Roughness Grade Analysis on Fitness Landscape for Optimization Problem of Multi-Dimensional Function

Shi Hui Wu

Abstract—**The roughness grade analysis on fitness landscape is helpful for obtaining the difficulty of the multi-dimensional function optimization problem, improving the optimization algorithms, and finding all local minima. Firstly, comparison studies are carried out on several commonly used indicators that depict the roughness of fitness landscape, such as autocorrelation function index, the improved fitness distance correlation (FDC) coefficient index, which are calculated using samples instead of differentiability of the function. A comprehensive index called roughness grade (RG) is constructed to measure the roughness of the fitness landscape by utilizing indices such as total variation of the function, rate of decline, FDC, etc. The advantages and disadvantages of the roughness indicators are summarized according to the results of experiments, which show that the improved FDC index and RG index are qualified for measuring different aspects of the roughness characteristics, and the improved FDC index has advantages over RG on fixed value range, less samples required, and simple calculation, thus can be used as main index, while RG index can be used as aided index for designing roughness grade based optimization algorithms of multi-dimensional function.**

Index Terms—**Fitness landscape, fitness distance correlation (FDC), multi-dimensional function, roughness grade.**

I. INTRODUCTION

The fitness landscape is one of the most influential concepts in evolutionary biology, which is a mapping from a set of genotypes to fitness, where the set of genotypes is organized according to which genotypes can mutate from one to another [1]. In 1990, Weinberger[2] brought the concept of fitness landscape to the performance analysis of heuristic algorithms, where the fitness landscape is used to help understand the mechanism of heuristic algorithm and predict its performance, as well as help design efficient algorithms. Today, lots of studies build relationships between fitness landscape and the difficulty of heuristic algorithm [2], [3], while some other studies utilize the fitness landscape to help find all local minima of multi-dimensional functions [4]. This paper is for the second case, that is, we study the roughness of the fitness landscape to help search for local minima and improve algorithm. In many practical problems, such as multi-parameter design, optimization via simulation, we hope to obtain not only all global minima, but also those good local minima to help us make decisions [5]. However, the commonly used optimization algorithms, such as genetic algorithm, ant colony algorithm, simulated annealing algorithm, can only find the global minima and some local minima, but not all local minima. Understanding the roughness of fitness landscape is helpful for designing efficient algorithm in generating initial search points and setting search step length, which are key issues for finding all local minima with least time and resources.

The indices that depict the fitness landscape are mainly statistic features of the fitness landscape surface, such as G-measure index proposed by [4] and fitness distance correlation (FDC) index by [6]. In [4], a gradient based quality measure, called G-measure, is designed to measure the local minima distribution of a multi-dimensional continuous and differentiable function, and they allocated more initial search points in the region with higher G-measure, which means the fitness landscape of the region is possibly rougher and more likely to have more local minima. However, during the calculation of G-measure, the function is limited to a continuous and differentiable function because G-measure needs to calculate the first derivative and second derivative, e.g., the amount of convexity and concavity requires second derivative. This is not available for many cases where the function is not explicit, such as black-box function, simulation function, etc. In [3], fitness distance correlation (FDC) coefficient has been shown to be a reasonable measure to quantify problem difficulty in genetic algorithm and genetic programming for a wide set of problems, and GA problems can be classified in three types depending on the value of FDC coefficient. At the same time, the limitations of FDC are also pointed out: the calculation of FDC asks for the global optima to be known a priori, which is not available in many optimization problems, and for this reason, they think FDC is not a predictive measure, but can only be used as a theoretical indication. As a result, we have to improve the FDC coefficient firstly so that it can be used as a predictive measure to predict the roughness grade distribution of fitness landscape.

In this work, we study the roughness grade of fitness landscape for optimization problem of undifferentiable multi-dimensional function, including black-box function, simulation function. Based on samples from the function, we compare several roughness grade indicators, such as autocorrelation function index, the improved FDC coefficient index, total variation, and rate of decline of the function, and the result shows that the improved FDC coefficient can effectively depict the roughness distribution of fitness landscape in most cases. In order to better describe the roughness grade of fitness landscape, a comprehensive index called roughness grade (RG) is designed, and experimental results show that the improved FDC coefficient and RG are effective in representing the roughness distribution of fitness

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landscape.

The remainder of this paper is organized as follows:

In the next section, we introduce the concept of fitness landscape. This is followed by definition of several roughness grade indicators in Section III. Our computational results are presented in Section IV, and in the last section we summarize our findings and highlight some research issues.

II. Fitness Landscape

Viewing the search space (generally the set of all possible solutions) as a landscape, the height of a point in the search space reflects the fitness (objective) of the solution associated with that point, and a heuristic algorithm can be thought of as hill-climbing through it in order to find the highest peak of the landscape (for maximization problems) [7], or the lowest valley (for minimization problems). The hill-climbing process is shown in Fig.1, where initial points navigate on fitness landscape with certain step length and search rules to find their local minima. We can also find from Fig.1 that as the landscape becomes rougher, the hill-climbing to the global optimum will be more difficult. In this article, we will mainly tackle with minimization problems.

