's}^l = 1 \Rightarrow S_s^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L^t, \forall s \in S \qquad S_s^l \geq \sum_{s' \in \hat{S}_s} Z_{s's}^l \qquad (3)$$

- If a circulation leaves a section, this circulation must pass through this **section**:

$$\forall t \in T, \forall l \in L^t, \forall s \in S \qquad \sum_{s' \in S_s} Z_{ss'}^l = 1 \Rightarrow S_s^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L^t, \forall s \in S \qquad S_s^l \geq \sum_{s' \in S_s} Z_{ss'}^l \qquad (4)$$

Note: The inequality in constraints (3) and (4) represents the case of external sections and platforms. For example, a circulation can pass through an external section but it cannot enter this external section in case that this external section is the first section in the route of this circulation.

- If a circulation travels from section s to section s', it must use the **connector** $c_{ss'}$ between these two sections:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \in S_s \qquad Z_{ss'}^l = 1 \Rightarrow C_{c_{ss}}^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \in S_s \qquad Z_{ss'}^l \leq C_{c_{ss'}}^l \qquad (5)$$

Constraints of external sections:

- **Entering** circulation l must pass through and leave the external section given e_{in}^l:

$$\forall t \in T, \forall l \in L_{ent}^t \qquad S_{e_{in}^l}^l = 1 \qquad (6)$$

$$\forall t \in T, \forall l \in L_{ent}^t \qquad \sum_{s' \in S_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \qquad (7)$$

- This entering circulation l must not pass through others external sections:
-

$$\forall t \in T, \forall l \in L_{ent}^t, \forall s \in E \backslash \{e_{in}^l\} \qquad S_s^l = 0 \qquad (8)$$

- **Leaving** circulation l must enter and pass through the external section given e_{out}^l:

$$\forall t \in T, \forall l \in L_{leav}^t \qquad S_{e_{out}^l}^l = 1 \qquad (9)$$

$$\forall t \in T, \forall l \in L_{leav}^t \qquad \sum_{s' \in \hat{S}_{e_{out}^l}} Z_{s' e_{out}^l}^l = 1 \qquad (10)$$

- This leaving circulation l must not pass through others external sections:

$$\forall t \in T, \forall l \in L_{leav}^t, \forall s \in E \backslash \{e_{out}^l\} \qquad S_s^l = 0 \qquad (11)$$

- Crossing circulation l must pass through and leave the external **section** given e_{in}^l and it must enter and pass through the external section given e_{out}^l:

$$\forall t \in T, \forall l \in L_{cross}^t \qquad S_{e_{in}^l}^l = 1 \qquad (12)$$

$$\forall t \in T, \forall l \in L_{cross}^t \qquad \sum_{s' \in S_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \qquad (13)$$

$$\forall t \in T, \forall l \in L_{cross}^t \qquad S_{e_{out}^l}^l = 1 \qquad (14)$$

$$\forall t \in T, \forall l \in L_{cross}^t \qquad \sum_{s' \in \hat{S}_{e_{out}^l}} Z_{s' e_{out}^l}^l = 1 \qquad (15)$$

- This crossing circulation l must not pass through others external **sections**:

$$\forall t \in T, \forall l \in L_{cross}^t, \forall s \in E \backslash \{e_{in}^l, e_{out}^l\} \qquad S_s^l = 0 \qquad (16)$$

Constraints of internal sections: If a circulation enters an internal section, it must leave this internal section. Conversely, if this circulation leaves this internal section, it must enter this internal section.

$$\forall t \in T, \forall l \in L^t, \forall s \in I \qquad \sum_{s' \in \hat{S}_s} Z_{s's}^l = \sum_{s'' \in S_s} Z_{ss''}^l \quad (17)$$

