Asymmetric and Symmetric Spherical Product Surfaces with both Implicit and Parametric Representations

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Abstract—**This paper proposes spherical product functions to define implicit spherical product surfaces. A spherical product function is composed of a contour function and a profile function and its iso-surface's shape is generated by modulating and translating the iso-curve of the contour function through the points on the iso-curve of the profile function. This paper also shows if contour and profile functions are ray-linear, then an implicit spherical product surface can be parameterized and hence have both the advantages of implicit and parametric surfaces. Moreover, this paper proposes ray-linear two-branch and one-branch linear and super-hyperbolic functions that can be used to construct new contour and profile functions with asymmetric or symmetric iso-curves. Based on them, an implicit spherical product surface can has asymmetric or symmetric contour and profile and also has a parametric representation.**

Index Terms—**Implicit surface, blending operations, parametric surface.**

I. Introduction

In object modeling, implicit surface and parametric surface have their own advantages. Parametric surface is more popular than implicit surface due to its easier rendering and free-form surface generation, but implicit surface is attracting more and more attention because a complex implicit surface can be constructed easily from primitive implicit surfaces by blending operations [1]-[4] such as union and intersection.

In implicit surface modeling, primitive implicit surfaces are defined as a level surface of a defining function, which decides the shapes of primitive implicit surfaces to be blended for creating a complex object. In the literature, many defining functions were developed, including generalized distance functions [5], super-quadrics [6], generalized distance metrics [7], super-ellipsoids [8]-[10], cylinders [11], sweep objects [12] and hyper-quadrics [13]. Among these functions above, super-quadrics [2] can be represented parametrically, too, and briefly they have both the advantages of implicit and parametric surfaces.

To create new primitive defining functions with shapes more diverse than or different from existing defining functions, this paper proposes spherical product function. It is composed of a contour function and a profile function and its iso-surface, called implicit spherical product surface, is obtained by translating and modulating the iso-curve of the contour function from the points on the iso-curve of the profile function. In addition, this paper:

1) Shows that if both the contour and profile functions are ray-linear, then an implicit proposed spherical product surface can also be represented parametrically, that is, it has the advantages of implicit and parametric surfaces.
2) Proposes ray-linear two-branch lineal and super-hyperbolic functions, which can be used to construct ray-linear contour and profile functions with symmetric shapes by an intersection blend. Thus, a spherical product surface can have a symmetric contour or profile.
3) Proposes ray-linear one-branch lineal and super-hyperbolic functions, which can be used to construct ray-linear contour and profile functions with asymmetric shapes by an intersection blend and hence a spherical product surfaces can have an asymmetric contour or profile.

This paper is organized as follows. Surface definitions are presented in Section II. Implicit spherical product surfaces are described in Section III. Symmetric contour and profile curves are presented in Section IV. Asymmetric contour and profile curves are presented in Section V. Conclusion is given in Section VI.

II. Surface Definitions

This section reviews parametric and implicit surfaces.

A. Definition of Parametric Surface

Parametric surface is defined by a parametric formula $P(\alpha, \beta):[0, 1]^2\rightarrow R^3$,

$$P(\alpha, \beta) =[X(\alpha, \beta), Y(\alpha, \beta), Z(\alpha, \beta)],$$

where X, Y, and Z are polynomials of parameters α and β. Parametric surface needs less computing time in calculating the surface than implicit surface does.

B. Definition of a Primitive Implicit Surface

A primitive implicit surface is defined using primitive defining functions $f_i(x, y, z):R^3\rightarrow R_+$, i=1,2,…, by the point set:

$$\{ (x, y, z)\in R^3 \mid f_i(x, y, z) =1 \}.$$

In this paper, $f_i(x,y,z)=1$ denotes an primitive implicit surface for short and R_+ stands for $[0, \infty]$ in R

Because primitive defining functions $f_i(x,y,z)$ control the shapes of primitive implicit surfaces $f_i(x,y,z)=1$ to be blended for a complex implicit surface, presented in *Subsection C*, many defining functions were proposed in [5]-[13]. For example, super-quadrics [6] are written by:

$$F(x, y, z)=((x/a_1)^{2/n_1}+(y/a_2)^{2/n_1})^{n_1/n_2}+(z/a_3)^{2/n_2})^{n_2/2}=1,$$

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where n_1 and n_2 are squareness parameters of the shape in east-west and north-south directions, respectively. Some existing primitive implicit surfaces are displayed in Fig. 1.