Definition 1 A fitness landscape is defined as (S, f, d) that consists of three ingredients: the search space S, element distance $d(\boldsymbol{x}_1, \boldsymbol{x}_2)$ where $\boldsymbol{x}_1, \boldsymbol{x}_2 \in S$, and fitness function $f(\boldsymbol{x})$: $S \to R$, $\boldsymbol{x} \in S$.

There are two potential applications of the roughness grade analysis on fitness landscape:

1) Reduce the initial points. Multi-start algorithm [8] has been widely used to solve all local optima. With less initial points, search time on finding all local optima can be greatly shortened. Generally, the regions with high roughness grade may have lots of peaks and valleys, which means more initial points are needed in these regions to allocate the local optima, e.g., the regions S_1 and S_3 in Fig.2 are rougher than S_2 and S_4, thus more initial points should be allocated in S_1 and S_3 when using multi-start algorithm.
2) Allocate search step length according to the roughness grade. The regions with higher roughness grade may have lots of peaks and valleys, which means more detailed search should be taken to find those local optima, e.g., the region S_1 and S_3 in Fig.2 are rougher than S_2 and S_4, thus more detailed search are needed, i.e., the search step length d_1 and d_3 should be relatively smaller than that of d_2 and d_4 to find all local optima in S_1 and S_3.

III. Roughness Indicators of Fitness Landscape

Researchers have designed several indices to measure the structure features of fitness landscape. The roughness grade is one of the most important structure features.

There are many factors that influence the roughness grade, which include correlation coefficient of neighbour points on fitness landscape, number of local minima, amount of convexity and concavity, etc. Generally, the rougher the fitness landscape is, the more difficult for algorithms to find the optimum.

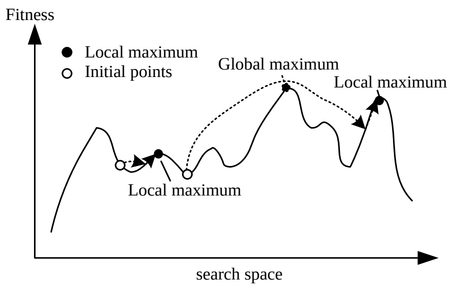


Fig. 1. Hill-climbing process on fitness landscape.

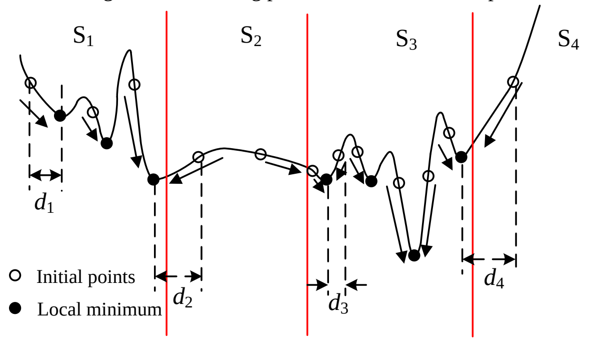


Fig. 2. Hill-climbing on fitness landscape regions with different roughness.

It is easy to imagine the fitness landscape of one or two dimensional functions, e.g., fitness landscape of one dimensional function is in fact a curve in two-dimensional space, and two dimensional function is a surface in three-dimensional space. However, for function with dimension greater than three, the fitness landscape cannot be depicted as visualized image[1]. For such situations, the question is how we could reflect the structure features to a visualized index. We will introduce several indices in earlier studies, and make necessary modifications to tailor for undifferentiable function.

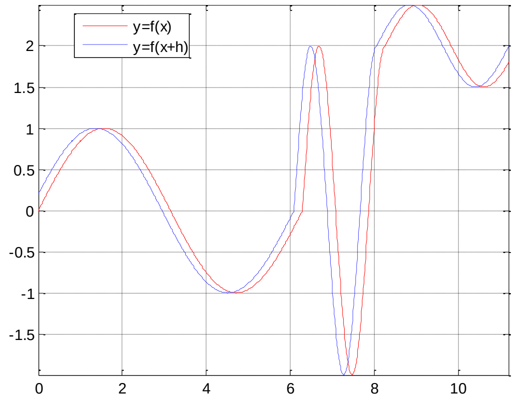
A. Autocorrelation Function Index

Definition 2 [6] For function $y=f(x)$, let $\{f(x_0), f(x_1), \cdots f(x_n)\}$ denotes the time series of the fitness value when a navigator point moves step by step on the fitness landscape, the autocorrelation function index r is defined as

