Fuzzy Inventory Models of Perishable Multi-Items for Integrated and Non-integrated Businesses with Possibility/Necessity Measure of Trapezoidal Fuzzy Goal

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***Abstract*—A multi–objective inventory models of deteriorating items have been developed with Weibull rate of decay allowing shortages, in which demand is taken as a function of time and production is proportional to demand rate. Here objectives are to maximize the profit from different items with space constraint on infinite planning horizon for non-integrated and integrated business. Objectives are also made fuzzy in nature for non- integrated business. The compromised solutions of the optimization problem are obtained by the application of Zimmermann's technique and Fuzzy Additive Goal Programming technique. Crisp and fuzzy weights are used to incorporate the relative importance of the objective and constraint goals. The models are illustrated numerically and the results of those models each with crisp and fuzzy weights are compared. The results for the model assuming them to be Single House Integrated Business (SHIB) are obtained by using Generalized Reduced Gradient method. The costs like cost per unit items, holding costs, set up costs, shortage costs, selling prices are taken in fuzzy environment as triangular fuzzy numbers and trapezoidal fuzzy numbers also. When costs are imprecise, optimistic and pessimistic equivalent of fuzzy objective function is obtained by using credibility measure of fuzzy event by taking fuzzy expectation. The problems have been solved by formulating them as a single objective with fuzzy costs. The results of fuzzy SHIB model is illustrated with numerical example and those are compared with the best possible solution of the non- integrated business.**

***Index Terms*—Multi–objective, crisp/fuzzy weights, multi–item, expected value with possibility/necessity.**

I. Introduction

In most of the inventory model, the production rate and demand rate are uniform throughout the period.

But it is usually observed in the market that sales of the fashionable goods, electronic gadgets, seasonable products, food- grains etc. change with time. So, the production rate and demand rate is varied. Demand varies according to the time, quality of items, festivals, weathers etc. For these reasons, dynamic models of production inventory systems have been considered and solved by assuming that demand is a continuous function of time which may increase or decrease with time and production depends on many factors like man power, introducing new technology, availability of new material, power supply, time, demand etc. A number of research papers have already been published on the topic in which production of many retail items is proportional to demand [1]-[3] and others]. Marketing researchers recognize that the production of many retail items is proportional to the demand along with factors like man power, new technology, power supply, time etc. [4]. Many times, the holding cost of perishable items increases with time. [5] developed an inventory model for deteriorating items with price-dependent demand and time-varying holding cost.

Recently much work has been done regarding inventory models for deteriorating items. The life–time of perishable items like perfumes, medicines, blood etc. are fixed and they cannot be used after the date of expiry. [6] have reviewed inventory models for deteriorating items. [7]-[9] have been developed some inventory models for perishable items like perfumes, medicines, blood etc. which have the fixed lifetime and cannot be used after the expired date. [10] developed an inventory model with Weibull rate of decay having selling price dependent demand. However, they considered the case of instantaneous replenishment. In many practical situations like food processing industries, photochemical industries, the production is not instantaneous. [11] discussed a perishable inventory model with finite rate of replenishment having Weibull lifetime and price dependent demand. [12] developed an inventory model for items with Weibull ameliorating. [13] gave a model on Weibull distributed deterioration. [14], [15] and others presented inventory models with Weibull distribution deterioration, time-varying demand and shortages.

In many problems more than one objective can be considered. But there is a multi item multi objective inventory model for integrated and non–integrated business developed by [16]. [17] Developed a multi–objective fuzzy inventory model of deteriorating items with available storage area. To give the relative importance to the objective they have assigned cardinal weights (crisp/fuzzy). [19] and others presented a model with necessity/ possibility constraint by using expected value of fuzzy variable. Muti item classical inventory model under resource constraints such as capital investment, available storage area, number of orders and available setup time etc. are presented in well known books [19], [20] and others]. Taking space limitation as constraints several workers [21], [22] have considered multi item inventory models in crisp and fuzzy environments. [4], [23] have developed two inventory models, in the first model, the production rate is assumed to be a function of the on hand inventory level and in the second model, the production rate is assumed to be a function of demand rate.

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Nowadays, almost every important real world problem involves more than one objective. So, decision makers try to model them as multi criteria decision making (MCDM) problems identifying the different criteria. The importance of such models is to produce the best alternative satisfying the objectives and the constraint which best fulfills the requirement of decision makers. Recently various new methods [cf. [24]-[26] have been outlined to find the compromise solutions of MCDM problems.

In many realistic situations, it is difficult to assign precise aspiration levels to objectives. Moreover, in some cases, it is not even possible to articulate precise boundaries of the constraints. In such situations a fuzzy goal model is more appropriate. In these cases, normally, linear and non- linear shapes for the membership functions of the fuzzy objective and constraint goals are proposed. To reflect the decision maker's performances regarding the relative importance of each objective goal, crisp / fuzzy weights are used [cf. [27]. The fuzzy priorities may be "linguistic variables" such as "very important", "moderately important" and "important". Membership functions can be defined for these fuzzy priorities in order to develop a combined measure of the degree to which the different goals are attended. Recently, [28] presented a multi-objective inventory model of deteriorating items with space constraint in a fuzzy environment. Till now the cost of items are taken as constant, but in real life situations these costs may be of imprecise type. The costs of the items may decrease or increase according to the demand and stock etc. That's why the costs (units holding setup shortage deterioration) are taken in fuzzy environment as triangular and trapezoidal fuzzy numbers. The possibility/necessity and credibility measures of objective are also considered. [29], [30] introduced the necessity and possibility constraints which are very relevant to the real life decision making problems and presented the process of defuzzification for their constraints. Roy et.al [31], [32] and [18] have developed models with necessity and possibility constraints by using expected value of fuzzy variables [cf.[33]].

In this paper, a multi–objective inventory model of deteriorating items have been developed with Weibull rate of decay allowing shortages, in which demand is taken as a function of time, and production is proportional to demand rate. Here the objectives are to maximize the profit from different deteriorating items with space constraint on infinite planning horizon for non- integrated and integrated business. It is assumed that the deteriorating rates of different items follow the two parameter Weibull distributions and observed that the deteriorating cost along with distribution parameters have a tremendous influence on the optimal profit for both type of businesses. The objectives for profit maximization for each item are separately formulated. These objectives are also made fuzzy in nature for non-integrated business. The impreciseness of inventory parameters and goals for non-integrated business has been expressed by linear membership functions. The compromised solutions of the multi–objective non-linear optimization problem are obtained by the application of two different fuzzy optimization methods – (i) Zimmermann's technique and (ii) Fuzzy Additive Goal Programming technique (FAGP) based on gradient method. Crisp and fuzzy weights are used to incorporate the relative importance of the objective and constraint goals. The models are illustrated numerically and the results of those models each with crisp and fuzzy weights are compared.