Constraints of non-stopping platforms: We consider that trains can pass through some platforms but might not stop at these platforms. If a circulation enters an non-stopping platform, it must leave this platform. Conversely, if this circulation leaves this platform, it must enter this platform:

$$\forall t \in T, \forall l \in L^t, \forall p \in P \quad P_p^l = 0 \Rightarrow \sum_{s' \in S_p} Z_{ps'}^l = \sum_{s'' \in \hat{S}_p} Z_{s''p}^l$$

These constraints are expressed using the linear constraints below:

$$\forall t \in T, \forall l \in L^t, \forall p \in P$$
$$\begin{cases} \sum_{s' \in S_p} Z_{ps'}^l - \sum_{s'' \in \hat{S}_p} Z_{s''p}^l \leq H \cdot P_p^l \\ \sum_{s'' \in \hat{S}_p} Z_{s''p}^l - \sum_{s' \in S_p} Z_{ps'}^l \leq H \cdot P_p^l \end{cases} \qquad (18)$$

Note: If $P_p^l = 0$, the inequation (20) implies that $\sum_{s' \in S_p} Z_{ps'}^l - \sum_{s' \in \hat{S}_p} Z_{s''p}^l \leq 0$ and $\sum_{s'' \in \hat{S}_p} Z_{s''p}^l - \sum_{s' \in S_p} Z_{ps}^l \leq 0$, it means that $\sum_{s' \in S_p} Z_{ps}^l = \sum_{s'' \in \hat{S}_p} Z_{s''p}^l$. If $P_p^l = 1$, the inequation is always true because H is a sufficiently big constant.

Constraints of stopping platforms:

- There is only one stopping platform for entering circulation and leaving circulation:

$$\forall t \in T, \forall l \in L_{ent}^t \cup L_{leav}^t \qquad \sum_{p \in P} P_p^l = 1 \tag{19}$$

- There is no stopping **platform** for crossing circulation:

$$\forall t \in T, \forall l \in L_{cross}^t \qquad \sum_{p \in P} P_p^l = 0 \tag{20}$$

- • Entering circulations and leaving circulations of the same train must have the same platform:

$$\forall t \in T, \forall l, l' \in L_{ent}^t \cup L_{leav}^t, \forall p \in P \qquad P_p^l = P_p^{l'} \tag{21}$$

- An entering **circulation** must enter the stopping platform:
-

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P \qquad P_p^l = 1 \Rightarrow \sum_{s \in \hat{S}_p} Z_{sp}^l = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P \qquad \sum_{s \in \hat{S}_p} Z_{sp}^l \geq P_p^l \tag{22}$$

- An entering **circulation** must not leave the stopping platform:

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P \qquad P_p^l = 1 \Rightarrow \sum_{s \in S_p} Z_{ps}^l = 0$$

The constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P \qquad \sum_{s \in S_p} Z_{ps}^l \leq (1 - P_p^l) \tag{23}$$

- A leaving circulation must leave a stopping platform:

$$\forall t \in T, \forall l \in L_{leav}^t, \forall p \in P \qquad \sum_{s \in S_p} Z_{ps}^l \geq P_p^l \tag{24}$$

- A **leaving** circulation must not enter the stopping platform:

$$\forall t \in T, \forall l \in L_{leav}^t, \forall p \in P \qquad \sum_{s \in \hat{S}_p} Z_{sp}^l \leq (1 - P_p^l) \tag{25}$$

Constraints of relations between sections and connectors: We consider that circulations are not allowed to pass through a connector many time in this model. Circulations can pass a connector only one time. If a circulation l passes through a connector c, there must be two sections, connected to this connector, which are in the route of circulation l:

$$\forall t \in T, \forall l \in L^t, \forall c \in C \qquad C_c^l = 1 \Rightarrow \sum_{s \in S_c} S_s^l = 2$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L^t, \forall c \in C \qquad \begin{cases} \sum_{s \in S_c} S_s^l - 2 \leq H \cdot (1 - C_c^l) \\ 2 - \sum_{s \in S_c} S_s^l \leq H \cdot (1 - C_c^l) \end{cases} \tag{26}$$

Note: We remind that S_c is a set of sections connected with connector c.

