Modeling Interstate War Combat Deaths

Vaughn H. Standley, Frank G. Nuño, and Jacob W. Sharpe

Abstract—**A prolonged campaign of peaceful interstate competition is an ideal strategic application of artificial intelligence. Monte Carlo simulation, based on validated war analytics, must be at the heart of this capability. Otherwise the system will not know how to assess the potential consequences of failed solutions, chief among them combat fatalities resulting from interstate war. Although the power law has been used since 1960 to model the statistical distribution of deaths resulting from violent conflict, it is not a valid candidate for use in Monte Carlo simulation because it is mathematically divergent for the case of interstate war. Probing Correlates of War Project data, investigators found that combat fatalities in interstate war follow log-gamma or log-normal distributions, depending on whether a state is attacking or defending. Both distributions are valid for use in Monte Carlo simulations. Moreover, they are strong quantitative evidence that war should be modeled as a zero-sum, non-cooperative, high-risk game.**

Index Terms—**Combat deaths, interstate war, log-gamma, log-normal, monte carlo, power-law.**

I. INTRODUCTION

Russian President Vladimir Putin, speaking about artificial intelligence (AI), said "whoever is the leader will become the ruler of the world" [1]. Whether hyperbolic or prophetic, his assertion is difficult to dismiss. At the very least, effective innovation and use of AI systems is essential to global leadership in the 21st century. Thus far, however, most discussed AI applications fall far outside the strategic level of decision-making, to include killer robots, antimissile systems, and even an AI-enabled "dead hand" [2].

AI-assisted systems, where humans actively play a role in decision-making, can have more impact at the strategic level. We are a significant distance, both technologically and ethically, from complete AI control of strategic retaliatory strikes. Even taking advice from an AI system about whether to launch on an attack warning seems unlikely. There is not enough time to crosscheck the validity of the AI's solution for so consequential a decision given the possibility of a false alarm. Conversely, complex and prolonged periods of interstate competition are ideal for strategic application of AI because there is time to understand AI advice that is difficult to understand. Although each decision is not as consequential, the true power of AI will be in its ability to ensure that cumulative results are optimized.

Monte Carlo simulation, based on validated war analytics, are essential to decision-enhancing systems. The Monte Carlo method, widely used in many fields, relies on repeated random sampling and statistical analysis. After carrying out and comparing many simulations of possible courses of action, an AI system can chart the best path. Moreover, Monte Carlo is one of the computing methods that can be accelerated by quantum computers [3]. Large and complex Monte Carlo problems that are currently intractable will be solved in reasonable times. However, without validated analytics describing the risk associated with interstate competition, Monte Carlo analysis will not be able to correctly estimate the potential consequences of failed solutions.

The most straightforward and tangible consequence of the failure of peaceful interstate competition is combat fatalities. Beginning in 1960 with Lewis Fry Richardson's famous "The Statistics of Deadly Quarrels" [4], the power law has been used almost exclusively to model the statistical distribution of war fatalities. A phenomenon may be probabilistically distributed according to the power law if the logarithm of the exceedance probability $P(S>s)$ plotted against the logarithm of severity s (in deaths) appears as a straight line with a negative slope $-q$. This is written as $P(C > c) = c^{-q}$. Intuitively, the power law states that the probability of exponentially increasing consequences is observed to decrease exponentially.

An exceedance probability indicates how often a random variable S will exceed a value of s, written as $P(S>s)$. Exceedance probabilities are relevant to disasters, natural or manmade, because disaster conditions exist for all values above a value of s [5]. To construct a building to survive strong earthquakes, for example, the architect cares about the probability that the earthquake will be greater than some specified Richter value, such as "9". Above that severity the building will be damaged. We would write this as $P(S>9)$. Similarly, in war the probability of losing soldiers in an amount greater than some value is much more important than the probability of losing a specific number of soldiers. There are, of course, a multitude of issues that contribute to nations building armies and engaging in war, but for the purposes of the present research, soldier deaths as a measure of severity s is the focus. The probability density function is the derivative of the exceedance probability function. Likewise, the exceedance probability is the complement (i.e. subtracted from one) of the integral of the probability density function from zero to S.