The results for the model assuming them to be Single House Integrated Business (SHIB) are obtained by using Generalized Reduced Gradient method (GRG). Till now the cost of items are taken as constant. But in real life situation these cost may be of imprecise type. So uncertainty is to be imposed, that's why in this paper the costs like cost per unit items, holding costs, set up costs, shortage costs, selling prices are taken in fuzzy environment as triangular fuzzy numbers and for more realistic situations, these are taken as trapezoidal fuzzy numbers also. The fuzzy SHIB model with imprecise inventory cost is formulated to optimize the possibility necessity measure of fuzzy goal of the objective function. When costs are imprecise, optimistic and pessimistic equivalent of fuzzy objective function is obtained by using credibility measure of fuzzy event by taking fuzzy expectation. Also the problems have been solved by formulating them as a single objective with crisp and fuzzy costs. The results of crisp and fuzzy SHIB model is illustrated with numerical example and those are compared with the best possible solution of the non- integrated business.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations have been used in developing the model:

1) n = number of items.
2) Shortages are allowed and backlogged.
3) The lead time is zero.
4) Planning horizon is infinite.
5) W= available floor or shelf – space (sq. ft.),
   For i – th (i = 1, 2, 3………, n) item.
6) W^i = storage space required per unit item (sq. ft.),
7) K_i = stock level at time t_{1i},
8) S_i = shortages level at time t_{3i},
9) Items are deteriorated and its rate is $\theta_i(t)$ i.e.,
   $\theta_i(t) = \alpha_i \beta_i t^{(\beta_i - 1)}$, $0 < \alpha_i < 1$, $t > 0$, $\beta_i \geq 1$.
   Holding cost $h_i(t)$ units time is linearly time dependent i.e., $h_i(t) = h_{1i} + h_{2i}t$ where h_{1i} and h_{2i} are positive constants ($).
10) T_i = the time period for each cycle (years),
11) H_i = the shortage cost per unit per time ($),
12) C_i = the ordering cost per cycle ($),
13) Time dependent demand at a time t is defined as $R_i(t) = a_i - b_i t$, $a_i > 0$ and b_i ($0 < b_i \leq 1$) are constants.
14) N_i = the purchase cost per unit per time ($). Also, selling price per unit is $m_i N_i$, where $m_i > 1$,
15) I_i = the total average profit ($),
16) $q_i(t)$ be the inventory level at time t in cycle (0, T_i),
17) Demand dependent production at time t is
    $P_i = \gamma_i R_i(t)$, $\gamma_i > 1$.
18) K= space covered by all the items (sq. ft.) i.e. $K = \sum_{i=1}^{n} K_i w^i$,
19) I = profit in totality ($) i.e. $I = \sum_{i=1}^{n} I_i$.

III. Model and Formulations

Here, inventory model for i-th deteriorating item is shown in Fig.1. Initially, the stock is assumed to be zero. Demand dependent production starts at t=0 and simultaneously supply also starts to satisfy the time dependent demand $R_i(t)$. At $t = t_{1i}$, the stock level reaches K_i units. The production is then stopped. The inventory accumulated during the production period (0, t_{1i}) after meeting the demand during the period and the deterioration, the inventory reaches to the zero level at time $t = t_{2i}$. Now, the shortages are accumulated to the level S_i at time $t = t_{3i}$ and demand dependent production starts with the time dependent demand $R_i(t)$. The backlog is filled during the time (t_{3i}, T_i), till the backlog becomes zero. The cycle then repeats itself after time T_i.

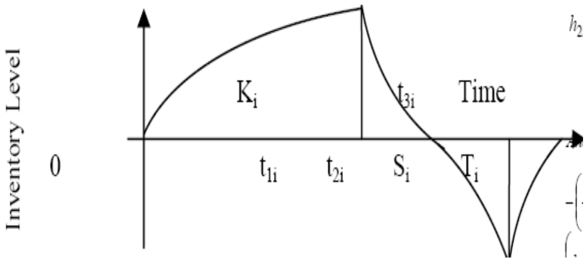

Fig. 1. Pictorial representation of the inventory system for the i-th item.

The differential equations describing the inventory level $q_i(t)$ of i-th item in the interval $0 \le t \le T_i$, is given by:

$$\frac{dq_i(t)}{dt} + \theta(t)q_i(t) = (\gamma_i - 1)R_i(t), 0 \le t \le t_{1i} \tag{1}$$

$$\frac{dq_i(t)}{dt} + \theta(t)q_i(t) = -R_i(t), t_{1i} \le t \le t_{2i} \tag{2}$$

$$\frac{dq_i(t)}{dt} = -R_i(t), t_{2i} \le t \le t_{3i} \tag{3}$$

$$\frac{dq_i(t)}{dt} + \theta(t)q_i(t) = (\gamma_i - 1)R_i(t), t_{3i} \le t \le T_i \tag{4}$$

The conditions are , $q_i(t) = 0$ at $t = 0$, t_{2i} and T_i ; $q_i(t) = K_i$ at $t = t_{1i}$. Using the conditions, the solution of (1) is

where

$$K_i = (\gamma_i - 1)\left(a_i t_{1i} - \frac{b_i t_{1i}^2}{2} - \frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+1)}}{(\beta_i+1)} + \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+2)}}{2(\beta_i+2)}\right) \tag{6}$$

$$and, q_i(t) = \left(1 - \alpha_i t^{\beta i}\right)\left(a_i t_{2i} - \frac{b_i t_{2i}^2}{2} + \frac{a_i\alpha_i\beta_i t_{2i}^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i\alpha_i\beta_i t_{2i}^{(\beta_i+2_i)}}{2(\beta_i+2)}\right) - \left(a_i t - \frac{b_i t^2}{2} - \frac{a_i\alpha_i\beta_i t^{(\beta_i+1)}}{(\beta_i+1)} + \frac{b_i\alpha_i\beta_i t^{(\beta_i+2_i)}}{(\beta_i+2)}\right) \tag{7}$$

Now, the solutions of (3) and (4) are:

$$q_i(t) = \left(a_i t_{2i} - \frac{b_i t_{2i}^2}{2}\right) - \left(a_i t - \frac{b_i t^2}{2}\right) \tag{8}$$

$$q_i(t) = (\gamma_i - 1)\left(a_i t - \frac{b_i t^2}{2} - \frac{a_i\alpha_i\beta_i t^{(\beta_i+1)}}{(\beta_i+1)} + \frac{b_i\alpha_i\beta_i t^{(\beta_i+2_i)}}{2(\beta_i+2)}\right) - (\gamma_i - 1)\left(1 - \alpha_i t^{\beta i}\right)\left(a_i Ti - \frac{b_i Ti^2}{2} + \frac{a_i\alpha_i Ti^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i\alpha_i Ti^{(\beta_i+2_i)}}{(\beta_i+2)}\right) \tag{9}$$

Deteriorating units = DE_i = (DE_{1i} + DE_{2i}), where deteriorating units in (0, t_{1i}) and (t_{1i}, t_{2i}) are :

$$DE_{1i} = \int_{0_{1i}}^{t_{1i}} \alpha_i\beta_i t^{(\beta_i-1)} q_i(t)dt = (\gamma_i - 1)\left(\frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+2)}}{2(\beta_i+2)}\right) \tag{10}$$

$$DE_{2i} = \int_{t_{1i}}^{t_{2i}} \alpha_i\beta_i t^{(\beta_i-1)} q_i(t)dt, = \frac{a_i\alpha_i t_{2i}^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i\alpha_i t^{(\beta_i+2_i)}}{(\beta_i+2)} - \alpha_i t_{1i}^{(\beta_i)}\left(a_i t_{2i} - \frac{b_i t_{2i}^2}{2}\right) + \left(\frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+2)}}{2(\beta_i+2)}\right) \tag{11}$$

