On the Solution of Nonlinear Time-Fractional Generalized Burgers Equation by Homotopy Analysis Method and Modified Trial Equation Method

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***Abstract*—In this paper, we have executed the Homotopy Analysis Method and Modified Trial Equation Method which has newly been submitted to the literature for obtaining analytical solution of the nonlinear time-fractional generalized Burgers equation occuring in various areas of physics, chemistry, applied sciences, applied mathematics such as modeling of gas dynamics and traffic flow. Then, we have formed a table which includes numerical conclusions for time-fractional generalized Burgers equation. Finally, we have obtained the 2D and 3D surfaces by means of programming language Mathematica 9 in order to interpret in the sense of physical phenomena for analytical solution and approximate solution which have been obtained.**

***Index Terms*—Nonlinear time-fractional generalized Burgers equation, homotopy analysis method, modified trial equation method.**

I. Introduction

In the last three decades, Fractional Differential Equations and Systems (FDEs, FDES) have investigated keen interests among mathematicians and research scientists, for their analytical and numerical solutions [1]-[15]. To date, methodological studies have been shown not limited to, the numerical difference Differential Transform Method (DTM), the Homotopy Analysis and Homotopy Perturbations Methods (HAM, HPM), the Sumudu Transform Method (STM) and Variational Iteration Method (VIM) used to achieve solutions for various proposed Fractional Differential Equations and Systems.

Liu has introduced to the literature Trial Equation Method for solving nonlinear differential equation [16]-[18]. Gurefe has developed this approach as a Extended Trial equation method [19]. In this paper, we have modified this technique as a Modified Trial equation method (MTEM) for solving nonlinear fractional differential equations [20]-[22].

After applying the MTEM to the nonlinear time-fractional generalized Burgers equation, and producing related surfaces of graphics by means of the symbolic Algebra package, Mathematica Release 7, should we obtain the closer numerical result to analytical solution than by implementing other methods, we would say that HAM is a preferable method of choice for its accuracy on these equations. So, to the best of our knowledge, the modified trial equation method has not been widely applied for studying the invariance properties of fractional PDEs.

In this research, we consider nonlinear time-fractional generalized Burgers equation described as following and occur in different areas in mathematical physics;

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} - u_{xx} - \beta u^{p} u_{x} = 0 \tag{1}$$

where $0 < \alpha \le 1,\ p > 0$.

The purpose of this paper is to obtain numerical solutions of the nonlinear time-fractional generalized Burgers equation with fractional order by using HAM [21].

II. Fractional Calculus Basic Definitions, and Homotopy Analysis Method Properties

First of all, we recall some fundamental properties of fractional calculus, and then show the main prospects of the HAM and MTEM. We take into account the HAM and MTEM technique below, for solving nonlinear time-fractional generalized Burgers equation with fractional order [23].

A. Preliminaries

In this chapter, we recall some definitions and properties of the fractional calculus theory in the sense of Riemann -Liouville derivative proposed by Jumarie. Let $f : [0,1] \to \Re$ be a continuous function and $\alpha \in (0,1)$. The Jumarie modified fractional derivative of order α and f may be defined by expression of as follows [23]:

$$D_x^{\alpha} f(x) = \begin{cases} \dfrac{1}{\Gamma(-\alpha)} \displaystyle\int_0^x (x-\xi)^{-\alpha-1} \left[f(\xi) - f(0) \right] d\xi, & \alpha < 0, \\ \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dx} \displaystyle\int_0^x (x-\xi)^{-\alpha} \left[f(\xi) - f(0) \right] d\xi, & 0 < \alpha < 1, \\ \left(f^{(n)}(\xi) \right)^{\alpha - n} & , n \le \alpha \le n+1, n \ge 1. \end{cases} \tag{2}$$

As well as this equality, we may recall the fractional Riemann-Liouville derivative which is used in this study as following [23]:

$$D_x^{\alpha} k = 0$$

$$D_x^{\alpha} x^{\mu} = \begin{cases} 0, & \mu \le \alpha - 1, \\ \dfrac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)} x^{\mu - x}, & \mu > \alpha - 1. \end{cases} \tag{3}$$

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B. Modified Trial Equation Method (MTEM)

In this paper, a new trial equation method will be given. In order to apply this method to fractional nonlinear partial differential equations, we consider the following steps.