The power law, sometimes also referred to as the Pareto distribution, is mathematical divergent when its parameter q is less than one. Unfortunately, most research reports q to be less than one for interstate war combat deaths. The present study sought one or more valid distributions to enable the

application of Monte Carlo simulation to strategic competition and to draw inferences from the results to advance the field of strategic deterrence.

II. LITERATURE REVIEW

Recent literature about the distribution of war combat deaths fit into one of three areas: application of the power law, alternative distributions, and use in Monte Carlo simulation.

A. Power Law

Researchers continue to hold that "Richardson's finding that the severity of interstate wars is power law distributed belongs to the most striking empirical regularities in world politics" [6], many reporting q values near 0.5 [7]. A power law fit with $q = 0.5$ is shown in Fig. 1 with a dashed red line. Unless the use is qualified, applications of the power law to interstate war when $q<1$ are in error [8]. A value less than one indicates that the exceedance probability decreases slower than the increase in number of deaths, implying that the number of deaths increases arbitrarily such that the mean is mathematically divergent. A proper qualification will recognize that "data held to be power-law distributed represent samples from some underlying population. As these samples often cover a narrower scale range than that of the population as a whole they are truncated" [9]. Some researchers use q when it is less than one to estimate the likelihood of war and terrorist attacks [10], [11], an application that is in error even with the aforementioned qualification. It is noted that there is some overlapping research where war is reported to have a q value greater than one [12] and are, therefore, not divergent.

B. Alternative Distributions

Curvature in log-log data suggest the applicability of logarithmic distributions other than the power law, such as the log-normal distribution [13]. A log-normal (LN) distribution is a normal distribution applied to the logarithm of the statistic. Curvature is evident in many of the plots meant to demonstrate the applicability of the power law to war death statistics [6], [7]. The log-normal is a symmetric distribution containing values less than zero, which is problematic because the number of deaths should be greater than or equal than one. A non-symmetric distribution favoring higher statistics is the log-gamma distribution. The log-gamma (LG) distribution, conspicuously absent from the literature about interstate combat deaths, is the gamma distribution applied to the logarithm of the statistic. It may be written as follows:

$$\mathrm{LG}[\log(x);a,b] = \frac{1}{\llbracket(a)\beta^a} \log(x)^{a-1} \varepsilon^{-\log(x)/\beta} \qquad (1)$$

The gamma function is the exponential distribution when $\alpha=1$.

C. Use in Monte Carlo Simulation

Distributions governing violent conflict are being combined with the Monte Carlo method to study conflict and cooperation. However, research continues to rely exclusively on the power law for the distribution of combat deaths [14]. While some research recognizes the applicability of the exponential and log-normal distribution to the times between violent events [15], their use has not been incorporated into Monte Carlo methods for modelling severity (i.e. deaths).

III. METHOD

Quantitative methods are necessary to build validated war analytics. The Correlates of War (COW) Project began publishing war data in 1963 and has continuously improved and added to this data ever since. Its data is quantitative and subject to careful quality control. Of interest to the present research is COW's interstate classification of wars based upon the status of territorial entities and focusing on those classified as members of the state system. This dataset exists within COW's War Data, 1816 - 2007 (v4.0) [16]. It includes wars that took place between or among recognized states where there are at least 1,000 fatalities. The data exists as rows of named wars that include start and end dates, combat deaths, outcome, and which state was the initiator. Because the focus of this study is strategic war, which requires a level of resources achievable only by nation states, other datasets were not used. The interstate dataset contains 91 named interstate wars from 1816 to 2007, most of which involve more than two states. Accordingly, COW data includes a row for each state in a named war, indicating when the state entered the war and on which side it fought as well as corresponding deaths.