Holding cost = $HC_i = HC_{1i} + HC_{2i}$, where holding costs in (0, t_{1i}) and (t_{1i}, t_{2i}) are:

$$HC_{1i} = \int_{0_{1i}}^{t_{1i}} h_i(t)q_i(t)dt = h_{1i}(\gamma_i - 1)\left(\frac{a_i t_{1i}^2}{2} - \frac{b_i t_{1i}^3}{6} - \frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+2)}}{(\beta_i+1)(\beta_i+2)} + \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+3)}}{2(\beta_i+2)(\beta_i+3)}\right) + h_{2i}(\gamma_i - 1)\left(\frac{a_i t_{1i}^3}{3} - \frac{b_i t_{1i}^4}{8} - \frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+3)}}{(\beta_i+1)(\beta_i+3)} + \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+4)}}{2(\beta_i+2)(\beta_i+4)}\right) \tag{12}$$

$$\begin{aligned} And, HC_{2i} &= \int_{t_{1i}}^{t_{2i}} h_i(t)q_i(t)dt = \frac{h_{1i}a_i t_{2i}^2}{2} - \left(\frac{b_i h_{1i} t_{2i}^3}{3} - \frac{a_i h_{2i} t_{2i}^3}{6}\right) + \frac{b_i h_{2i} t_{1i}^4}{8} + \frac{h_{1i}a_i\alpha_i\beta_i t_{2i}^{(\beta_i+2)}}{(\beta_i+1)(\beta_i+2)} \\ &- \left(\frac{h_{1i}a_i\alpha_i\beta_i}{(\beta_i+1)(\beta_i+2)} - \frac{h_{2i}a_i\alpha_i\beta_i}{2(\beta_i+2)(\beta_i+3)}\right)t_{2i}^{(\beta_i+3)} - \frac{h_{2i}b_i\alpha_i\beta_i t_{2i}^{(\beta_i+4)}}{2(\beta_i+2)(\beta_i+4)} - \\ &\left(h_{1i}t_{1i} - \frac{h_{1i}\alpha_i t_{1i}^{(\beta_i+1)}}{(\beta_i+1)} + \frac{h_{1i}t_{1i}^2}{2} - \frac{h_{1i}\alpha_i t_{1i}^{(\beta_i+2)}}{(\beta_i+2)}\right)\left(a_i t_{2i} - \frac{b_i t_{2i}^2}{2} + \frac{a_i\alpha_i t_{2i}^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i\alpha_i t_{2i}^{(\beta_i+2)}}{(\beta_i+2)}\right) \\ &+ h_{1i}\left(\frac{a_i t_{1i}^2}{2} - \frac{b_i t_{1i}^3}{6} - \frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+2)}}{(\beta_i+1)(\beta_i+2)} + \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+3)}}{2(\beta_i+2)(\beta_i+3)}\right) \\ &+ h_{2i}\left(\frac{a_i t_{1i}^3}{3} - \frac{b_i t_{1i}^4}{8} - \frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+3)}}{(\beta_i+1)(\beta_i+3)} + \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+4)}}{2(\beta_i+2)(\beta_i+4)}\right) \end{aligned} \tag{13}$$

Total shortage cost in (t_{2i}, T_i) = SC_i = $H_i(SC_{1i})$, where inventory level in (t_{3i},T_i) are :

$$SC_{1i} = H_i\int_{t_{2i}}^{t_{3i}} q_i(t)dt = H_i\left(\frac{a_i t_{3i}^3}{2} - \frac{b_i t_{3i}^3}{3}\right) - \left(a_i t_{3i} - \frac{b_i t_{3i}^2}{2}\right)t_{2i} + \frac{a_i t_{2i}^2}{2} + \frac{b_i t_{2i}^3}{6} \tag{14}$$

Relations between t_{1i}, t_{2i}, t_{3i} and T_i (by equality conditions at t_{1i} and t_{3i}) are:

$$\gamma_i\left(a_i t_{1i} - \frac{b_i t_{1i}^2}{2} - \frac{a_i\alpha_i\beta_i t_{1i}^{(\beta_i+1)}}{(\beta_i+1)} + \frac{b_i\alpha_i\beta_i t_{1i}^{(\beta_i+2)}}{2(\beta_i+2)}\right) = \left(1 - \alpha_i t_{1i}^{\beta_i}\right)\left(a_i t_{2i} - \frac{b_i t_{2i}^2}{2} + \frac{a_i\alpha_i t_{2i}^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i\alpha_i t_{2i}^{(\beta_i+2)}}{(\beta_i+2)}\right) \tag{15}$$

$$And, \left(a_i t_{2i} - \frac{b_i t_{2i}^2}{2}\right) = (\gamma_i - 1)\left(-\frac{a_i \alpha_i \beta_i t_{1i}^{(\beta_i+1)}}{(\beta_i+1)} + \frac{b_i \alpha_i \beta_i t_{1i}^{(\beta_i+2)}}{2(\beta_i+2)}\right) + \gamma_i \left(a_i t_{3i} - \frac{b_i t_{3i}^2}{2}\right) - (\gamma_i - 1)\left(a_i T_i - \frac{b_i T_i^2}{2} + \frac{a_i \alpha_i T_i^{(\beta_i+1)}}{(\beta_i+1)} - \frac{b_i \alpha_i T_i^{(\beta_i+2)}}{(\beta_i+2)}\right) + (\gamma_i + 1)\left(a_i T_i - \frac{b_i T_i^2}{2}\right)\alpha_i t_{3i}^{\beta_i} \tag{16}$$

Now, selling price - production cost = $(m_i - 1) N_i (R_i + Q_i)$ (17)

$$where, R_i = \int_0^{t_{1i}} \gamma_i R_i(t) dt = \gamma_i \left(a_i t_{1i} - \frac{b_i t_{1i}^2}{2}\right) \tag{18}$$

$$and, Q_i = \int_{t_{3i}}^{T_i} \gamma_i R_i(t) dt = \gamma_i \left(a_i T_i - \frac{b_i T_i^2}{2}\right) - \gamma_i \left(a_i t_{3i} - \frac{b_i t_{3i}^2}{2}\right) \tag{19}$$

Total average profit per unit time,

I_i = (selling price- production cost–holding cost–shortage cost–deteriorating cost– set up cost)/T_i; (20)

Here, our objective is to maximize the above profit for three items with the following types of models.

IV. Types of Models

There are four types of models.

Model 1–Crisp, Integrated; Model 2-Fuzzy, Integrated; Model 3-Crisp, Non-integrated; and Model 4-Fuzzy, Non-integrated.

A. Model 1–Crisp, Integrated

Assuming that the items are dealt collectively as a single integrated business process, the corresponding single objective model is to

Maximize $\{I_1, I_2, I_3\}$

Subject to, equations (15), (16),

$$K \le W, and\ t_{2i} \ge 0, i = 1,2,3 \tag{21}$$

B. Model 2-Fuzzy, Integrated

In practical situation, every cost is imprecise, so we take $\{N_i, C_i, H_i, h_{1i};\ i = 1, 2 \text{ and } 3\}$ as fuzzy numbers i.e. as $\{\tilde{N}_i, \tilde{C}_i, \tilde{H}_i, \tilde{h}_{1i}; i = 1,2, and 3\}$. Then due to this assumption, the crisp functions (I, I_1, I_2, I_3) will become the fuzzy functions $(\tilde{I}, \tilde{I}_1, \tilde{I}_2, \tilde{I}_3)$. The optimization of a fuzzy objective is not well defined. So instead of $(\tilde{I}, \tilde{I}_1, \tilde{I}_2, \tilde{I}_3)$ one can optimize its equivalent optimistic and pessimistic returns. Accordingly optimization of this model - 2 can be described as follows:

To optimize the optimistic and pessimistic equivalent of $(\tilde{I}, \tilde{I}_1, \tilde{I}_2, \tilde{I}_3)$ by lemma-1 (c.f. **Appendix-A**), the problem reduces to,

Maximize $E(\tilde{I}) = \sum_{i=1}^{3} E(\tilde{I}_i)$ Subject to, equations (15), (16),

$$K \le W, and\ t_{2i} \ge 0, i = 1,2,3 \tag{22}$$

in which, $\{E(\tilde{N}_i), E(\tilde{C}_i), E(\tilde{H}_i), E(\tilde{h}_{1i}); i = 1,2, and 3\}$ are expected values of $\{\tilde{N}_i, \tilde{C}_i, \tilde{H}_i, \tilde{h}_{1i}; i = 1,2, and 3\}$ respectively. If $\tilde{N}_i$ be a triangular fuzzy number i.e. $\tilde{N}_i = (N_{i1}, N_{i2}, N_{i3})$ then E ($\tilde{N}_i$) = (½) $[(1-\rho)N_{i1} + N_{i2} + \rho N_{i3}]$ and if $\tilde{N}_i$ be a trapezoidal fuzzy number i.e. $\tilde{N}_i = (N_{i1}, N_{i2}, N_{i3}, N_{i4})$, E ($\tilde{N}_i$) = (½) $[(1-\rho)(N_{i1} + N_{i2}) + \rho(N_{i3} + N_{i4})]$,

where ρ (0 < ρ < 1) is the managerial attitude factor. ρ = 1 and ρ = 0 represent most optimistic and pessimistic attitude respectively and ρ = 0.5 represents the credibility measure. Similar will be the case for $\{\tilde{C}_i, \tilde{H}_i, \tilde{h}_{1i}; i = 1,2, and 3\}$.

C. Model 3-Crisp, Non-Integrated

In crisp environment multi-objective production inventory problem with space constraint is to

Maximize $\{I_1, I_2, I_3\}$

Subject to, equations (15), (16),

$$K \le W, and\ t_{2i} \ge 0, i = 1,2,3 \tag{23}$$

D. Model 4-Fuzzy, Non-Integrated

When the above average profit of every item and availability of space area become fuzzy, the said crisp model (20) is transformed to a fuzzy model as:

Maximize $\{I_1, I_2, I_3\}$

Subject to, equations (15), (16) and

$$K \le \tilde{W}, and\ t_{2i} \ge 0, i = 1,2,3 \tag{24}$$

V. Multi-Objective Mathematical Programming

A general multiple objective non-linear programming problems are of the following form:

Minimize $F(x) = [f_1(x), f_2(x), \text{----------}, f_k(x)]$

Subject to $g_t(x) \le c_t$, t=1,2,…..,T

$h_j(x) = b_j$, j=1,2,…..,J

$x \in S$, where $S = [x / x \in Rn]$.

Here, $x = [x_1, x_2, \text{----------}, x_n]^T$ is an n-dimensional vector of decision variables, $f_1(x)$, $f_2(x)$, --------, $f_k(x)$ are k distinct objective functions. S is the set of feasible solutions. An optimal solution of a single objective problem is defined as one that minimizes the objective function $f_i(x)$, subject to the constraint set $x \in S$. To define a vector minimal point at which all components of the objective function vector $f(x)$ are simultaneously minimized, is not an adequate generalization. Since such points are seldom attainable. Zimmerman [1978] showed that fuzzy programming technique can be used effectively to solve the multi-objective programming problem as follows:

VI. Fuzzy Programming Technique To Solve Crisp Multi-Objective Problem

The above multi-objective programming problem (22) is defined completely in crisp environment. To solve this crisp problem by fuzzy technique, we first have to assign two values U_k and L_k as upper and lower bounds of the k-th objective for each k = 1, 2, 3. Here L_k = aspired level of achievement, U_k= higher acceptable level of achievement and $d_k = U_k - L_k$ = degradation allowance. The steps of the fuzzy programming technique are as follows:

Step-1: Each objective function I_1, I_2 and I_3 of the multi–objective programming problem (22) is optimized separately subject to the constraint of the problem (22). Let these optimum values be I_1^*, I_2^* and I_3^*.

Step-2: At each optimal solution of the three single-objectives programming problem solved in step-1 find the value of the remaining objective functions and construct a pay-off matrix of order 3 × 3 as follows:

$$\begin{array}{c|ccc} & I_1 & I_2 & I_3 \\ \hline t_{21} & I_1^*(t_{21}) & I_2(t_{21}) & I_3(t_{21}) \\ t_{22} & I_1(t_{22}) & I_2^*(t_{22}) & I_3(t_{22}) \\ t_{23} & I_1(t_{23}) & I_2(t_{23}) & I_3^*(t_{23}) \end{array}$$

From the pay-off matrix, find lower bounds LI_1, LI_2, LI_3, and upper bounds UI_1, UI_2, UI_3 as follows: the lower bounds $LI_1 = \text{Min}\{ I_1(t_{21}), I_1(t_{22}), I_1(t_{23})\}$, $LI_2 = \text{Min}\{ I_2(t_{21}), I_2(t_{22}), I_2(t_{23})\}$, $LI_3 = \text{Min}\{ I_3(t_{21}), I_3(t_{22}), I_3(t_{23})\}$. And the upper bounds $UI_1 = \text{Max}\{ I_1(t_{21}), I_1(t_{22}), I_1(t_{23})\}$, $UI_2 = \text{Max}\{ I_2(t_{21}), I_2(t_{22}), I_2(t_{23})\}$, $UI_3 = \text{Max}\{ I_3(t_{21}), I_3(t_{22}), I_3(t_{23})\}$.

Step-3: To solve this crisp problem by Zimmermann [1978] method, we take the membership functions μ_{I_1}, μ_{I_2}, and μ_{I_3} respectively of the objective functions I_1, I_2, I_3 in the linear form as follows:

$$\mu_{I_i} = \begin{cases} 1 \; for \; I_i > UI_i; \\ \dfrac{I_i - LI_i}{UI_i - LI_i} \quad for \; LI_i \le I_i \le UI_i; \\ 0 \; for \; I_i < LI_1; \end{cases} \quad i=1,2,3.$$

Step-4: Using the above membership functions, the crisp non-linear programming model (22) is formulated and solved by Zimmermann's technique and Additive Goal Programming technique.

VII. Fuzzy Non-Linear Programming (FNLP) Algorithm To Solve Fuzzy Multi–Objective Problem

Taking the profit goal as B_i with tolerance P_i (i = 1,2,3) and space constraint goal as W with tolerance P_w the linear membership functions μ_i (i = 1,2,3) and μ_w for three objectives and one constraint are as follows:

$$\mu_{w_i} = \begin{cases} 0 \; for \; I_i < B_i - P_i; \\ 1 - \dfrac{B_i - I_i}{P_i} \quad for \; B_i - P_i \le I_i \le B_i; \\ 1 \; for \; I_i > B_i; \end{cases}$$

$$\mu_{w_4} = \begin{cases} 1 \; for K < W; \\ 1 - \dfrac{K - W}{P_W} \quad for W \le K \le W + P_W; \\ 0 \; for K > W + P_W; \end{cases}$$

where, $K = K_1 w^1 + K_2 w^2 + K_3 w^3$,

Using the above membership functions, the fuzzy non-linear programming model (23) is formulated and solved by following the methods of Zimmermann (1978).