Step 1. We consider time fractional partial differential equation in two variables and a dependent variable u and take the wave transformation

$$P\left(u, D_t^{\alpha}u, u_x, u_{xx}, u_{xxx}, \cdots\right)=0, \tag{4}$$

$$u(x,t)=u(\eta), \qquad \eta=kx-\frac{\lambda t^{\alpha}}{\Gamma(1+\alpha)}, \tag{5}$$

where $\lambda \neq 0.$ Substituting (5) into (4) yields a nonlinear ordinary differential equation

$$N\left(u, u', u'', u''', \cdots\right)=0. \tag{6}$$

Step 2. Take trial equation as follows:

$$u'=\frac{F(u)}{G(u)}=\frac{\sum_{i=0}^{n} a_i u^i}{\sum_{j=0}^{l} b_j u^j}=\frac{a_0+a_1u+a_2u^2+\cdots+a_nu^n}{b_0+b_1u+b_2u^2+\cdots+b_lu^l}, \tag{7}$$

And

$$u''=\frac{F(u)\left(F'(u)\,G(u)-F(u)\,G'(u)\right)}{G^3(u)}, \tag{8}$$

where $F(u)$ and $G(u)$ are polynomials. Substituting above relations into (6) yields an equation of polynomial $\Omega(u)$ of u:

$$\Omega(u)=\rho_s u^s+\cdots+\rho_1 u+\rho_0=0. \tag{9}$$

According to the balance principle, we can get a relation of n and l. We can compute some values of n and l.

Step 3. Let the coefficients of $\Omega(u)$ all be zero will yield an algebraic equations system:

$$\rho_i=0, \qquad i=0,\cdots,s. \tag{10}$$

Solving this system, we will specify the values of $a_0,\cdots,a_n$ and $b_0,\cdots,b_l$.

Step 4. Reduce (5) to the elementary integral form

$$\pm(\mu-\mu_0)=\int\frac{G(u)}{F(u)}du. \tag{11}$$

Using a complete discrimination system for polynomial to classify the roots of $F(u)$, we solve (11) with the help of Mathematica 9 and classify the exact solutions to (6). In addition, we can write the exact traveling wave solutions of (4). For a better interpretations of results obtained in this way, we plotted 3D surfaces of analytical and approximate solution by taking into consideration suitable parameter.

III. The Approximate Solution of Time-Fractional Generalized Burgers Equation by Using HAM

The analytical solution of nonlinear time-fractional generalized Burgers equation by using MTEM has been obtained as following in [20]:

$$u(x,t)=\left[\alpha_1\pm\frac{\alpha_1-\alpha_2}{\exp\left[B_1\left(x-\lambda_1 t^{\alpha}\right)\right]-1}\right]^{1/p}, \tag{12}$$

where $k, p, \beta, \alpha_1 \neq \alpha_2, b_0$ are constants and A_1, B_1, λ_1 are defined by

$$A_1=-\frac{k(1+p)}{p\beta}, B_1=\frac{k(\alpha_1-\alpha_2)}{A_1}, \qquad \lambda_1=\frac{-k\alpha_1}{pb_0\Gamma[1+\alpha]}. \tag{13}$$

When we regulate the (1) for $p=\beta=1$, we get equation as following:

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}-u_{xx}(x,t)-u(x,t)u_x(x,t)=0. \tag{14}$$

The analytical solution of (14) by using MTEM has been obtained for $p=\beta=1, b_0=a_1=0.1, a_2=0.01,$ $\lambda_1=0.110326, B_1=0.45$ as following:

$$u(x,t)=0.1+\frac{0.09}{-1+e^{\left(0.45x-\frac{1}{10\Gamma[1+\alpha]}t^{\alpha}\right)}}. \tag{15}$$

The initial condition of (14) is as following:

$$u(x,0)=0.1+\frac{0.09}{-1+e^{0.45x}}. \tag{16}$$

We take into consideration the linear operator

$$L\left[\Phi(x,t;p)\right]=D_t^{\alpha}\left[\Phi(x,t;p)\right], \tag{17}$$

With the property

$$L[c]=0. \tag{18}$$

We can consider the nonlinear operator N for (14) as following:

$$N\left[\Phi(x,t;p)\right]=\frac{\partial^{\alpha}\Phi(x,t;p)}{\partial t^{\alpha}} -\frac{\partial^{2}\Phi(x,t;p)}{\partial x^{2}}-\Phi(x,t;p)\frac{\partial\Phi(x,t;p)}{\partial x}. \tag{19}$$