Based merely on casualties, "asking who won a given war… is like asking who won the San Francisco earthquake" [17]. There is more to victory in war than simple casualty numbers. But to prevent judgements from intermingling with potential conclusions of the study, investigators removed themselves from that aspect and relied specifically on the determinations of the COW Project. Moreover, the Monte Carlo method requires that whatever factors causing a war and its severity are stipulated and will result in a statistical distribution. Despite being probabilistic, one can still learn about the severity of war to better understand the concept of war, its causes, and decision makers' willingness to begin a war or how they react to war. Critics of COW often state that the project fails to consider many of the classical – or qualitative – approaches to studying warfare and its numerous variables. Despite these alleged shortcomings, the investigators believe that COW data supports a quantitative approach to studying a large problem that may indeed have no immediate comprehensive explanation.

Common sense suggests that all combat deaths of all nations participating in a single named interstate war be treated as one statistic. However, this turns out to be wrong. In a Nash [18] zero-sum game where the objective is to minimize the maximum number of casualties, the so-called "minimax" strategy where there is no cooperation, each nation decides unilaterally which strategy to employ. Speaking about this situation, Schelling observed that a zero-sum "minimax strategy converts the situation into one involving two essentially unilateral decisions" [19]. Interstate war is thus strictly a unilateral risk calculation involving multiple belligerents as each state makes its own risk decision. As a result, each nation's losses in the war, not simply the total of a named war, are considered separate. A war in which three nations combat two enemy nations

therefore results in five rows of data rather than one. Thus, the 91 named COW interstate wars become 337 "unilateral decisions". The veracity of this claim is borne out by the results of the study.
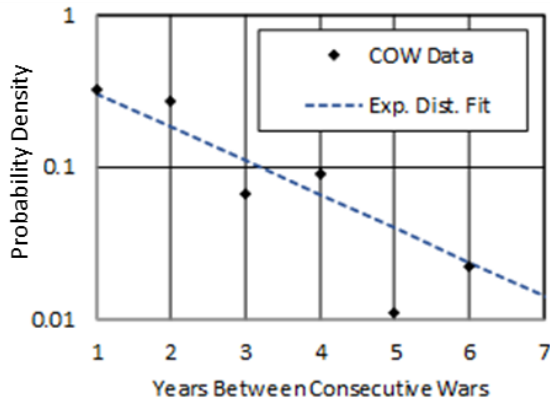

Fig. 1. Exponential distribution fit ($\lambda = 0.51$ wars/yr) to COW [16] Inter-State war data where fit goodness $R^2 = 0.93$.
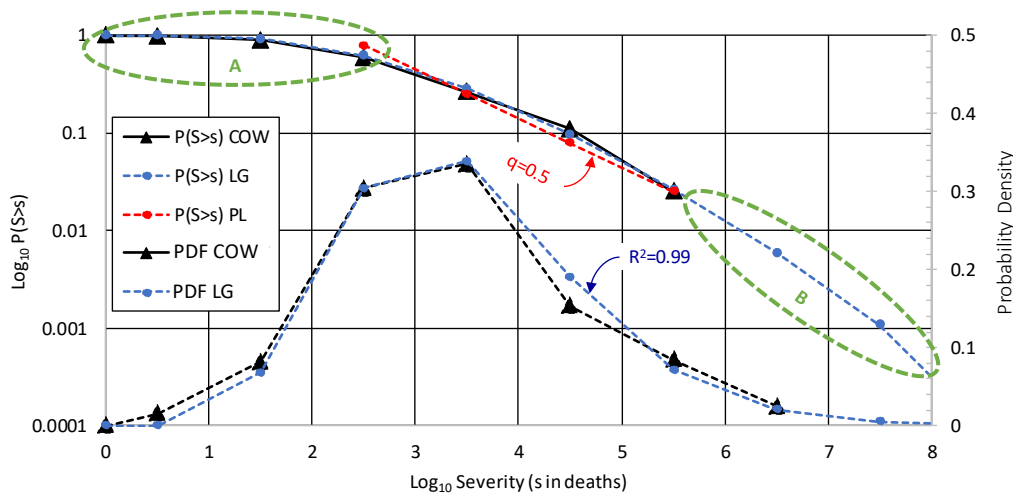
IV. DATA ANALYSIS AND DISCUSSION