VIII. Crisp Weights

Sometimes decision makers are able to provide crisp relative weights for objective goals to reflect their relative importance. Here, positive crisp weights w_i (i = 1,2,…….., n) for crisp model are used which can be normalized by taking $\sum_{i=1}^{n} w_i = 1$. To achieve more importance of the objective goal we chose suitable inverse weight in the fuzzy non-linear programming technique. Similarly, in fuzzy inventory model we may choose the smallest of the inverse weighted membership function corresponding to the most important objective goal.

IX. Fuzzy Weights

When the decision maker can only provide linguistic or imprecise weight (e.g. profit of first objective is very important, profit of second objective is very important etc), we may use fuzzy weights according to Narsimhan [1980]. Here, membership functions of fuzzy weights are introduced to develop a combined measure of the degree to which objective goals are attended. Let μ_{w_i} $\{\mu_i(x)\}$ represent the weighted contribution of the i-th goal to the overall aggregated objective, where μ_{w_i} $(\mu_i(x))$ is the membership function corresponding to the fuzzy weights associated with the i-th goal. Then by using min operation, the membership function μ_D (x) of the decision (D) is: $\mu_D(x) = \mu_{w_1}(\mu_1(x)) \wedge \mu_{w_2}(\mu_2(x)) \wedge \text{--------} \wedge \mu_{w_n}(\mu_n(x)) = \min\{\mu_{w_1}(\mu_1(x)), \mu_{w_2}(\mu_2(x)), \text{---------} \mu_{w_n}(\mu_n(x))\}$

The maximized decision x^* is obtained by: $\mu_D(x^*) = \max\{\min\{\mu_{w_i}(\mu_i(x))\}\}$, i= 1, 2,……,n.

Note that the membership functions of fuzzy weights are functions of the membership function of the goal. The rationality for constructing these membership functions is that the more important the goals are, the higher are the degrees of their membership, and so the higher are the membership grade of their fuzzy weights.

X. Classification and Formulation of Model 3 (Crisp, Non-Integrated)

This model is further developed with crisp and fuzzy weights as follows:

Crisp weighted crisp model,

Fuzzy weighted crisp mode

A. Crisp Weighted Crisp Model

Let w_1, w_2, and w_3 are the intuitive crisp weights for the first item, second item and third item respectively. Then the model in (23) can be formulated by using two techniques: Zimmermann's technique and Additive Goal Programming technique as follows:

(i) Zimmermann's model:

Maximize λ

Subject to, $w_i\left(\dfrac{I_i-LI_i}{UI_i-LI_i}\right)\ge\lambda;$ equations (15), (16),

$$K\le W, and\ t_{2i}\ge 0, i=1,2,3; \sum_{i=1}^{3} w_i=1,\ 0\le\lambda\le 1. \tag{25}$$

(ii) Additive Goal Programming model:

Maximize $V(\lambda_1,\lambda_2,\lambda_3)=w_1\lambda_1+w_2\lambda_2+w_3\lambda_3$

Subject to, $\dfrac{I_i-LI_i}{UI_i-LI_i}\ge\lambda_i$, equations (15), (16),

$$K\le W, and\ t_{2i}\ge 0, i=1,2,3, \sum_{i=1}^{3} w_i=1,\ 0\le\lambda_i\le 1. \tag{26}$$

Observing the optimal results of Zimmermann's model and Additive Model with crisp weights, we have developed further only Zimmermann's model with crisp and fuzzy weights in fuzzy environments and with fuzzy weights in crisp environments.

B. Fuzzy Weighted Crisp Model

let w1, w2, and w3 are the intuitive fuzzy weights for the first item, second item and third item respectively, then the model (23) can be formulated by using Zimmermann's technique as follows:

Maximize λ

Subject to, $\mu_{w_i}\left(\dfrac{I_i-LI_i}{UI_i-LI_i}\right)\ge\lambda;$ equations (15), (16),

$$K\le\tilde{W}, and\ t_{2i}\ge 0, i=1,2,3,\ 0\le\lambda\le 1. \tag{27}$$

XI. Classification and Formulation of Model 4 (Fuzzy, Non-Integrated)

This model is further developed with crisp and fuzzy weights as follows:

(a) Crisp Weighted Fuzzy Model

(b) Fuzzy Weighted Fuzzy Model

A. Crisp Weighted Fuzzy Model

Let w_1, w_2, w_3 and w_4 are the intuitive crisp weights for the first item, second item, third item and floor space respectively, and then the model (24) can be formulated by using Zimmermann's technique as follows:

Maximize λ

Subject to, $w_i\left(1-\dfrac{B_i-I_i}{P_i}\right)\ge\lambda; w_4\left(1-\dfrac{K-W}{P_W}\right)\ge\lambda;$

$w_1+w_2+w_3+w_4=1$, equations (15), (16),

$$K\le W, and\ t_{2i}\ge 0, i=1,2,3,\ 0\le\lambda\le 1. \tag{28}$$

B. Fuzzy Weighted Fuzzy Model

Let w_1, w_2, w_3 and w_4 are the intuitive fuzzy weights for the first item, second item, third item and floor space respectively, then the model (24) can be formulated by using with crisp weights i.e. (25) and (26) are presented in TABLES III and IV respectively.

Zimmermann's technique as follows:

Maximize λ

Subject to, $\mu_{w_i}\left(1-\dfrac{B_i-I_i}{P_i}\right)\ge\lambda; \mu_{w4}\left(1-\dfrac{K-W}{P_W}\right)\ge\lambda;$

$K_1w^1+K_2w^2+K_3w^3\le W$, equations (15), (16), $0\le\lambda\le 1$,

$$t_{2i}\ge 0,\ i=1, 2, 3. \tag{29}$$

XII. Illustration of the Models

To illustrate the above crisp and fuzzy models for integrated and non-integrated businesses, we assume the following input data shown in Table I.

Optimal results of non-integrated businesses Using LINGO Software are given below:

For the above data, the following pay-off matrix (cf. TABLE II) is constructed and then the optimum results for the different representations of the crisp inventory model

Here, optimum results of the crisp model by two different methods are presented. In each method, four different cases have been made out, depending upon the relative importance given among the three objectives. In case-1 equal weightage to all objectives; in case–2, more importance to 1st objective than the other two objectives; in case-3, more care to maximization of 2nd objective than others; and similarly in case-4, 3rd objective received more weightage than others. As expected, case-2 gives maximum return when maximum attention is paid to the 1st objective; similarly case–3 and case–4 give better results if the decision maker gives maximum importance to the maximization of 2nd and3rd objectives respectively.

Now, we find the optimum results of the crisp inventory model with fuzzy weights i.e. (26), which are shown in TABLE V.

Fuzzy data: $\tilde{I}_1$ = ($219,$243), $\tilde{I}_2$ = ($663,$781), $\tilde{I}_3$ = ($1630,$1660), $\tilde{W}$ =(sq.ft.175,sq.ft.225)with input data.

Here, two different cases have been made out, depending upon the relative importance given among the three objectives. In case-1 equal fuzzy weightage to all the objectives; in case–2, more importance to 1st objective than the other two objectives. As expected, both case 1 and 2 of fuzzy weighted crisp model give better return than the crisp weighted crisp model **(**Zimmermann's Model) in totality and when special attention is paid to a particular objective respectively.