Therefore, we construct the zero-order deformation equation as follows:

$$(1-p)L\left[\Phi(x,t;p)-u_0(x,t)\right] = p\hbar N\left[\Phi(x,t;p)\right]. \tag{20}$$

Absolutely, if p is $p=0$, this gives us initial condition

$$\Phi(x,t;0)=u_0(x,t)=u(x,0), \tag{21}$$

And if p is $p=1$, this gives us analytical solution

$$\Phi(x,t;1)=u(x,t). \tag{22}$$

When we expand the solution function of $\Phi(x,t;p)$ to the tylor series, we obtain as following:

$$\Phi(x,t;p)=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t)\,p^m, \tag{23}$$

where

$$u_m(x,t)=\frac{1}{m!}\frac{\partial^m\Phi(x,t;p)}{\partial p^m}\bigg|_{p=0}. \tag{24}$$

If the auxiliary linear operator, the initial condition and the auxiliary parameter $\hbar$ are properly chosen, the above series converges at $p=1$, and one can have

$$u(x,t)=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t). \tag{25}$$

When we perform (20) m times by differentiating with respect to p, we obtain m th-order deformation equation

$$L\left[u_m(x,t)-\chi_m u_{m-1}(x,t)\right]=\hbar R_m\left[\vec{u}_{m-1}(x,t)\right], \tag{26}$$

where $R_m\left[\vec{u}_{m-1}(x,t)\right]$ is defined by

$$R_m\left[\vec{u}_{m-1}(x,t)\right]=\frac{\partial^{\alpha}u_{m-1}(x,t)}{\partial t^{\alpha}}-\frac{\partial^{2}u_{m-1}(x,t)}{\partial x^{2}} -\sum_{n=0}^{m-1}u_n(x,t)\frac{\partial u_{m-1-n}(x,t)}{\partial x}, \tag{27}$$

And

$$\chi_m=\begin{cases}0, & m\le 1,\\ 1, & m>1.\end{cases} \tag{28}$$

The solution of m th-deformation equation for $m\ge 1$ give rise to

$$u_m(x,t)=\chi_m u_{m-1}(x,t)+\hbar J_t^{\alpha}\left[R_m\left[\vec{u}_{m-1}(x,t)\right]\right]. \tag{29}$$

When we use the initial condition (16) along with (29), we attain the first three terms of (16) as following

$$u_0(x,0)=0.1+\frac{0.09}{-1+e^{0.45x}},$$
$$m=1\Rightarrow u_1(x,t)=\chi_1u_0(x,t)+\hbar J_t^{\alpha}\left[R_1\left[\vec{u}_0(x,t)\right]\right],$$
$$=\hbar J_t^{\alpha}\left[\frac{\partial^{\alpha}u_0}{\partial t^{\alpha}}-\frac{\partial^{2}u_0}{\partial x^{2}}-\sum_{n=0}^{0}u_0(x,t)\frac{\partial u_0}{\partial x}\right],$$

$$u_1(x,t)=\frac{891ht^{\alpha}}{16\times10^{6}\alpha\Gamma[\alpha]}C\,sch^{2}\left(\frac{9x}{400}\right), \tag{30}$$

$$m=2\Rightarrow u_2(x,t)=\chi_2u_1(x,t)+\hbar J_t^{\alpha}\left[R_2\left[\vec{u}_2(x,t)\right]\right],$$
$$=u_1+\hbar J_t^{\alpha}\left[\frac{\partial^{\alpha}u_1}{\partial t^{\alpha}}-\frac{\partial^{2}u_1}{\partial x^{2}}-\sum_{n=0}^{1}u_n\frac{\partial u_n}{\partial x}\right],$$
$$u_2(x,t)=\frac{891ht^{\alpha}}{16\times10^{6}\alpha\Gamma[\alpha]}C\,sch^{2}\left(\frac{9x}{400}\right) +\frac{891\times4^{-7-\alpha}e^{9x/200}h^{2}t^{\alpha}}{9765625\alpha\Gamma[\alpha]\Gamma[1/2+\alpha]}\left(-1+e^{9x/200}\right)^{-2} \times\left(99t^{\alpha}\sqrt{\pi}Coth\left(\frac{9x}{400}\right)+625\times4^{3+\alpha}\Gamma[1/2+\alpha]\right), \tag{31}$$
$$\vdots$$