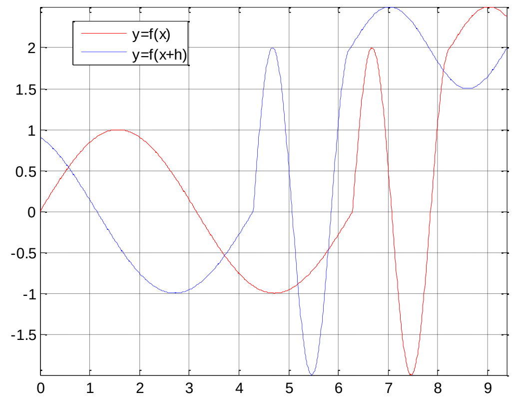
$$r=\frac{E[f(x_{t+h})f(x_t)]-E[f(x_t)]^2}{E[f(x_t)^2]-E[f(x_t)]^2}=\frac{\sum_{t=1}^{T}(y_{t+h}\cdot y_t)-\left(\sum_{t=1}^{T}y_t\right)^2}{\sum_{t=1}^{T}y_t^2-\left(\sum_{t=1}^{T}y_t\right)^2} \quad (1)$$

where $E[f(x_t)]$ denotes the mathematical expectation of $f(x_t)$, and h denotes the step length between two consecutive neighbor points in the time series.

Autocorrelation function index r actually measures the correlation degree of current surface and a future surface moved by a step length h. For example, in Fig.3(a), r denotes the correlation coefficient between $f(x)$ and $f(x+h)$. As a result, r is between -1 to 1, and the closer it is to 0, the lower correlation the neighboring points will have, thus the fitness landscape is rougher. On the contrary, the closer r is to 1, the closer the neighbor points will be, thus the fitness landscape is smoother. However, r is not fixed, but varies greatly with step length h. As shown in Fig.3(b), when h changes from 0.2 to 2, the value of r reduces from 0.9212 to 0.2032. Generally speaking, for a continuous function, r becomes smaller as h grow bigger, which means it will be meaningless to compare the roughness grade of two functions by *r* when they don't have the same *h*. In other words, the roughness grade of two functions are comparable by *r* only when *h* is the same.



(a) When h=0.2, we have r=0.9212



(b) When h=2, we have r=0.2032

Fig. 3. Examples of *r* with different *h*.

Meanwhile, before we calculate *r* for a multi-dimensional function, the direction of time series should be known. For example, for a two dimensional function $y=f(x_1, x_2)$, if the direction of time series follows x_1 axis, i.e., x_2 keeps unchanged and *y* becomes $f(x_1+h, x_2)$ after a time step; if the direction of time series follows x_2 axis, i.e., x_1 keeps unchanged and *y* becomes $f(x_1, x_2+h)$ after a time step; or if the direction of time series follows certain angle between x_1 and x_2 axis, e.g., *y* becomes $f(x_1+h/\sqrt{2}, x_2+h/\sqrt{2})$ after a time step. Apparently, different directions of time series will result in different *r*, which is like cutting through an irregular ball, different cutting directions will get different cutting surfaces, thus get different roughness grade. However, cutting directions are infinite, as a result, *r* cannot be determined if the direction of time series is unknown. Therefore, although *r* is capable of describing the structure feature of the fitness landscape, it is not suitable to be a roughness grade index for a multi-dimensional function because of the limitations by *h* and cutting directions.

B. Fitness Distance Correlation Coefficient

Fitness distance correlation (FDC) coefficient, firstly proposed by Jones[9], is used to describe the correlation degree between distance to optimum and fitness of points on the fitness landscape. The essence of FDC is Pearson correlation coefficient, which means FDC has the feature of a normal Pearson correlation coefficient: if the fitness landscape is smooth and continuous, FDC should be closer to one; if it is rough and fluctuant, FDC should be small, which means little relationship exists between distance to optimum and fitness. As stated earlier, the calculation of FDC asks for the global optima to be known a priori[3], which is not available in many optimization problems, thus we cannot directly use FDC as an index to measure the roughness grade distribution of fitness landscape. Here, we make some improvement to FDC in (2): firstly, a set of sample points are collected on the function, from which the best sample point is chosen to approximately represent the global optima. This may cause calculation error to FDC since the sample best may differ from global optima, however, the influence of such error should be small, because if the fitness landscape is rough, the correlation coefficient between distance to any benchmark point (including the global optima) and fitness will be relatively low, let alone sample best point may be near to global optima. Therefore, compared with traditional FDC, the improved FDC by (2) can not only reflect the roughness of fitness landscape, but also overcome the disadvantage of knowing the global optima beforehand.

Definition 3 For function $y=f(\boldsymbol{x})$, $\boldsymbol{x}\in\Omega$, let P={$\boldsymbol{x}_i$, i=1, 2,…, n} denotes a set of sample points on the fitness landscape, where $\boldsymbol{x}_{\min}$ is the sample point with greatest fitness (or best sample point). The FDC on P is defined as:

$$fdc(\Omega)=\frac{\sum_{i=1}^{n}(y_i-\bar{y})(d_i-\bar{d})}{\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2\sum_{i=1}^{n}(d_i-\bar{d})^2}} \tag{2}$$

where y_i denotes the fitness of $\boldsymbol{x}_i$; $\bar{y}$ denotes the mean of y_i, i.e., $\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$; d_i denotes Euclidean distance from $\boldsymbol{x}_i$ to $\boldsymbol{x}_{\min}$, i.e., $d_i=\|\boldsymbol{x}_i-\boldsymbol{x}_{\min}\|$; $\bar{d}$ denotes the mean of d_i, i.e., $\bar{d}=\frac{1}{n}\sum_{i=1}^{n}d_i$.