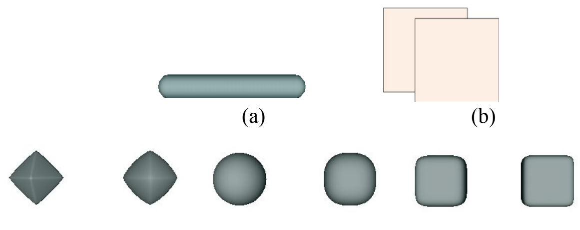


Fig. 1. (a). A cylinder. (b). Planes. (c). Super-quadrics $(x_2/n1+y_2/n_1+z_2/n_1)^{n_1/2}=1$ with parameter n1 varying from 1.9, 15, 1, 0.8, 0.4, to 0.25 for the objects from left to right.

C. Blending Operation to Construct a Complex Surface

Moreover, a more complex implicit surface is created easily by constructing k primitive implicit surfaces $f_i(x, y, z)=1$, $i=1,..., k$, through a blending operator $B_k(x_1,..., x_k): R_+^k \to R_+$ and is defined by the point set:

$$\{(x,y,z)\in R^3 \mid B_k(f_1(x, y, z),...,f_k(x, y, z)) = 1\}. \quad (1)$$

Some existing blending operators can be found in [1]-[4], such as Super-ellipsoidal intersection and union operators [4],

$$B_k(x_1,\ldots, x_k)=(x_1^{\,n}+\ldots+x_k^{\,n})^{1/p} \text{ and}$$
$$B_k(x_1,\ldots, x_k)=(x_1^{-n}+\ldots+x_k^{-n})^{-1/n}.$$

In (1), blending operation $B_k(f_1(x, y, z),..., f_k(x, y, z))$ can also be viewed as a new defining function and reused as a new primitive surface in other blending operations. This is seen in Fig. 2, which shows sequential union blends of four cylinders $f_i((x, y, z))=1$, $i=1, 2, 3, 4$, defined by $B_2(B_2(B_2(f_1(x, y, z), f_2(x, y, z)), f_3(x, y, z)), f_4(x, y, z))=1$.

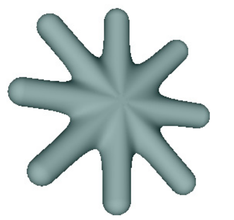


Fig. 2. Sequential union blends of four cylinders.

III. Implicit Spherical Product Surfaces

This section defines an implicit spherical product surface and describes their shapes.

A. Spherical Product Functions

Let $h(x, y)$ and $m(x, z)$, called contour and profile functions respectively, both map R^2 to R_+. Thus, if $h(x, y)=1$ and $m(x, z)=1$, called contour curve and profile curve respectively, are viewed as a horizonal curve and a vertical curve, then an spherical product function, denoted as $m(x, z)\otimes h(x, y)$, is defined by

$$M(x, z)\otimes h(x, y)= m(h(x, y), z), \quad (2)$$

Therefore, $m(x, z)\otimes h(x,y)$ can define an implicit spherical product surface by

$$F(x, y, z)=m(x, z)\otimes h(x, y)=1.$$

Surface $m(x, z)\otimes h(x, y)=1$ has a cross–section like contour curve $h(x, y)=1$ and has a profile like profile curve $m(x, z)=1$. Described geometrically, every point (x_0, z_0) that satisfies the equation $m(x_0, z_0)=1$ and $x_0\geq 0$ generates a contour curve, so $m(x, z)\otimes h(x, y)=1$ can be viewed as contour curve $h(x, y)=1$ translated along z-axis by $[0, 0, z_0]$ and modulated to be $h(x, y)=x_0$. That is, surface $m(x, z)\otimes h(x, y)=1$ is

1) Like a translational surface with contour $h(x, y)=1$ translated along z-axis and modulated by every point on $m(x, z)=1$, as shown in Fig. 3.
2) Like a rotational (revolution) surface with profile curve $m(x, z)=1$ revolved with respect to z-axis when $h(x, y)$ is given by $(x^2+y^2)^{0.5}$.