Actual time interval of occupation of sections and connectors.

The actual time interval of occupation of a section $s \in S$ by a circulation l is defined by $[\alpha_s^l, \beta_s^l]$ and the time taken for circulation l remaining stopped at section s is defined by variables W_s^l.

The actual time interval of occupation of a connector $c \in C$ by a circulation l is defined by $[\upsilon_c^l, \omega_c^l]$.

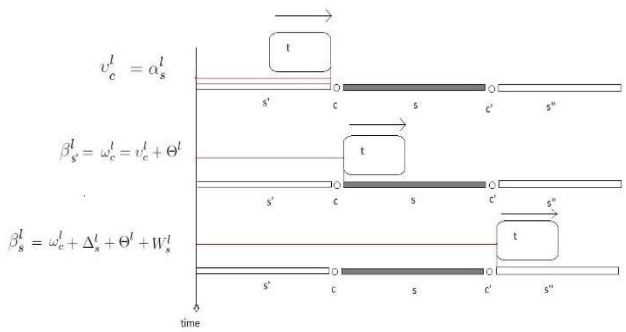

Fig. 7. Actual time intervals of occupation of sections and connectors.

- The actual time intervals of occupations of sections and connectors are represented in Fig. 7. The constraints of all connectors are expressed as follows:

$$\forall t \in T, \forall l \in L^t, \forall c \in C \qquad \omega_c^l = \upsilon_c^l + \Theta^l \tag{27}$$

- The constraints of all sections which are not the stopping platform are expressed as follows:
-

$$\forall t \in T, \forall l \in L^t, \forall s \in I \quad \beta_s^l = \alpha_s^l + \Delta_s^l + 2\Theta^l + W_s^l \tag{28}$$

$$\forall t \in T, \forall l \in L^t, \forall s \in E \quad \beta_s^l = \alpha_s^l + \Delta_s^l + \Theta^l + W_s^l \tag{29}$$

$$\forall t \in T, \forall l \in L^t, \forall p \in P$$
$$P_p^l = 0 \Rightarrow \beta_p^l = \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l$$

This constraint is expressed using the linear constraints below:

$$\forall t \in T, \forall l \in L^t, \forall p \in P$$
$$\begin{cases} \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l - \beta_p^l \leq H \cdot P_p^l \\ \beta_p^l - \alpha_p^l - \Delta_p^l - 2\Theta^l - W_p^l \leq H \cdot P_p^l \end{cases} \tag{30}$$

To pass through an internal section or a platform, trains pass through two connectors (constraints (28) and (30)). To pass through an external section, trains pass through only one connector (constraint (29)).

Succession of sections:

The actual time intervals of occupations of two consecutive sections are represented in Fig. 8.

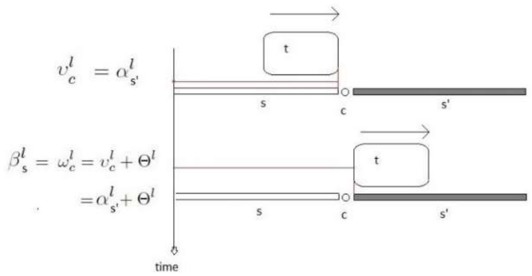

Fig. 8. Actual time intervals of occupation of two consecutive sections.