The probability of war is distinct from the severity of war, the latter being dependent on the former. First, the temporal statistics of the 91 named interstate wars is analyzed, specifically counting the frequency of the number of years between consecutive wars. The result is that the time between wars follows an exponential distribution where there is on average one interstate war every two years, yielding an exponential distribution parameter $\lambda=0.5$ wars/yr. The data and exponential fit to the data is shown in Fig. 1. Because of a mathematical connection between the exponential and Poisson distributions, it is known that the probability of there being one or more wars per year follows the Poisson distribution with the same parameter. A Poisson distribution based on COW data predicts that in any given year there is a 31% chance that there will be one interstate war somewhere in the world. The average deaths and number of states participating in wars has remained nearly constant in the last 200 years. Thus, consideration of additional temporal changes across the dataset is not warranted.

Fig. 2. Probability density and exceedance probability *P(S>s)* curves comprised of COW [16] interstate war data and a parameterized log-gamma (LG) for all interstate wars from 1816 to 2007. The probability density functions (PDF) track the right-side scale and the exceedance curves track the left-side scale. A power-law (PL) fit with q=0.50 is shown for comparison. Circled areas A and B indicate where the power law is deficient.

A log-gamma probability density with parameters $\alpha = 9$ and $\beta = 0.39$ fits the COW data for all 337 data points as is indicated in Fig. 2 [20]. This is written as LG[$\alpha=9$, $\beta=0.39$]. The R^2 of the probability density fit compared to the COW data is 0.99, indicating an excellent fit. Equally important, the log-gamma fit holds for $s > 10^6$. This can be seen in the exceedance probability *P(S>s)* curve, derived from the probability density, that follows the logarithmic scale on the right-side of Fig. 2. A side effect of converting 91 named wars to 337 state wars is that the maximum number of deaths for a given war falls below 10^7. Because the *P(S>s)* curve follows the complement of the integral of density, there are also no data points for *P(S>10^7)*. This asymmetric log-gamma distribution, which favors the higher statistics, fits the data better than any log-normal distribution. Even though a log-normal provides an overall excellent fit, it fails to adequately fit the high-consequence portion of the data.

To check the fit for the most extreme values of combat deaths, the average slope of the power law and log-gamma curves between severities $s=10^4$ and $s=10^8$ were compared.

They are found to be in good agreement (0.62 versus 0.55). Thus, the log-gamma distribution fits the entire range of severity covered by the COW data when the wars are analyzed only by state (i.e. treating wars as unilateral decisions). Critically different than the power law, however, is that the slope of the log-gamma increases in negativity so that the distribution is valid for higher death values. Specifically, the slope of the *P(S>s)* curve between 10^8 and 10^9 is -3.0. As this slope is less than negative one, and subsequent slopes are decreasing, the fit is valid. The same cannot be said for the power law.

Much is gained by correcting the distribution of combat deaths. First and foremost, the log-gamma is valid as a probability distribution. The area under the curve and its mean are bounded and convergent with increasing deaths. The log-gamma covers all ranges of combat deaths where the power law does not. Specifically, the power law does not provide results for regions A and B identified by dashed green lines in Fig. 2. Region A covers combat deaths below 1,000. Certainly, one can expect and does observe that states suffer

interstate combat deaths below 1,000. However, they were not intended to be included in the COW interstate war dataset and only appear because the named wars were broken apart into their state components. Region B is the estimated combat deaths above ten million. Being able to estimate this region has important implications about the probability of wars not yet fought. The log-gamma fit can be used to estimate the probability of wars having 10M, 100M, and even 1B causalities. If there is an interstate war, it turns out that the probability of this many deaths is 2%, 0.5%, and 0.1%, respectively. The power law cannot be used to predict these probabilities. The fact that the log-gamma fits the data for all ranges of combat deaths is a strong indication of its correctness over the use of the power law.