Now, we find the optimum results of the fuzzy objectives i.e. (27) with the crisp weights, which are shown below:

The pay-off matrix is made (by equating t_{2i} of the rest two objectives with t_{2i} of the objective considered) in TABLE II. The optimum results of the crisp weighted crisp models are presented in TABLE III and IV respectively. TABLE III: in case-1 equal weightage to all objective; case-2 gives maximum return when maximum attention is paid to the 1st objective; similarly case-3 and case-4 give better results if DM gives maximum importance to the 2nd and 3rd objectives respectively. TABLE V: as expected, gives maximum returns when maximum attention is paid to the 1st, 2nd and 3rd objective in case 2, 3 and 4 respectively than TABLE III.

Models presented in TABLES VI and VII give maximum return when maximum attention is paid to the 1st objective in case-1, than case-2 of Table III and V respectively. Fuzzy weights: fuzzy model gives more profit than crisp model in both the cases. In terms of total profit, TABLE VIII gives better result than TABLE VII. This model can be extended to include discount, random planning horizon, salvage of deteriorated qualities; etc. Determination of exact weights for multi-item multi-objective fuzzy model and their solution may be the topic of the further research.

Obviously, crisp model for integrated businesses gives best result than all type of models for the non-integrated businesses. Now, for integrated businesses, all costs are taken as triangular fuzzy numbers (TFNs) as shown in TABLE IX.

$$\tilde{N}_1=(14,18,22), \tilde{C}_1=(145,165,185), \tilde{H}_1=(6,9,12), \tilde{h}_{11}=(1,1.5,2), \tilde{N}_2=(16,20,14), \tilde{C}_2=(155,175,195), \tilde{H}_2=(7,10,13),$$
$$\tilde{h}_{12}=(1.2,1.7,2.2), \tilde{N}_3=(19,23,27), \tilde{C}_3=(165,185,205), \tilde{H}_3=(8,11,14), \tilde{h}_{13}=(1.5,2,2.5)$$

The optimum results for the integrated representation of the fuzzy inventory model i.e. (21) are presented in TABLE IX.

For the more realistic situations, we here consider all the costs to be trapezoidal fuzzy numbers (TrFNs) as shown below:

$$\tilde{N}_1=(14,17,19,22), \tilde{C}_1=(145,160,170,185), \tilde{H}_1=(6,7,10,12), \tilde{h}_{11}=(1,1.3,1.7,2), \tilde{N}_2=(16,19,21,14), \tilde{C}_2=(155,170,180,195),$$
$$\tilde{H}_2=(7,9,11,13), \tilde{h}_{12}=(1.2,1.5,1.9,2.2), \tilde{N}_3=(19,22,24,27), \tilde{C}_3=(165,180,190,205), \tilde{H}_3=(8,10,12,14), \tilde{h}_{13}=(1.5,1.8,2.1,2.5)$$

And the corresponding optimum results for the integrated representation of the fuzzy inventory model given in (21) are presented in TABLE X.

Here, optimal results of fuzzy model for integrated businesses are presented with possibility, necessity and credibility measures with triangular and trapezoidal fuzzy numbers shown in TABLE IX and X. Decision Makers can take decision according to the available situations.

XIII. Conclusion

Till now, in the field of inventory, some multi-objective models of deteriorating items with two or more objectives are available in crisp and fuzzy environment. Here, inventory models having Weibull rate of decay with three objectives allowing shortages have been presented in crisp and fuzzy environments for integrated and non-integrated businesses. The models of non-integrated businesses have been solved by FNLP techniques and the model of integrated businesses has been solved by Generalized Reduce Gradient Method by analyzing possibility/necessity measure. The results have been presented with different types of weights admissible to the objectives for non-integrated businesses. Each weight, which implies the relative importance of the objective goals, can be determined through the practical experiences. Though the problem has been formulated in the field of inventory, the present methodology in formulation and solution can be adopted for a fuzzy non-linear decision making problem in any discipline. Moreover, in this paper, model has been formulated with time-dependent demand, demand-dependent production and time-varying holding cost allowing shortages for integrated and non-integrated businesses. All costs are taken as triangular fuzzy numbers and trapezoidal fuzzy numbers in the case of integrated business. The present analysis can be easily extended to other types of inventory models with constant demand, infinite replenishment, fixed -time horizon etc.

Our proposed model has a wide range of application in the field of businesses and management in which the economic conditions can also be developed. This model can be further extended to include discount, random planning horizon, salvage of deteriorated qualities; etc. Determination of exact weights for mult-item mult-objective fuzzy model and their solution may be the topic of the further research.

TABLE I: Input Data (For Three Items)

Items	m_i	N_i ($)	a_i	H_i ($)	C_i ($)	b_i	h_{1i}	h_{2i}	α_i	β_i	γ_i	w^i	W (Sq. Ft.)
1	1.6	18	30	9	165	0.85	1.5	0.003	0.14	1.3	1.2	5	200
2	1.7	20	65	10	175	0.9	1.7	0.004	0.12	1.2	1.3	4	
3	1.8	23	100	11	185	0.95	2.0	0.005	0.10	1.1	1.4	3	

TABLE II: Pay-off Matrix

	I_1 ($)	I_2 ($)	I_3($)
t_{21}	241.4155	701.3331	1632.692
t_{22}	220.5831	779.8763	1651.422
t_{23}	233.5547	664.4949	1658.836

TABLE III: (Crisp Weighted Crisp Model) ZIMMERMANN'S Model

Case	w_1	w_2	w_3	I_1($)	I_2($)	I_3($)	T	I($)	SC (Sq. Ft.)
1	1/3	1/3	1/3	234.73	748.17	1650.45	2.5645	2633.34	115.01
2	0.2	0.3	0.5	238.37	737.97	1641.62	2.8940	2617.96	122.71
3	0.5	0.2	0.3	233.59	779.85	1650.12	2.5116	2663.56	158.01
4	0.3	0.5	0.2	231.94	778.63	1654.07	2.3895	2664.62	150.70

TABLE IV: (CRISP WEIGHTED CRISP MODEL) ADDITIVE MODEL\

Case	w_1	w_2	w_3	I_1($)	I_2($)	I_3($)	T	I($)	SC (Sq. Ft.)
1	1/3	1/3	1/3	233.28	779.83	1652.55	2.4679	2665.65	163.29
2	0.6	0.2	0.2	238.33	778.32	1641.76	2.889	2658.41	184.87
3	0.2	0.6	0.2	233.38	779.83	1657.18	2.4746	2665.63	163.65
4	0.2	0.2	0.6	227.50	778.30	1657.18	2.1774	2662.98	147.60

TABLE V: CRISP WEIGHTED FUZZY MODEL (ZIMMERMANN'S MODEL)

Case	w_1	w_2	w_3	w_4	I_1($)	I_2($)	I_3($)	T	I($)	SC (Sq. Ft.)
1	1/4	1/4	1/4	1/4	235.0	774.53	1650.0	2.5838	2659.53	186.89
2	.1	.2	.3	.4	239.16	776.31	1638.40	2.9977	2653.87	181.34
3	.2	.1	.3	.4	232.50	779.85	1651.50	2.5116	2663.84	154.77
4	.3	.2	.1	.4	226.37	777.80	1657.63	2.1317	2661.80	145.21

TABLE VI: FUZZY WEIGHTED CRISP MODEL (ZIMMERMANN'S MODEL)

Case	w_1	w_2	w_3	I_1($)	I_2($)	I_3($)	T	I($)	SC (Sq. Ft.)
1	[.12,.42]	[.12,.42]	[.12..42]	235.25	778.81	1636.33	2.6662	2650.38	200.0
2	[.8,1]	[.4,.8]	[0,.2]	238.64	737.69	1634.44	2.9271	2610.76	145.94