B. Parameterization of Implicit Spherical Product Surface

The most important property of an implicit spherical product surface is that it can be parameterized. This is described in Theorem 1:

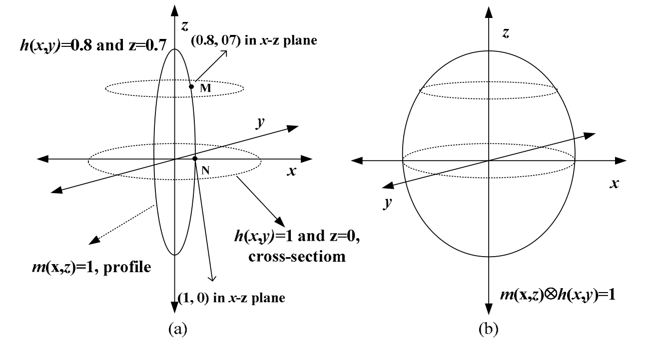


Fig. 3. (a) Dotted curves show the contours generated by points M (0.8, 07) and N (1, 0) on the curve $m(x, z)=1$. (b) The shape of $m(x, z)\otimes h(x, y)=1$ has a cross-section like contour curve $h(x, y)=1$ and a profile like profile curve $m(x, z)=1$.

- **Theorem 1:** If contour and profile functions $m(x, z)$ and $h(x, y)$ both have the ray-linear property stated below:

  Non-negative ray-linear property: "A function $f(x, y)$: $R^2 \to R_+$ is called non-negative ray-linear if condition $f(ax, ay)=af(x, y)$ holds for any $(x, y)\in R^2$ and $a\in R_+$"

  , then an implicit surface $m(x, z)\otimes h(x, y)=1$ is able to be represented parametrically by:

$$P(\alpha,\beta)\begin{bmatrix}\cos\alpha\cos\beta/(h(\cos\alpha,\sin\alpha)m(\cos\beta,\sin\beta)) \\ \sin\alpha\cos\beta/(h(\cos\alpha,\sin\alpha)m(\cos\beta,\sin\beta)) \\ \sin\beta/m(\cos\beta,\sin\beta)\end{bmatrix} \quad (3)$$

where $\alpha\in[-\pi, \pi]$ and $\beta\in[-\pi, \pi]$.

Proof: Since $m(x, z)$ and $h(x, y)$ are ray-linear, then curves $m(x, z)=1$ and $h(x, y)=1$ can be parameterized by

$$M(x, z)=1 \equiv \begin{bmatrix} m_x(\beta) \\ m_z(\beta)\end{bmatrix}=\begin{bmatrix} cos\beta/m(cos\beta,sin\beta) \\ sin\beta/m(cos\beta,sin\beta)\end{bmatrix} \text{ and}$$

$$H(x, y)=1 \equiv \begin{bmatrix} h_x(\alpha) \\ h_y(\alpha)\end{bmatrix}=\begin{bmatrix} cos\alpha/h(cos\alpha,sin\alpha) \\ sin\alpha/h(cos\alpha,sin\alpha)\end{bmatrix}.$$

Because $h(x, y)$ is ray-linear, $h(x, y)=x_0$ is the same as $h(x/x_0, y/x_0)=1$. It follows that $m(x, z)\otimes h(x, y)=1$ can be parameterized by

$$\begin{bmatrix} h_x(\alpha)m_x(\beta) \\ h_y(\alpha)m_x(\beta) \\ m_z(\beta) \end{bmatrix}.$$

This is equal to (3) after being expanded.