Thus, the series solution expressed by the HAM can be written in the form of following:

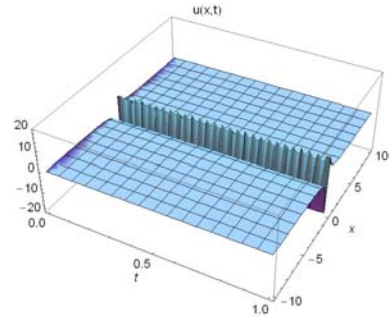
$$u(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots. \tag{32}$$

Then, we can write the approximate solution obtained by HAM of nonlinear time-fractional generalized Burgers equation as following:

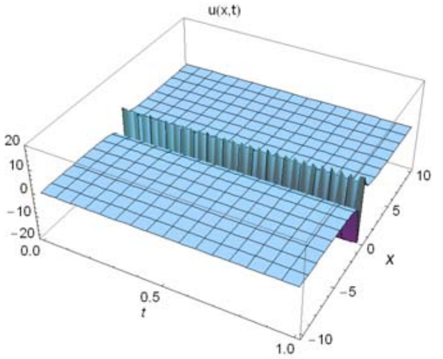
$$u(x,t)=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t)$$
$$=u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots,$$

$$=0.1+\frac{0.09}{-1+e^{0.45x}}+\frac{891ht^{\alpha}}{16\times10^{6}\alpha\Gamma[\alpha]}C\,sch^{2}\left(\frac{9x}{400}\right) +u_2(x,t)=\frac{891ht^{\alpha}}{16\times10^{6}\alpha\Gamma[\alpha]}C\,sch^{2}\left(\frac{9x}{400}\right) +\frac{891\times4^{-7-\alpha}e^{9x/200}h^{2}t^{\alpha}}{9765625\alpha\Gamma[\alpha]\Gamma[1/2+\alpha]}\left(-1+e^{9x/200}\right)^{-2} \times\left(99t^{\alpha}\sqrt{\pi}Coth\left(\frac{9x}{400}\right)+625\times4^{3+\alpha}\Gamma[1/2+\alpha]\right). \tag{33}$$

The 2D and 3D surfaces of the approximate solution obtained by HAM and analytical solution founded by MTEM for nonlinear time-fractional generalized Burgers equation by using Mathematica 9 programming as following:

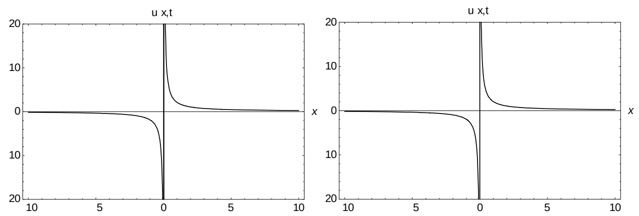


$a:\alpha=0.25$



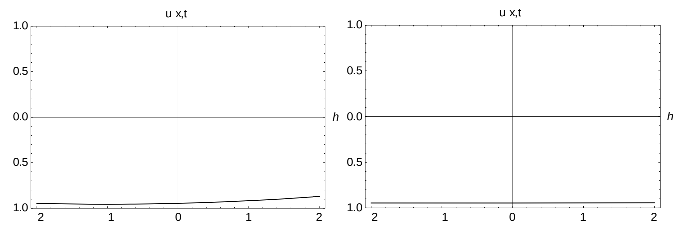
$b:\alpha=0.95$

Fig. 1. The 3D surfaces of (15) analytical solution obtained by means MTEM for $0<t<1, -10<x<10, \lambda_2=0.110326, B_4=0.45,\ a_2=0.01, b_0=a_1=0.1, p=\beta=1,$



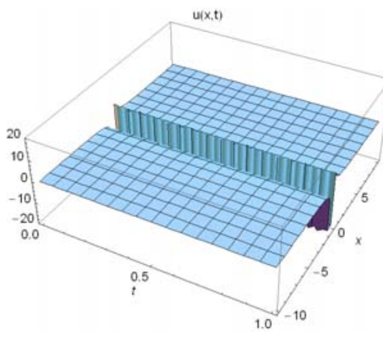
$a:\alpha=0.25$
$b:\alpha=0.95$

Fig. 2. The 2D surfaces of (15) analytical solution obtained by means MTEM for $t=0.01, -10<x<10, \lambda_2=0.110326,\ B_4=0.45, a_2=0.01, b_0=a_1=0.1, p=\beta=1.$