- If circulation l travels from section s to section s' by connector c, we consider that the section s' is reserved when connector c is occupied by circulation l. Thus, we have the constraint below:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \in S_s \qquad Z_{ss'}^l = 1 \Rightarrow v_{c_{ss'}}^l = \alpha_{s'}^l$$

This constraint is expressed using the linear constraints below:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \in S_s$$
$$\begin{cases} v_{c_{ss'}}^l - \alpha_{s'}^l \le H \cdot (1 - Z_{ss}^l) \\ \alpha_{s'}^l - v_{c_{ss'}}^l \le H \cdot (1 - Z_{ss'}^l) \end{cases} \tag{31}$$

- According to Fig. 8, if a circulation l travels from section s to section s', their corresponding occupation times must respect the constraint below:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \in S_s \quad Z_{ss'}^l = 1 \Rightarrow \beta_s^l = \alpha_{s'}^l + \Theta^l$$

This constraint is expressed using the linear constraints below:

$$\forall t \in T, \forall l \in L^t, \forall s \in S, \forall s' \in S_s$$
$$\begin{cases} \alpha_{s'}^l + \Theta^l - \beta_s^l \le H \cdot (1 - Z_{ss'}^l) \\ \beta_s^l - \alpha_{s'}^l - \Theta^l \le H \cdot (1 - Z_{ss}^l) \end{cases} \tag{32}$$

Actual time interval of occupation of stopping platform: The time interval of occupation of stopping platform of an entering circulation must respect the preferred arrival time A^l which can be adjusted within a time interval L (Fig. 9).

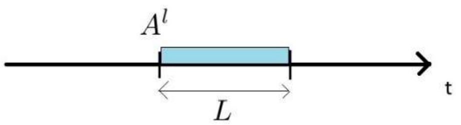

Fig. 9. Deviation L.

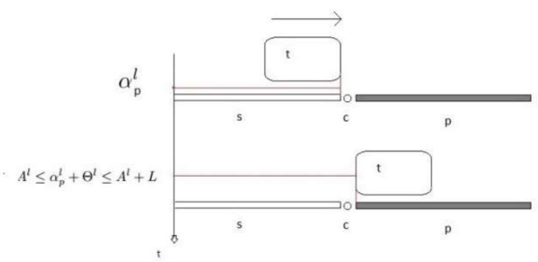

Fig. 10. Actual time interval of occupation of a platform.

The actual time interval of occupation of a platform is represented in Fig. 10, we assume that the entering circulation of a train arrives at a stopping platform when the train leaves the connector connected with the platform. Hence, the entering circulation allows passengers to board or unboard the train. After that, the leaving circulation of this train will pass through and leaves the platform.

- The time interval of occupation of a stopping platform of an entering circulation must respect the preferred arrival time of platform A^l, see Fig. 9:

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P$$
$$P_p^l = 1 \Rightarrow A^l \le \alpha_p^l + \Theta^l \le A^l + L$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P$$
$$\begin{cases} A^l - \alpha_p^l - \Theta^l \le H \cdot (1 - P_p^l) \\ \alpha_p^l + \Theta^l - A^l \le L + H \cdot (1 - P_p^l) \end{cases} \tag{33}$$

- The time interval of occupation of stopping platform of an entering circulation must respect the stopping time at platform D^l:

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P \qquad P_p^l = 1 \Rightarrow \beta_p^l = \alpha_p^l + \Theta^l + D^l$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L_{ent}^t, \forall p \in P$$
$$\begin{cases} \beta_p^l - \alpha_p^l - \Theta^l - D^l \le H \cdot (1 - P_p^l) \\ \alpha_p^l + \Theta^l + D^l - \beta_p^l \le H \cdot (1 - P_p^l) \end{cases} \tag{34}$$

Note: If trains have two entering circulations, each entering circulation has its own arrival time A^l and stopping time D^l at platform.

- If trains have one entering circulation and one leaving circulation (train type denoted by T_{11}), their corresponding occupation times must respect the constraint below:

$$\forall t \in T_{11}, \forall l_1 \in L_{ent}^t, \forall l_2 \in L_{leav}^t, \forall p \in P \;\; P_p^{l_1} = 1 \Rightarrow \alpha_p^{l_2} = \beta_p^{l_1}$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T, l_1 \in L_{ent}^t, l_2 \in L_{leav}^t, \forall p \in P$$
$$\begin{cases} \beta_p^{l_1} - \alpha_p^{l_2} \le H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_2} - \beta_p^{l_1} \le H \cdot (1 - P_p^{l_1}) \end{cases} \tag{35}$$

- If trains have two entering circulations l_1, l_2 with $A^{l_1} < A^{l_2}$ and one leaving circulation l_3 (train type denoted by T_{21}), see Fig. 11, the time interval of occupation of stopping platform of leaving circulation l_3 must depend on the time interval of occupation of a stopping platform of the second entering circulation l_2.