Since FDC is Pearson correlation coefficient, we have $fdc\in[-1, 1]$. When fdc=1, the relationship between y_i and d_i is linear, that is, the closer to the best sample point (d_i is smaller), the better fitness it becomes (y_i is smaller). In other words, there exists certain route on the fitness landscape from sample point to optimum point. When $|fdc|$ is smaller, it means that fitness doesn't become better as sample points approaching optimum point, because there may be some peaks and valleys blocking the route to optimum point. Therefore, we can conclude that as $|fdc|$ is small, fitness landscape will be rough.

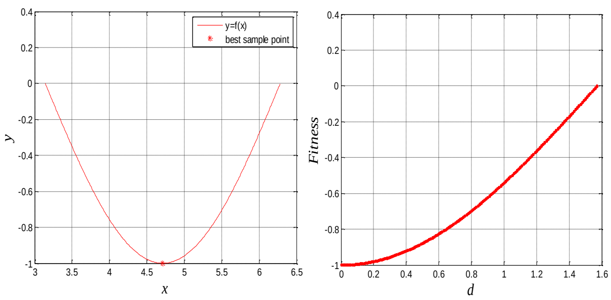
Examples of FDC for several typical function curves are given as follow.

1) For a simple one dimensional function, $y=\sin x$, $x\in[\pi,2\pi]$, the function curve is shown in Fig.4(a). We collect 100 sample points uniformly on the feasible region, and the set of sample points P is obtained as P={π, 1.01π, 1.02π,…, 2π}. The best sample point is $\boldsymbol{x}_{\min}$=3π/2. As shown in Fig.4(b), the closer to $\boldsymbol{x}_{\min}$ (d is smaller), the better fitness we have. Apparently, it is the most ideal situation where we can

see a clear and smooth route to optimum point from any sample point on P. By (2), *fdc*=0.9791, which is very close to one. It means that the roughness of the landscape is very low, and it is very easy to navigate to optimum point.

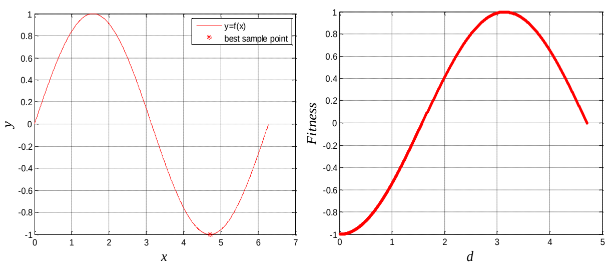
2) For the same function but with different feasible region, $y = \sin x$, $x \in [0, 2\pi]$, the function curve is shown in Fig.5(a). Similarly, 100 sample points are collected where $\boldsymbol{x}_{\min}=3\pi/2$. As shown in Fig. 5(b), the relationship between d and fitness can be divided into two parts: when distance to $\boldsymbol{x}_{\min}$ is smaller than π, the closer to $\boldsymbol{x}_{\min}$, the better fitness we have; when distance to $\boldsymbol{x}_{\min}$ is greater than π, the closer to $\boldsymbol{x}_{\min}$, the worse fitness we have, that's because the route to optimum point is blocked by a peak on [0, $\pi/2$]. However, the roughness of fitness landscape in Fig.5(a) is generally not so complicated, and we obtain *fdc*=0.8376 by (2). Although *fdc* is relatively lower than in Fig.4, the correlation of *d* and *fitness* is still great since in most region they are correlative positively. As a result, the roughness of the landscape is still acceptable, and not difficult to navigate to optimum point.

Obviously, as the fitness landscape becomes rougher, the route to optimum point will encounter more and more blocks, thus *fdc* will be reduced gradually. The problem is to what extent the *fdc* value being reduced that we can consider the roughness of fitness landscape acceptable. We define a threshold acceptable *fdc*, denoted as fdc_0: when *fdc* is higher than fdc_0, the roughness of fitness landscape is considered acceptable, thus local minima are easy to obtain by algorithms; on the contrary, when *fdc* is lower than fdc_0, the roughness of fitness landscape is considered unacceptable, thus local minima on this region are difficult to find, or at least some local minima may be missed. By lots of tests and experiments, the results show that it is highly suggested to set fdc_0 in [0.4, 0.6].



(a) Function curve (b) Fitness varies with *d*

Fig. 4. Roughness and *fdc* of typical function curve.



(a) Function curve (b) Fitness varies with *d*

Fig. 5. Roughness and *fdc* of typical function curve.

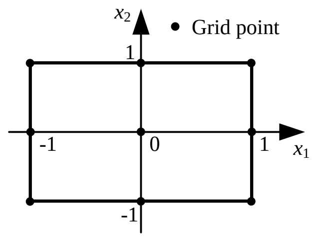


Fig. 6. Example of TV for a two dimensional function.