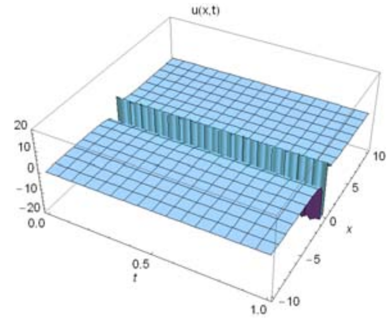


$a:\alpha=0.25\ b:\alpha=0.95$

Fig. 3. The 2D surfaces of *h* being (33) obtained solution by means HAM when $t=0.01, x=-2, -2<h<2$.

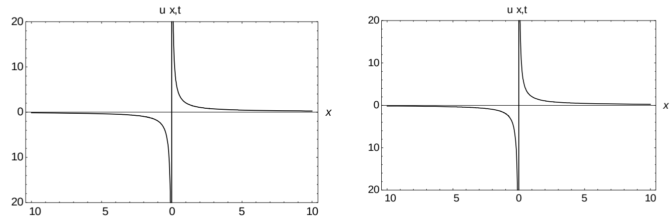


$a:\alpha=0.25$



$b:\alpha=0.95$

Fig. 4. The 3D surfaces of (33) approximate solution obtained by means HAM when $-10<x<10,\ 0<t<1, h=-1/120,$



$a:\alpha=0.25$

$b:\alpha=0.95$

Fig. 5. The 2D surfaces of (33) approximate solution obtained by means HAM when $t=0.01, -10<x<10, h=-1/120,$

Remark 1**:**

The solution (33) obtained by using the Homotopy Analysis method for (14) have been checked by Mathematica Program 9. To our knowledge, these analytical and approximate solutions that we find in this paper have been newly submitted to literature. According to these datas, we can comment that these techniques are very convenient for the solutions by showing Fig. 1, Fig. 2, Fig. 3, Fig. 4, and Fig. 5.

TABLE I: THE RESULT OF ERRORS OF (33) APPROXIMATE SOLUTION OBTAINED BY MEANS HAM AND ANALYTICAL SOLUTION FOUNDED BY MTEM FOR $\alpha=0.25,\ t=0.01,\ h=-1/120, -8<x<12$

x	$\alpha=0.25$		
	$u_{Analitik}$	u_{HAM}	$u_{Analitik}-u_{HAM}$
-8	-1.96620E-1	-1.97704E-1	1.08367E-3
-4	-4.42037E-1	-4.46388E-1	4.35126E-3
4	5.60736E-1	5.56309E-1	4.42734E-2
8	3.08777E-1	3.07684E-1	1.09317E-3
12	2.26171E-1	2.25692E-1	4.78683E-4

TABLE II: THE RESULT OF ERRORS OF (33) APPROXIMATE SOLUTION OBTAINED BY MEANS HAM AND ANALYTICAL SOLUTION FOUNDED BY MTEM FOR $\alpha=0.95,\ t=0.01,\ h=-1/120, -8<x<12$

x	$\alpha=0.95$		
	$u_{Analitik}$	u_{HAM}	$u_{Analitik}-u_{HAM}$
-8	-1.97654E-1	-1.97694E-1	4.00745E-5
-4	-4.46189E-1	-4.46350E-1	1.61574E-4
4	5.56509E-1	5.56347E-1	1.61678E-4
8	3.07733E-1	3.07693E-1	4.00874E-5
12	2.25714E-1	2.25696E-1	1.75776E-5

Remark-2:

When we take into consideration Table I and Table II, the numerical results of analytical solution and approximate solution are very closer. The numerical errors are very smaller, and therefore, we can underline that these methods very suitable for such a fractional differential equations.

IV. Conclusion

In this paper, firstly, the modified trial equation method has been applied to obtain the analytical solution of nonlinear time-fractional generalized Burgers equation. Secondly, the approximate solution of nonlinear time-fractional generalized Burgers equation by using analytical solution attained by using MTEM has been gained by means of Homotopy analysis method. Finally, after we submitted 2D and 3D surfaces for both solutions, we formed a table including numerical results and errors.

As a result of data obtained and the proposed methods in this study, we can say that they can also be applied to other generalized fractional nonlinear differential equations.

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