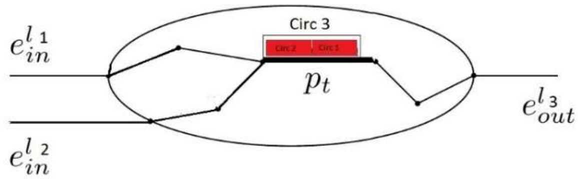

Fig. 11. Coupling system of trains.

The entering circulation l_1 must enter a platform before l_2 ($A^{l_1} < A^{l_2}$). The constraint of the time interval of occupation of a stopping platform of entering circulations and leaving circulation is expressed below:

$$\forall t \in T_{11}, \forall l_1 \in L^t_{ent}, \forall l_2 \in L^t_{leav}, \forall p \in P \;\; P^{l_1}_p = 1 \Rightarrow \alpha^{l_2}_p = \beta^{l_1}_p$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T_{21}, l_1, l_2 \in L^t_{ent}, l_3 \in L^t_{leav}, \forall p \in P$$
$$\begin{cases} \beta^{l_2}_p - \alpha^{l_3}_p \leq H \cdot (1 - P^{l_1}_p) \\ \alpha^{l_3}_p - \beta^{l_2}_p \leq H \cdot (1 - P^{l_1}_p) \end{cases} \quad (36)$$

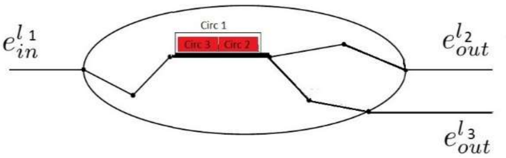

Fig. 12. Decoupling system of trains.

- If trains have one entering circulation l_1 and two leaving circulations l_2, l_3 (train type denoted by T_{12}), see Fig. 12, the time interval of occupation of a stopping platform of the first leaving circulation l_2 (circulation l_2 must leave the platform before l_3) must depend on the time interval of occupation of a stopping platform of the entering circulation l_1. This constraint is expressed below:

$$\forall t \in T_{12}, \forall l_1 \in L^t_{ent}, \forall l_2, l_3 \in L^t_{leav}, \forall p \in P$$
$$P^{l_1}_p = 1 \Rightarrow \alpha^{l_2}_p = \beta^{l_1}_p$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T_{12}, l_1 \in L^t_{ent}, l_2, l_3 \in L^t_{leav}, \forall p \in P$$
$$\begin{cases} \beta^{l_1}_p - \alpha^{l_2}_p \leq H \cdot (1 - P^{l_1}_p) \\ \alpha^{l_2}_p - \beta^{l_1}_p \leq H \cdot (1 - P^{l_1}_p) \end{cases} \quad (37)$$

The second leaving circulation can begin to occupy the platform only after the first leaving circulation leaves the stopping platform. The constraint of the time interval of occupation of a stopping platform of two leaving circulations l_2, l_3 is expressed below:

$$\forall t \in T_{12}, \forall l_1 \in L^t_{ent}, \forall l_2, l_3 \in L^t_{leav}, \forall p \in P$$
$$P^{l_1}_p = 1 \Rightarrow \alpha^{l_3}_p = \beta^{l_2}_p$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T_{12}, l_1 \in L^t_{ent}, l_2, l_3 \in L^t_{leav}, \forall p \in P$$
$$\begin{cases} \beta^{l_2}_p - \alpha^{l_3}_p \leq H \cdot (1 - P^{l_1}_p) \\ \alpha^{l_3}_p - \beta^{l_2}_p \leq H \cdot (1 - P^{l_1}_p) \end{cases} \quad (38)$$