C. Total Variation and Rate of Decline

A gradient based comprehensive measure called G-measure is designed to describe the roughness of fitness landscape in [4]. Actually, G-measure is the function of total variation, rate of decline and the amount of convexity and concavity. Since the amount of convexity and concavity is unavailable for an undifferentiable functions, such as black box function, G-measure is also unavailable. However, the other two indices, total variation and rate of decline, are capable of describing the roughness of fitness landscape and available for any given functions. Suppose the feasible region Ω is equally divided by grids with step length h, and $\overline{x^i}$ (i=1,2,…,k) denotes points on the grids, or we call grid points.

Definition 4 For function $y = f(\boldsymbol{x}) = f(x_1, x_2, \cdots, x_n)$, $\boldsymbol{x} \in \Omega$, total variation (TV) is a measure that reflects the sum total of variations on the region. TV is defined as integrals of gradient mode on the region Ω, which can be approximated by sum of gradient modes of the grid points. Thus TV(Ω) is defined as follow.

$$\mathrm{TV}(\Omega) = \int_{\Omega} |\nabla f| d\Omega \approx \frac{m(\Omega)}{k} \sum_{i=1}^{k} \left|\nabla f(\overline{x^i})\right| \tag{3}$$

where $m(\Omega)$ denotes the measure of the region Ω, e.g., $m(\Omega)$ represents length for one dimensional region, area for two dimensional region, and volume for three dimensional region, etc., the gradient $\nabla f(\overline{x^i})=\left\{\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n}\right\}_{x=\overline{x^i}}$ on grid point $\overline{x^i}$ is approximated as

$$\left.\frac{\partial f}{\partial x_j}\right|_{x=\overline{x^i}} = \frac{f(\overline{x^i_{j+1}}) - f(\overline{x^i_{j-1}})}{2h} \tag{4}$$

Take a two dimensional function $f(x_1, x_2)$ for example, as shown in Fig.6, the step length h=1, and Ω is $x_1 \in [-1,1]$, $x_2 \in [-1,1]$. Then we have $m(\Omega)$=4, and TV(Ω)= $\frac{4}{3}\left(\sum_{i\in\{-1,0,1\}}\left|\frac{f(1,i)-f(-1,i)}{2}\right| + \sum_{i\in\{-1,0,1\}}\left|\frac{f(i,1)-f(i,-1)}{2}\right|\right)$ by (3) and (4).

Definition 5 For function $y = f(\boldsymbol{x}) = f(x_1, x_2, \cdots, x_n)$, $\boldsymbol{x} \in \Omega$, rate of decline (RD) is a measure that reflects the decline speed of the function value on the region. $\max \overline{x_i}$ and $\min \overline{x_i}$ (i=1,2,…,k) denotes the grid point with maximum and minimum function value respectively, where $\overline{x^i}$ (i=1,2,…,k) denotes points on the grids. RD(Ω) is defined as follow.

$$\mathrm{RD}(\Omega)=\frac{f(\max\overline{x_i})-f(\min\overline{x_i})}{\left\|\max\overline{x_i}-\min\overline{x_i}\right\|} \tag{5}$$

where $\left\|\max\overline{x_i}-\min\overline{x_i}\right\|$ is the Euclidean distance between the grid points with maximum and minimum function values.

D. Roughness Grade

Generally, TV and RD should be used together to reflect the roughness of fitness landscape. Specifically, when TV is great and RD is small at the same time, that is, the sum of gradient modes is great but the average decline speed of the function value is small, it means there should have great fluctuates on the region, perhaps lots of peaks and valleys, thus it is relatively rougher on the region. As mentioned in Section Ⅲ(C), $m(\Omega)$ denotes the measure of the region Ω. Obviously, for two regions with different $m(\Omega)$, the roughness measure, such as TV and RD, will be incommensurable. To deal with such situation, we should compare these indicators on unit measure. Also, as mentioned before, fitness landscape will be rough as $|fdc|$ is small. Above all, a comprehensive roughness measure called roughness grade (RG) is defined as

$$RG(\Omega)=\frac{10}{m(\Omega)}\frac{\mathrm{TV}(\Omega)}{\mathrm{RD}(\Omega)}\left[1-|fdc(\Omega)|\right] \tag{6}$$

where the coefficient 10 is used to avoid too low value of RG. Obviously, the fitness landscape on the region Ω will be rougher as RG(Ω) becomes greater.

IV. Illustrative Examples

Two simple examples are given below to compare whether the roughness indicators listed in Section Ⅲ can reflect the roughness degree of the fitness landscape. Note that we use *fdc* to represent the improved FDC in Section Ⅲ.

Example 1. A piecewise function is given as below.