TABLE VII: FUZZY WEIGHTED FUZZY MODEL (ZIMMERMANN'S MODEL)

w_1	w_2	w_3	W_4	I_1($)	I_2($)	I_3($)	T	I($)	SC (Sq. Ft.)
[0,1]	[.25,1]	[.434,.999]	[.55,.98]	232.99	779.79	1652.90	2.4506	2665.68	161.23
[.8,1]	[.4,.8]	[.1,.5]	[.5,1]	239.57	775.77	1636.41	3.0589	2651.75	184.58

TABLE VIII: INTEGRATED MODEL (GRG METHOD)

I_1($)	I_2($)	I_3($)	T	I($)	SC (Sq. Ft.)
232.58	779.76	1653.38	2.4262	2665.72	161.08

TABLE IX: INTEGRATED FUZZY MODEL FOR TFNS (NECESSITY/POSSIBILITY ATTITUDE)

ρ	I_4($)	T	SC (Sq. Ft.)	ρ	I_4($)	T	SC(Sq. Ft.)	ρ	I_4($)	T	SC (Sq. Ft.)
0	2413.44	2.5008	164.20	0.4	2615.23	2.4399	161.66	0.8	2817.28	2.3883	159.46
0.1	2463.86	2.4845	163.53	0.5	2665.72	2.4262	161.08	0.9	2867.82	2.3767	158.95
0.2	2514.30	2.4690	162.89	0.6	2716.23	2.4131	160.52	1	2918.38	2.3655	158.46
0.3	2564.76	2.4541	162.26	0.7	2766.75	2.4005	159.98				

TABLE X: INTEGRATED FUZZY MODAL FOR TRFNS (NECESSITY/POSSIBILITY ATTITUDE)

ρ	I_4($)	T	SC (Sq. Ft.)	ρ	I_4($)	T	SC (Sq. Ft.)	ρ	I_4($)	T	SC (Sq. Ft.)
0	2352.04	2.5340	165.65	0.4	2603.21	2.4471	162.11	0.8	2854.86	2.3775	159.17
0.1	2414.78	2.5103	164.70	0.5	2666.08	2.4283	161.33	0.9	2917.84	2.3622	158.51
0.2	2477.55	2.4880	163.80	0.6	2728.99	2.4105	160.58	1	2980.83	2.3476	157.88
0.3	2540.37	2.4670	162.93	0.7	2791.91	2.3936	159.86				

APPENDIX A

A.1: Any fuzzy subset $\tilde{a}$ of $\Re$ (where $\Re$ represents a set of real numbers) with membership function $\mu_{\tilde{a}}(x)$: $\Re \rightarrow [0,1]$ is called a fuzzy number. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy quantities with membership functions $\mu_{\tilde{a}}$ (x) and $\mu_{\tilde{b}}$ (y) respectively. Then according to Dubois and Prade (1987), Liu and Iwamura (1998), Maiti and Maiti (2006)

$$Pos\left(\tilde{a}*\tilde{b}\right)=\left\{\sup\left(\min\left(\mu_{\tilde{a}}(x),\mu_{\tilde{b}}(y)\right)\right), x,y\varepsilon\Re, x*y\right\}$$

$$Nes\left(\tilde{a}*\tilde{b}\right)=\left\{\inf\left(\max\left(1-\mu_{\tilde{a}}(x),\mu_{\tilde{b}}(y)\right)\right), x,y\varepsilon\Re, x*y\right\}$$

where the abbreviation `Pos' represents possibility and `Nes' represents necessity and `*' is any of the relations >, <, =, ≤, ≥.

The dual relationship of possibility and necessity requires that

$$Nes\left(\tilde{a}*\tilde{b}\right)=1-Pos\left(\overline{\tilde{a}*\tilde{b}}\right)$$

Also necessity measures satisfy the condition,

$$MinNes\left(\tilde{a}*\tilde{b}\right), Nes\left(\overline{\tilde{a}*\tilde{b}}\right)=0$$

The relationships between possibility and necessity measures satisfy the following conditions (cf. Dubois and Prade (1988)}: $Pos\left(\tilde{a}*\tilde{b}\right) \geq Nes\left(\tilde{a}*\tilde{b}\right)$, then

$$Nes\left(\tilde{a}*\tilde{b}\right) > 0 \Rightarrow Pos\left(\tilde{a}*\tilde{b}\right)=1 \text{ and}$$

$$Pos\left(\tilde{a}*\tilde{b}\right)<1 \Rightarrow Nes\left(\tilde{a}*\tilde{b}\right)=0.$$

If $\tilde{a},\tilde{b} \in \Re$ and $\tilde{c} = f\left(\tilde{a},\tilde{b}\right)$ where f : $\Re \times \Re \rightarrow \Re$

Be a binary operation, then membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is defined as:

$$\mu_{\tilde{c}}(z)=\begin{cases}\sup\left(\min\left(\mu_{\tilde{a}}(x),\mu_{\tilde{b}}(y)\right)\right), x,y\in\Re \\ and z=f(x,y) \forall z\in\Re\end{cases}$$

Recently based on possibility measure and necessity measure, the third set function Cr, called credibility measure, analyzed by Liu and Liu (2002) is as follows:

Cr ($\tilde{A}$) =1/2[Pos($\tilde{A}$)+Nes($\tilde{A}$)] for any $\tilde{A}$ in $2^{\Re}$, where $2^{\Re}$ is the power set of $\Re$.

It is easy to check that Cr satisfies the following conditions:

(i) $Cr(\phi)=0 and Cr(\Re)=1$;

(ii) $Cr(\tilde{A})\le Cr(\tilde{B}) whenever A,B\in 2^{\Re} and A\subseteq B$

Thus Cr is also a fuzzy measure defined on $\Re,\left(2^{\Re}\right)$. Besides, Cr is self dual, i.e. $Cr(\tilde{A})=1-Cr(\tilde{A}^{c})\ for any \tilde{A} in 2^{\Re}$.

In this paper, based on the credibility measure the following form is defined as

$$Cr(\tilde{A})=\left[\rho Pos(\tilde{A})+(1-\rho)Nes(\tilde{A})\right]$$

(cf. Liu and Liu (2002) for any $\tilde{A}$ in $2^{\Re}$ and $0<\rho<1$. It also satisfies the above conditions.

A.2. Triangular Fuzzy Number:

Triangular fuzzy number (TFN) $\tilde{A}$ (see Fig. A - 1) is the fuzzy number with the membership function $\mu_{\tilde{A}}(x)$, a continuous mapping: $\mu_{\tilde{A}}(x)=\Re\rightarrow[0,1]$,

$$\mu_{\tilde{A}}(x)=\begin{cases}0\ for -\infty\le x\le a_1\\ \dfrac{x-a_1}{a_2-a_1}\ for\ a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}\ for\ a_2\le x\le a_3\\ 0\ for\ a_3\le x\le\infty\end{cases}$$

Lemma 1: The expected value of triangular fuzzy number $\tilde{A}=(a_1,a_2,a_3)$ is E($\tilde{A}$) = (1/2) $\left[(1-\rho)a_1+a_2+\rho a_3\right]$

Proof 1. Let $\tilde{A}=(a_1,a_2,a_3)$ be a triangular fuzzy number.