Knowing a proper distribution for interstate war deaths, one can begin to ask questions about the underlying probabilities. In general, log-gamma distributions are the combination of different samples from an exponential distribution. The log-gamma distribution that fits all COW interstate war data, having two parameters α and β is generated by taking nine samples ($\alpha = 9.0$) from an exponential distribution of the logarithm of combat deaths having the same value for the beta rate parameter ($\beta = 0.39$). The significance is that the deaths in interstate war are a multiplicative combination of α independent random samples, or the addition of the logarithm of α samples. The exponential distribution used to create the log-gamma distribution is unique in that it is completely "memoryless", meaning that the past has no bearing on its future.
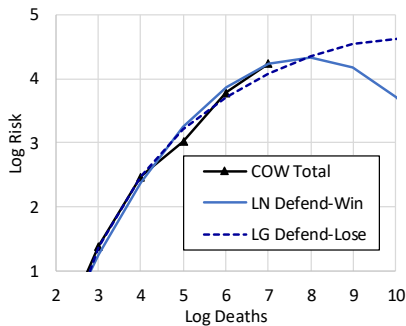

Fig. 3. Log of attack risk (expected deaths) vs. log deaths.

The statistics indicate that war behaves like a board game involving two different random spinners. Call the first the "war spinner" and the second the "severity spinner". The war spinner is governed by the Poisson distribution and the severity spinner is governed by the exponential distribution. Each year of the game, the first spinner is spun to decide if a war occurs. If war is the outcome, each player spins the severity spinner nine times. The results of the nine spins, which are the logarithm of the deaths, are added together to obtain the total combat deaths for that player. It does not matter who attacks or defends in this version of the game. On the other hand, if COW data is analyzed in terms of attack versus defend and win versus lose cases, one finds that the distribution of Fig. 2 is comprised of different distributions for each of these cases. Table I reports the individual distributions with win-lose and death statistics. When the game takes into account attack and defend strategies, it is the

same except that an attacker must be chosen, and the attacker and defender spin different severity spinners. The first row of the attack column in Table 1 indicates that the attacker would spin a log-gamma spinner with $\beta=0.39$ on average 8.6 times. What is more interesting is what probability distribution governs the severity spinner for the defend case. Overall, the best fit for defend is a log-gamma distribution. However, the defend-win case appears distinctly log-normal. The log-normal distribution is symmetric and, unlike the exponential distribution, is not "memoryless", suggesting that defend-win statistics are influenced by past wars and possible evidence of learning.

TABLE I: FITS TO COW [16] INTERSTATE WAR ATTACK AND DEFEND STATISTICS

		Attack 109 Wars (32%) 7.1M Deaths LG(α=8.6, β=0.39)	Defend 228 wars (68%) 25M Deaths LG(α=9, β=0.39)
Win 155 (44%)		61 (18%)	94 (28%)
		0.8M Deaths	17M Deaths (53%)
		LG(α=10,β=0.32)	LN(μ=3.5, σ=1.3)
Lose 119 (38%)		27 (9%)	92 (27%)
		5.4M Deaths	6.4M Deaths
		LG(α=8.0, β=0.48)	LG(α=11, β=0.31)
Other 63 (18%)		21 (6%)	42 (12%)
		0.9M Deaths	1.5M Deaths
		LG(α=17, β=0.20)	LG(α=12, β=0.24)
Total		337 Wars	
		32M Total Combat Deaths	
		LG(α=9, β=0.39)	

Despite the subtleties between attack and defend, the overall result is confirmed from the perspective of combat deaths: Strategic war is zero-sum, non-cooperative, and high-risk. Strategic war based on combat deaths is zero-sum because each participant starts with a number of combatants and loses some number of them. Neither side gains any combatants through enemy losses. The non-cooperative aspect of war is confirmed by having achieved excellent statistical results based on using all 337 rows of the 91 named interstate wars. The research team attempted but could not achieve these good results using only the 91 named wars. The high-risk nature is borne out by two results: (1) the distribution of combat deaths is based on the logarithm of war deaths and (2) the log of combat deaths follows a gamma distribution, which favors the right-side, high-severity part of the curve.