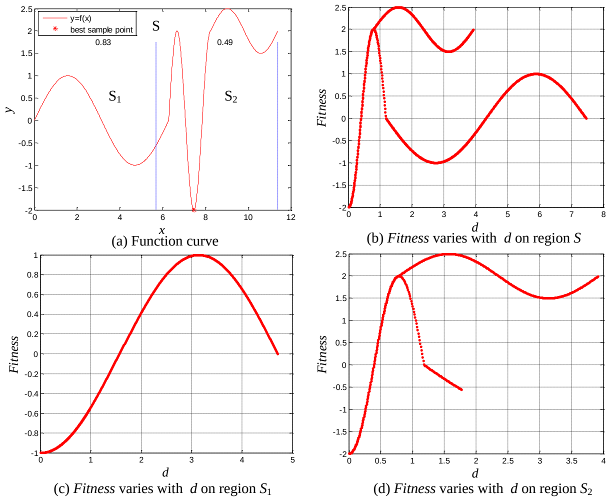
$$f(x)=\begin{cases}\sin x, & 0\le x<2\pi\\ 2\sin(4x-8\pi), & 2\pi\le x<2.625\pi\\ \frac{1}{2}\sin(2x-5.25\pi)+2, & 2.625\pi\le x<3.625\pi\end{cases}$$

We will discuss the roughness indicators on different sub regions. The sample points are collected uniformly on each region with step length 0.01 for (2), and each region is equally divided by grids with step length 0.01 for (3) to (5).

1) Divide the feasible region S: $x\in[0, 11.388]$ into two sub regions, S_1: $x\in[0, 5.6941]$, and S_2: $x\in[5.6941, 11.388]$, see Fig.7(a).

From the fitness landscape on region S, we can see that the landscape is rough with three peaks (or local maxima) and three valleys (or local minima), and the best sample point is $\boldsymbol{x}_{\min}$=7.4641. As shown in Fig.7(b), when d<0.785, the closer to $\boldsymbol{x}_{\min}$, the better fitness we have; on the contrary, when d>0.785, the relationship between *Fitness* and d becomes clutter because three peaks appear successively on that region, see Fig.7(a).

By (2), $fdc(S)$=0.0032, which means that the landscape is very rough with many peaks and valleys, and it will be tough to navigate to optimum if the initial points are far from $\boldsymbol{x}_{\min}$.



(a) Function curve

(b) *Fitness* varies with d on region S

(c) *Fitness* varies with d on region S_1

(d) *Fitness* varies with d on region S_2

Fig. 7. Function curve and *fdc* on different regions.

On region S_1, the best sample point is $\boldsymbol{x}_{1\min}$=4.7124. As shown in Fig.7(c), when $d<\pi$, the closer to $\boldsymbol{x}_{1\min}$, the better fitness we have; when $d>\pi$, the relationship between *Fitness* and d becomes negative because there is a peak in the region $[0, \pi/2]$, which lowers the *fdc* value. But overall, it is not so rough on S_1, see Fig.7(a), and $fdc(S_1)$= 0.8346 using (2).

On region S_2, the best sample point is $\boldsymbol{x}_{2\min}$= 7.4641. As shown in Fig.7(d), when d<0.785, the closer to $\boldsymbol{x}_{2\min}$, the better fitness we have; when d>0.785, the relationship between *Fitness* and d becomes clutter because two peaks appear successively on that region, see Fig.7(a), thus the landscape is rough and *fdc* should be low. By (2), $fdc(S_2)$= 0.4915, which is consistent with our prediction.

Meanwhile, several other roughness indicators are calculated by (1), (3), (4) and (5), as well as the comprehensive roughness index RG by (6), as shown in Table Ⅰ. Since TV and RD cannot be used to measure roughness degree independently, we will mainly compare three roughness indicators: r, *fdc* and RG.

From Table I, we can see that $fdc(S)$< $fdc(S_2)$ < $fdc(S_1)$, which means the roughness degree in descent order is $S\succ S_2\succ S_1$. The same roughness order can also be obtained by RG(S)> RG (S_2) > RG (S_1). However, for the indicator r, we have $r(S_2)$< $r(S)$ < $r(S_1)$ when h=0.5, so the roughest region is S_2 according to the indicator r. This is because r is not only decided by the roughness of the landscape, but also the step length h, and by its definition, r is greater when the landscape varies sharply within a smaller region, see Fig.3(a).

2) Divide the feasible region S into four sub regions, S_1: $x\in[0, 1.5708]$, S_2: $x\in[1.5708, 6.6759]$, S_3: $x\in[6.6759, 9.0321]$, and S_4: $x\in[9.0321, 11.388]$, see Fig.8.

Similarly, roughness indicators are calculated as shown in Table II. From Fig. 8, there are no peaks inside each region, which means the roughness of each region are very low. Compared with Table I, the *fdc* value of these four regions are much higher, while RG value are much lower, which can also demonstrate that *fdc* and RG can reflect the roughness of fitness landscape. Meanwhile, $r(S_3)$ in Table Ⅱ is very low, i.e., $r(S_3)$= 0.052342, but S_3 is not rough. Again, r is not fit for a roughness indicator, and we should use *fdc* and RG as roughness indicators.