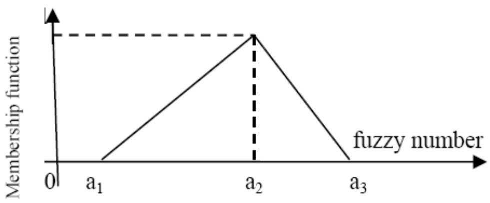

Fig. A – 1. Membership function of Triangular Fuzzy Number (TFN) $\tilde{A}=(a_1,a_2,a_3)$

$$Pos(\tilde{A}\ge r)=\begin{cases}1\ if\ r\le a_2\\ \dfrac{a_3-r}{a_3-a_2}\ if\ a_2\le r\le a_3\\ 0\ if\ r\ge a_3\end{cases}$$

$$Nes(\tilde{A}\ge r)=\begin{cases}1\ if\ r\le a_1\\ \dfrac{a_2-r}{a_2-a_1}\ if\ a_1\le r\le a_2\\ 0\ if\ r\ge a_2\end{cases}$$

$$Cr(\tilde{A}\ge r)=\begin{cases}1\ if\ r\le a_1\\ \dfrac{a_2-\rho a_1}{a_2-a_1}-\dfrac{(1-\rho)r}{a_2-a_1}\ if\ a_1\le r\le a_2\\ \rho\dfrac{a_3-r}{a_3-a_2}\ if\ a_2\le r\le a_3\\ 0\ if\ r\ge a_3\end{cases}$$

$$Cr(\tilde{A}\le r)=\begin{cases}0\ if\ r\le a_1\\ \rho\dfrac{r-a_1}{a_2-a_1}\ if\ a_1\le r\le a_2\\ \dfrac{\rho a_3-a_2+(1-\rho)r}{a_3-a_2}\ if\ a_2\le r\le a_3\\ 1\ if\ r\ge a_3\end{cases}$$

Based on the credibility measure, Liu and Liu (2002, 2003) presented the expected value operator of a fuzzy variable as: Let $\tilde{A}$ be a normalized fuzzy variable. The expected value of the fuzzy variable $\tilde{A}$ is defined by

$$E(\tilde{A})=\int_0^{\infty}Cr(\tilde{A}\ge r)dr-\int_{-\infty}^{0}Cr(\tilde{A}\le r)dr$$

When the right hand side of the above equation is of form $\infty-\infty$, the expected value cannot be defined. Also, the expected value operation has been proved to be linear for bounded fuzzy variable, i.e., for any two bounded fuzzy variables $\tilde{X} and \tilde{Y}$,we have $E(a\tilde{X}+b\tilde{Y})=aE(\tilde{X})+bE(\tilde{Y})$ for both real numbers a and b. Then the expected value of fuzzy variable $\tilde{A}$ is defined as:

$$E(\tilde{A})=\int_0^{a_1}Cr(\tilde{A}\ge r)dr+\int_{a_1}^{a_2}Cr(\tilde{A}\le r)dr+\int_{a_2}^{a_3}Cr(\tilde{A}\ge r)+\int_{a_3}^{a_4}Cr(\tilde{A}\ge r)$$

$$E(\tilde{A})=(1/2)\left[(1-\rho)a_1+a_2+\rho a_3\right]$$

A.3. Trapezoidal Fuzzy Number:

Trapedoidal fuzzy number (TrFNs) $\tilde{A}$ (see Fig. A - 2) is the fuzzy number with the membership function $\mu_{\tilde{A}}(x)$, a

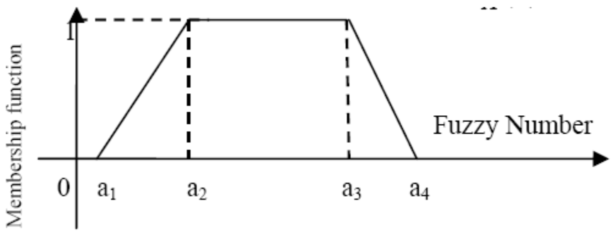

Fig. A – 2. Membership function of trapezoidal fuzzy

Number (TrFN) $\tilde{A}=(a_1,a_2,a_3,a_4)$ continuous mapping: $\mu_{\tilde{A}}(x)=\Re\rightarrow[0,1]$

$$\mu_{\tilde{A}}(x)=\begin{cases}0\ for -\infty\le x\le a_1\\ \dfrac{x-a_1}{a_2-a_1}\ for\ a_1\le x\le a_2\\ 1\ for\ a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3}\ for\ a_3\le x\le a_4\\ 0\ for\ a_4\le x\le\infty\end{cases}$$

$$Pos(\tilde{A}\ge r)=\begin{cases}1\ if\ r\le a_3\\ \dfrac{a_4-r}{a_4-a_3}\ if\ a_3\le r\le a_4\\ 0\ if\ r\ge a_4\end{cases}$$

$$Nes(\tilde{A}\ge r)=\begin{cases}1\ if\ r\le a_1\\ \dfrac{a_2-r}{a_2-a_1}\ if\ a_1\le r\le a_2\\ 0\ if\ r\ge a_2\end{cases}$$

The credibility measure for TrFNs can be defined as:

$$Cr\left(\tilde{A}\geq r\right)=\begin{cases} 1 \; if \; r\leq a_1 \\ \dfrac{a_2-\rho a_1}{a_2-a_1}-\dfrac{(1-\rho)r}{a_2-a_1} \; if \; a_1\leq r\leq a_2 \\ \rho \; if \; a_2\leq r\leq a_3 \\ \rho\dfrac{a_4-r}{a_4-a_3} \; if \; a_3\leq r\leq a_4 \\ 0 \; if \; r\geq a_4 \end{cases}$$

$$Cr\left(\tilde{A}\leq r\right)=\begin{cases} 0 \; if \; r\leq a_1 \\ \rho\dfrac{r-a_1}{a_2-a_1} \; if \; a_1\leq r\leq a_2 \\ \rho \; if \; a_2\leq r\leq a_3 \\ \dfrac{\rho a_4-a_3+(1-\rho)r}{a_4-a_3} \; if \; a_3\leq r\leq a_4 \\ 1 \; if \; r\geq a_4 \end{cases}$$

Based on the credibility measure, Liu and Liu(2002, 2003), as described in triangular fuzzy number. Then the expected value of trapezoidal fuzzy variable $\tilde{A}$ is defined as:

$$E\left(\tilde{A}\right)=(1/2)\left[(1-\rho)(a_1+a_2)+\rho(a_3+a_4)\right].$$

A.4. Multi – Objective Problem Under Fuzzy Expected Value Model:

A general multi - objective mathematical programming problem with fuzzy parameters in the objective function is of the following form:

$$Max\left[f_1(x,\xi), f_2(x,\xi), f_3(x,\xi)\ldots, f_n(x,\xi)\right]$$

Subject to, $g_i(x,\xi)\leq 0$, j=1, 2 , ……, k,

where x and ξ are decision vector and fuzzy vector respectively. To convert the fuzzy objective and constraints to their crisp equivalents, Liu & Liu (2002) proposed a new method to convert the problem into an equivalent multi-objective fuzzy expected value model i.e. the equivalent crisp model is:

$$Max\left[E\left[f_1(x,\xi)\right], E\left[f_2(x,\xi)\right], E\left[f_3(x,\xi)\right]\ldots, E\left[f_n(x,\xi)\right]\right]$$

Subject to, $E\left[g_i(x,\xi)\right]\leq 0$, j=1, 2 , ……, k.

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