- The constraint of the time interval of occupation of stopping platform of a leaving circulation is expressed below:

$$\forall t \in T, \forall l \in L^t_{leav}, \forall p \in P$$
$$P^l_p = 1 \Rightarrow \beta^l_p = \alpha^l_p + \Delta^l_p + \Theta^l + \Gamma^t + W^l_p$$

This constraint is expressed using the linear constraint below:

$$\forall t \in T, \forall l \in L^t_{leav}, \forall p \in P$$
$$\begin{cases} \alpha^l_p + \Delta^l_p + \Theta^l + \Gamma^t + W^l_p - \beta^l_p \leq H \cdot (1 - P^l_p) \\ \beta^l_p - \alpha^l_p - \Delta^l_p - \Theta^l - \Gamma^t - W^l_p \leq H \cdot (1 - P^l_p) \end{cases} \quad (39)$$

Security constraints. The security constraints ensure that two circulations cannot pass the same section or the same connector at the same time. We use the actual time interval variables and ordering variables defined previously to express these constraints.

- When two circulations using the same section, one circulation must be scheduled before the other:

$$\forall t, t' \in T, \forall l \in L^t, \forall l' \in L^{t'}, l \neq l', \forall s \in S \quad Y^{ll'}_s + Y^{l'l}_s = 1 \quad (40)$$

Occupation of sections: Two circulations passing through a common section cannot be scheduled during the same time interval.

If two circulations are not in the same train, the constraint is expressed below:

$$\forall t, t' \in T, t \neq t', \forall l \in L^t, \forall l' \in L^{t'}, \forall s \in S$$
$$\begin{cases} \beta^l_s \leq \alpha^{l'}_s + H \cdot (3 - S^l_s - S^{l'}_s - Y^{ll'}_s) \\ \beta^{l'}_s \leq \alpha^l_s + H \cdot (3 - S^l_s - S^{l'}_s - Y^{l'l}_s) \end{cases} \quad (41)$$

Note: If section s is in the route of both circulations l and l', so that $S^l_s = 1, S^{l'}_s = 1$ and either $Y^{ll'}_s = 1$ or $Y^{l'l}_s = 1$. It means that $3 - S^l_s - S^{l'}_s - Y^{ll'}_s = 0$ or $3 - S^l_s - S^{l'}_s - Y^{l'l}_s = 0$. In the first case, we have $\beta^l_s \leq \alpha^{l'}_s$, it means that circulation l leaves section s before the arriving of circulation l' at section s. The second constraint is trivially verified ($Y^{l'l}_s = 0$). In the other case, we have $\beta^{l'}_s \leq \alpha^l_s$, it means that circulation l' leaves section s before the arriving of circulation l at section s.

In case that two circulations are in the same train, the constraint is expressed below for all sections which are not platform:

$$\forall t \in T, \forall l, l' \in L^t, \forall s \in S \backslash P$$
$$\begin{cases} \beta^l_s \leq \alpha^{l'}_s + H \cdot (3 - S^l_s - S^{l'}_s - Y^{ll'}_s) \\ \beta^{l'}_s \leq \alpha^l_s + H \cdot (3 - S^l_s - S^{l'}_s - Y^{l'l}_s) \end{cases} \quad (42)$$

In case that section is a non-stop platform, the constraint is expressed below:

$$\forall t \in T, \forall l, l' \in L^t, \forall p \in P$$
$$\begin{cases} \beta^l_p \leq \alpha^{l'}_p + H \cdot (3 - S^l_p - S^{l'}_p - Y^{ll'}_p + P^l_p) \\ \beta^{l'}_p \leq \alpha^l_p + H \cdot (3 - S^l_p - S^{l'}_p - Y^{l'l}_p + P^l_p) \end{cases} \quad (43)$$

Note: The constraint of occupation of sections for the stopping platform is expressed in the constraints (33)-(39) in the previous part of this section.