TABLE I: ROUGHNESS INDICATORS OF DIFFERENT REGIONS IN FIG.7(A)

Region	r (with h=0.5)	*fdc*	RG	Roughness order by RG	Roughness order by *fdc*
S	0.6231	0.003166	4.8878	1	1
S_1	0.90187	0.8346	1.569	3	3
S_2	0.38513	0.4915	3.9167	2	2

TABLE II: ROUGHNESS INDICATORS OF DIFFERENT REGIONS IN FIG.8

Region	r (with h=0.5)	*fdc*	RG	Roughness order by RG	Roughness order by *fdc*
S_1	0.98946	0.97876	0.20998	3	3
S_2	0.60126	0.89875	0.64509	2	1
S_3	0.052342	0.90913	1.1297	1	2
S_4	0.80175	0.99204	0.079462	4	4

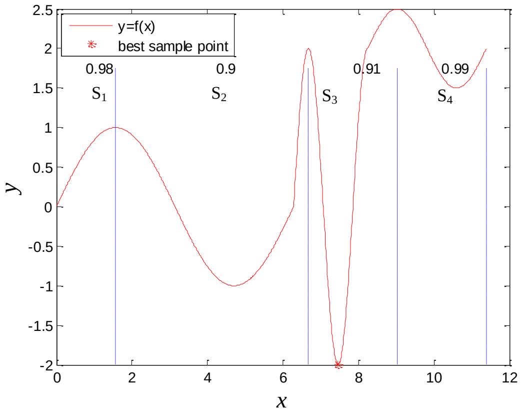


Fig. 8. Function curve and *fdc* on four different sub regions.

From Table II, we can see that the roughness order by RG and *fdc* are basically the same except for the regions S_2 and S_3. According to RG, the roughness on S_3 is much higher than S_2, while the roughness of S_2 and S_3 are nearly the same according to *fdc*. This is because RG mainly reflects the roughness on unit measure, e.g., unit length for one dimensional function, unit area for two dimensional function, etc. Thus, we can conclude that the roughness on unit length for S_3 is much higher than that of S_2, which is clearly shown in Fig.8 that the function curve on region S_3 is much more fluctuant than S_2 for unit length.

Above all, we can conclude that both RG and *fdc* can reflect the roughness of fitness landscape, and *fdc* mainly focuses on measuring the fluctuant of the landscape, the lower *fdc* is, the more peaks and valleys may exist on the region, while RG mainly focuses on measuring the fluctuant of unit measure landscape, the greater RG is, the sharper up and downs on a relatively smaller region may exist. Since RG>0, e.g., RG may be smaller than one, or in tens, in hundreds, it is capable of giving the roughness order of different regions, but not capable of indicating whether the landscape is with acceptable roughness. As mentioned before, since $fdc \in [-1, 1]$, there exits a threshold acceptable *fdc*, denoted as fdc_0, which can tell if the landscape is with acceptable roughness. By comparing the features of different roughness indicators, we suggest to use both RG and *fdc* to measure the roughness of different fitness landscapes to fully understand their roughness, and to use fdc_0 to judge if the landscape is with acceptable roughness for optimization algorithms.

Example 2. The six hump camel back function is given as below.

$$f(\boldsymbol{X}) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$$

where $-3 \le x_1, x_2 \le 3$.

The feasible range $[-3, 3]^2$ is equally divided into nine sub regions, S_1~S_9, as shown in Fig.9. The RG and *fdc* for those sub regions are calculated with different intervals of sample points (assuming that samples step length for (2) and grids step length for (3), (4) and (5) are equal to the intervals), as shown in Table Ⅲ, Table Ⅳ and Fig.9, and all six local/global minima P_1~P_6 are found, see Fig.9 and Table Ⅴ.

1) When sample points are collected with interval [0.01, 0.01], the calculation time increases greatly compared to interval [0.1, 0.1]. From Table Ⅲ and Table Ⅳ, we can see that both *fdc* and RG have not changed much as sample interval becomes smaller, where changes on *fdc* is even smaller. This means that *fdc* is robust to sample density, thus we don't have to collect too much samples to ensure a relatively accurate *fdc*.
2) As mentioned in Section Ⅲ(D), when TV(Ω) is great and RD(Ω) is small at the same time, the region Ω should have great fluctuates. Therefore, it is possible that TV(Ω)/(RD(Ω)**m*(Ω)) can reflect the roughness of the region Ω. On one hand, for the region S_5, there exist two global minima as shown in Fig.9 and Table Ⅴ, while the value of TV/(RD**m*) is also the greatest in nine sub regions; On the other hand, for the regions S_4 and S_6, there exist two local minima respectively, but their values of TV/(RD**m*) are smaller than that of S_3, on which there exists no local minimum. As a result, TV/(RD**m*) is not appropriate to measure the roughness independently, and that's why we have designed a comprehensive index RG in Section III(D).
3) The best value of RG and *fdc* are both on the region S_5, S_4 and S_6 in the same descent order, which is because there exist two local/global minima on these regions respectively, thus relatively rougher than the other regions with no local minima, see Fig. 9 and Table V. Hence RG and *fdc* are capable of measuring the roughness of fitness landscapes.