Monte Carlo simulation can be used to assess the risks of different strategies applied to interstate competition. However, the Monte Carlo technique depends on having valid probability distributions. The logarithmic distributions identified in Table I are valid in this context. In a Monte Carlo simulation, they can be used to estimate the combat deaths that are likely if a strategy fails to maintain peaceful competition. In turn, results from data-driven simulations can be used to train and test AI systems so they are reliable in recommending courses of action.

The importance of using the correct distribution is made clear in Fig. 3. It shows risk, the product of probability and deaths (i.e. expected deaths), as a function of deaths based on COW data. The LN and LG fits to this data are from Table 1. The risk for the LG fit to the Defend-Lose data continues to

rise beyond 10^{11}, whereas the LN fit to the Defend-Win data peaks at 10^8. Expected deaths in neither case, however, exceed 10^5.

The next phase of research will focus on the probability of war and possibility of it being conditional on factors such geographic proximity of states and/or their alliances. Results should allow Monte Carlo simulation of interstate competition strategies based on geography and/or alliances to estimate the risk of interstate war in terms of combat deaths.

V. CONCLUSION

Quantitative methods, not anecdotes, will be needed to train and test future military AI systems to help make decisions that avoid or minimize the risk of war. Richardson's use of the power law is such a quantitative method. However, use of the power law in this context is invalid. Here forward, it should be replaced by a log-gamma or log-normal distribution depending on the attack and defend strategy. Despite their shortcomings, COW Project and similar data seems underutilized. More emphasis should be placed on data-driven research because these are the foundation of future strategic AI technologies and made possible by the Monte Carlo technique. Research is ongoing to understand why the probability distributions for interstate war are logarithmic. In macroeconomic theory, wealth has a logarithmic utility, hinting that combat deaths exhibit the same sort of utility to a nation at war.

CONFLICT OF INTEREST AND DISCLAIMER

The authors declare no conflict of interest. The opinions, conclusions, and recommendations expressed or implied are the authors' and do not necessarily reflect the views of the Department of Defense or any other agency of the U.S. Federal Government, or any other organization.

AUTHOR CONTRIBUTION

V.H. Standley led technical content, F.G. Nuño led historic content, and J.W. Sharpe led strategy content. All authors approved the final version.
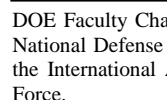
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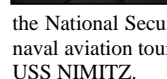
Vaughn H. Standley is from United States. He receive the Ph.D. physics Technical University of Vienna, Austria, the MS operations research George Mason University, MS nuclear engineering Air Force Institute of Technology, BS nuclear engineering Oregon State University.

He has been a program manager with the Department of Energy's (DOE) National Nuclear Security Administration since 2003, appointed DOE Faculty Chair to the College of Information in Cyberspace at the National Defense University in 2017, previously a nuclear inspector in the International Atomic Energy Agency and scientist in the U.S. Air Force.

Frank G. Nuño is from United States. He receive the MA history US Army Command and General Staff College, MS information and telecommunications systems Johns Hopkins University, BA history University of California Los Angeles.

He is active duty military faculty at the National Defense University's College of Information and Cyberspace. Prior to arriving at NDU, he served at the National Security Agency, the Defense Intelligence Agency, several naval aviation tours in nuclear command and control, and a tour onboard USS NIMITZ.

Jacob W. Sharpe is from United States. He receive the M.A. American University. He is a research assistant with the U.S. National Defense University College of Information and Cyberspace. Previously he worked with the Human Security Center and Atlantic Council. As a research assistant.