- With two circulations using the same connector, one circulation must be scheduled before the other:

$$\forall t, t' \in T, \forall l \in L^t, \forall l' \in L^{t'}, l \neq l', \forall c \in C \quad X^{ll'}_c + X^{l'l}_c = 1 \quad (44)$$

Occupation of connectors: Two circulations passing through a common connector cannot be scheduled during the same time interval:

$$\forall t, t' \in T, \forall l \in L^t, \forall l' \in L^{t'}, \forall c \in C$$
$$\begin{cases} \omega^l_c \leq \upsilon^{l'}_c + H \cdot (3 - C^l_c - C^{l'}_c - X^{ll}_c) \\ \omega^{l'}_c \leq \upsilon^l_c + H \cdot (3 - C^l_c - C^{l'}_c - X^{l'l}_c) \end{cases} \quad (45)$$

IV. Numerical Experiments

In this experiment, our topology is depicted in Fig. 13 and the correspondences between sections of this figure are listed in Table III.

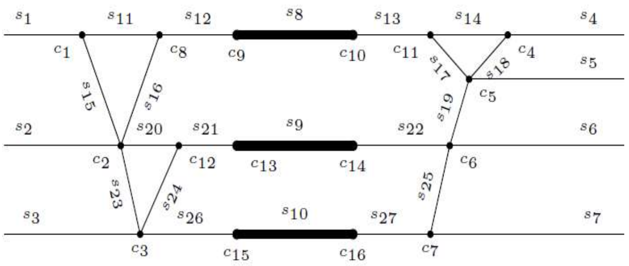

Fig. 13. Topology of the railway station.

TABLE III: Correspondences between Sections of Fig. 13

Section	External section	Platform
s_1	e_1	
s_2	e_2	
s_3	e_3	
s_4	e_4	
s_5	e_5	
s_6	e_6	
s_7	e_7	
s_8		p_1
s_9		p_2
s_{10}		p_3

TABLE IV: Example of Problem

Train	Circulation	Type	External section	A^t	D^t
1	1	ent	e_5	70	30
1	2	ent	e_4	90	40
1	3	leav	e_1		
2	4	ent	e_3	70	30
2	5	leav	e_4		
2	6	leav	e_6		
3	7	ent	e_2	65	30
3	8	leav	e_7		
4	9	ent	e_2	65	30
4	10	ent	e_3	80	40
4	11	leav	e_6		
5	12	ent	e_2	70	30
5	13	leav	e_5		
5	14	leav	e_6		
6	15	ent	e_1	90	30
6	16	leav	e_5		
7	17	ent	e_3	70	30
7	18	ent	e_1	90	40
7	19	leav	e_6		
8	20	ent	e_6	100	30
8	21	leav	e_2		
8	22	leav	e_3		
9	23	cros	$e_{in} = e_1$ $e_{out} = e_6$		

Seven external sections (e_1 to e_7) are considered for the arrival and departure of trains. There are three platforms (p_1 to p_3) which are used for the boarding or unboarding of passengers. There are a total of 27 sections and 16 connectors in this railway station. We assume that all sections are sections with two-way directions. For example, the set of doublets of connector c_1 is $K_{c_1} = \{(s_1, s_{11}), (s_{11}, s_1), (s_1, s_{15}), (s_{15}, s_1)\}$. All pairs of sections are not reachable (they are not doublets) even if two sections of these pairs are connected with a same connector. These pairs of unreachable sections are as follows: (s_{15}, s_{11}), (s_{11}, s_{16}), (s_{15}, s_{16}), (s_{15}, s_2), (s_{16}, s_{20}), (s_{16}, s_{23}), (s_{20}, s_{23}), (s_{20}, s_{24}), (s_{23}, s_3), (s_{23}, s_{24}), (s_{24}, s_{26}), (s_{14}, s_{18}), (s_{14}, s_{17}), (s_{17}, s_{18}), (s_{18}, s_5), (s_{17}, s_{19}), (s_{19}, s_6), (s_{22}, s_{25}), (s_{25}, s_7). For example, the pair of unreachable sections (s_{15}, s_{16}) means that trains are not allowed to travel from section s_{15} to section s_{16} and from s_{16} to s_{15}.