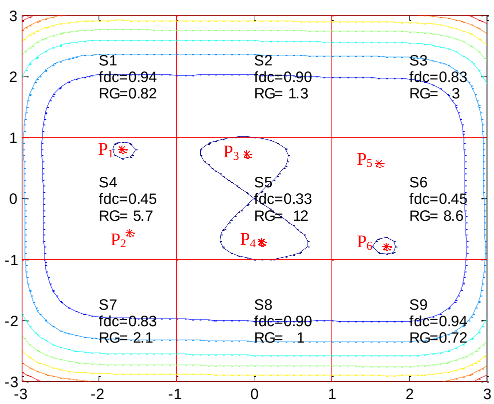


Fig. 9. Contour graph and roughness indicators (when interval is [0.1, 0.1]).

Comparing RG and *fdc*, both can reflect roughness degree, but *fdc* is much easier to obtain, and more robust to the sample interval *h*, which means less samples are needed. Besides, since $fdc \in [-1, 1]$, we can easily find a threshold acceptable *fdc*, denoted as fdc_0 to tell if the landscape is with acceptable roughness, which is not available for RG. Therefore, it is suggested that when analysing the fitness landscape for optimization algorithms, *fdc* can be regarded as main index, while RG as aided index.

TABLE III: THE ROUGHNESS INDICATORS (INTERVAL IS [0.1,0.1])

Sub regions	TV	RD	*m*	TV/(RD**m*)	*fdc*	RG
S_1	752.22	141.7	4	1.3271	0.93816	0.82073
S_2	574.16	108.62	4	1.3215	0.9034	1.2765
S_3	777.26	110.79	4	1.7539	0.83125	2.9597
S_4	209.03	50.494	4	1.0349	0.45312	5.6599
S_5	16.042	2.2163	4	1.8096	0.33025	12.12
S_6	208.05	33.191	4	1.5671	0.45312	8.57
S_7	778.85	156.95	4	1.2406	0.83125	2.0935
S_8	575.24	134.41	4	1.0699	0.9034	1.0335
S_9	752.81	161.15	4	1.1679	0.93816	0.72225

TABLE IV: THE ROUGHNESS INDICATORS (INTERVAL IS [0.01, 0.01])

Sub regions	TV	RD	*m*	TV/(RD**m*)	*fdc*	RG
S_1	776.75	160.6	4	1.2092	0.94028	0.72216
S_2	584.91	125.6	4	1.1642	0.90577	1.097
S_3	802.23	138.89	4	1.444	0.82899	2.4693
S_4	223.54	50.494	4	1.1068	0.45177	6.0676
S_5	17.076	2.2017	4	1.939	0.3036	13.503
S_6	223.45	48.501	4	1.1518	0.45177	6.3143
S_7	802.39	143.72	4	1.3958	0.82899	2.3869
S_8	585.02	128.58	4	1.1375	0.90577	1.0718
S_9	776.81	162.36	4	1.1961	0.94028	0.71439

TABLE V: ALL LOCAL/GLOBAL MINIMA OBTAINED

Local minima	x_1	x_2	y_{min}
P_1	-1.7036	0.7961	-0.2155
P_2	-1.6071	-0.5687	2.1043
P_3	-0.0898	0.7127	-1.0316
P_4	0.0898	-0.7127	-1.0316
P_5	1.6071	0.5687	2.1043
P_6	1.7036	-0.7961	-0.2155

V. CONCLUSIONS AND DISCUSSIONS

The fitness landscape analysis is meaningful for finding all local minima. In this paper, several roughness indicators are tested to see if they are qualified to measure the roughness of fitness landscapes, among which two indices, improved FDC and RG, are proved to be good for indicating the roughness. We find that the improved FDC, denoted as *fdc*, overcomes the limitation of traditional FDC which needs to know the global optima beforehand, and mainly focuses on measuring the fluctuant of the landscape: the lower *fdc* is, the more peaks and valleys may exist on the region; while RG is a comprehensive index that mainly focuses on measuring the fluctuant of unit measure landscape: the greater RG is, the sharper up and downs on a relatively smaller region may exist. Since the improved FDC has advantages over RG on fixed range ($fdc \in [-1, 1]$), robust to the sample interval *h* (or sample density), and easier to calculate, thus it can be regarded as main index, while RG as aided index. The proposed indices, improved FDC and RG, are calculated using the samples collected on the function, thus the differentiability of the function is not required, which is suitable for undifferentiable functions such as black box function, optimization function [10].

In the future, we would like to investigate how to utilize the proposed indices to practical applications and explore efficient algorithms by these indices on multi-dimensional optimization problems.

CONFLICT OF INTEREST

The author declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Shihui Wu conceived and designed the study, conducted experiments, and wrote this paper. The author approved the final version.

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