We run the experiments for 9 trains (3 type T_{21}, 3 type T_{12}, 2 type T_{11} et 1 type crossing train) which correspond to 23 circulations. The data of each train are presented in Table 4.

The following constants are used:

- Maximum permissible deviation for A^t: L=3
- Duration to traverse section by circulation Δ=20 for all.
- Duration to traverse connector by circulation Θ=2 for all.
- Duration for a coupling system or uncoupling system of trains Γ^t=5 for all.

Note: Times are counted in seconds.

We run the experiments with the objective function of minimizing the total of waiting times $\sum_{l\in L}\sum_{s\in S} W_s^l$ and minimizing the total of ending occupation time of sections $\sum_{l\in L}\sum_{s\in S} \beta_s^l$.

Objective function: MIN $\sum_{l\in L}\sum_{s\in S}(K_1 \cdot W_s^l + K_2 \cdot \beta_s^l)$

K_1: weight of total of waiting times

K_2: weight of total of ending occupation time of sections

In our experiments, we chose $K_1 = 0.6$, $K_2 = 0.4$

The computation study was conducted under C++ in Visual Studio 2017 and CPLEX version 12.8. The computer hardware runs Windows 64-bit operating system with Intel i7-870 CPU at 2.93 GHz and 4GB memory of RAM. The results are presented in Table 5. The time needed to solve the problem is 2.59 seconds. The results show that there are 6 interruptions of trains with a total waiting time of 94 seconds. The model considered has 1750 constraints and 995 variables after the presolve of CPLEX.

TABLE V: Results of Problem

Train	Circ	Route	Platform
1	1	5, 17, 13, 8	
1	2	4, 14, 13, 8	p_1
1	3	8, 12, 11, 1	
2	4	3, 24, 21, 9	
2	5	9, 22, 19, 18, 4	p_2
2	6	9, 22, 6	
3	7	2, 23, 26, 10	p_3
3	8	10, 27, 7	
4	9	2, 20, 21, 9	
4	10	3, 24, 21, 9	p_1
4	11	9, 22, 6	
5	12	2, 20, 21, 9	
5	13	9, 22, 19, 5	p_2
5	14	9, 22, 6	
6	15	1, 11, 12, 8	p_1
6	16	8, 13, 17, 5	
7	17	3, 26, 10	
7	18	1, 15, 23, 26, 10	p_3
7	19	10, 27, 25, 6	
8	20	6, 25, 27, 10	
8	21	10, 26, 23, 2	p_3
8	22	10, 26, 3	
9	23	1, 15, 20, 21, 9, 22, 6	

V. Conclusion

In this paper, we propose a mathematical model and a mixed-integer linear programming formulation to solve optimal train routing and scheduling for railway stations. The model is validated by an illustrative experiment. In the next work, we will make the experiments on real data related to a French railway station which was tested by [6]. The topology corresponding to this railway station has a total of 52 sections and 18 connectors. In this busy railway station, there are 247 trains and 504 circulations per day. We must add some working hypotheses that can be presumed still to correspond to the model of [6]. We will make a comparison of our results and their results to assess the performance of our method.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Dang conducted the research, analysed the data and wrote the paper. Bourdeaud'huy, Mesghouni and Toguyéni gived the guidance and direction on the research. Bourdeaud'huy, Mesghouni and Toguyéni validated the research. All authors had approved the final version.

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