The following theorem is proposed to help create new ray-linear contour and profile functions:

- **Theorem 2**: If all $f_i(x, y)$:$R^2 \rightarrow R_+$, i=1,..., k, and blending operator $B_k(x_1,..., x_k)$: $R_+^k \rightarrow R_+$ are ray-linear, then blending operation $B_k(f_1(x, y),...,f_k(x, y))$ is ray-linear, too.

Thus, from **Theorem 2** ray-linear contour and profile functions $h(x, y)$ and $m(x, z)$ can be developed by performing ray-linear super-ellipsoidal intersection blend on ray-linear two-branch or one-branch lineal and super-hyperbolic functions, presented in Sections IV and V, through the blending operation:

$$B_k(f_1,\ldots,f_k) = (f_1(x, y)^p + \ldots + f_k(x, y)^p)^{1/p}, \qquad (4)$$

where p>1 and $f_i(x, y)$, i=1,…,to k, are $f_h(x, y)$, $f_p(x, y)$, $f_{sh}(x, y)$ or $f_{sp}(x, y)$ in (5)-(8).

Subsequently, ray-linear contour and profile functions $h(x, y)$ and $m(x, z)$ developed from (4) can be used to develop implicit spherical product surface $m(x, z) \otimes h(x, y)$=1 with a parametric representation as in (3) of **Theorem 1**.

IV. Symmetric Contour and Profile Curves

According to **Theorems 1 and 2**, this section develops ray-linear contour and profile functions with symmetric iso-curves as in (4) by proposing:

1) Ray-linear two-branch lineal functions and
2) Ray-linear two-branch super-hyperbolic functions,

A. Ray-linear Two-Branch Lineal Function

A ray-linear two-branch function with two lineal iso-curves is defined by:

$$f_p(x, y) = |\underline{v} \bullet [x, y]| / d_v, \qquad (5)$$

where $\underline{v}$ is the unit normal vector of the line $f_p(x, y)$=1 and d_v controls the shortest distance from the origin to the line. Symbol • means dot product in this paper.

As shown in Fig. 4(a), $f_p(x, y)$ =1 is a pair of parallel and symmetrical lines. Besides, it is easy to prove $f_p(x, y)$ is non-negative ray-linear.

B. Ray-linear Two-branch Super-Hyperbolic Function

Let $\underline{v}$ and $\underline{u}$ be unit vectors in R^2, $\underline{v} \bullet \underline{u}$ be 0, d_v, d_u and m all be greater than 0. Then, a ray-linear two-branch function with symmetric super-hyperbolic iso-curves is defined by

$$f_h(x,y) = \begin{cases} \left(f_v(x, y)^m - f_u(x, y)^m \right)^{\frac{1}{m}} \\ 0 \quad if \quad (f_v(x, y))^m < f_u(x, y)^m \end{cases} \qquad (6)$$

where $f_v = |\underline{v} \bullet [x, y]| / d_v$ and $f_u = |\underline{u} \bullet [x, y]| / d_u$.

It is easy to prove $f_h(x, y)$ is non-negative ray-linear. As shown in Fig. 4(b). The shape $f_h(x, y)$=1 is a pair of super-hyperbolic and symmetrical curves bounded in specified regions; vectors $\underline{v}$ and $\underline{u}$ control the orientation of the curve; parameter d_v determines the shortest distance from the origin to the curve; and parameter m controls how much square the curve is, for example:

- When $m \approx 1$, $f_h(x, y)$ =1 degenerates toward two folded lines passing through "points f, e, and g" and "points f', e', and g' ", respectively, red dotted lines.
- When m>1, $f_h(x, y)$ =1 is two super-hyperbolic curves, black solid curves.

C. Symmetric Contour and Profile Curves

Fig. 5 demonstrates some contour or profile curves $h(x, y)$=1 or $m(x, z)$=1 defined by (4) where f_i are two-branch lineal and super-hyperbolic functions in (5)-(6).

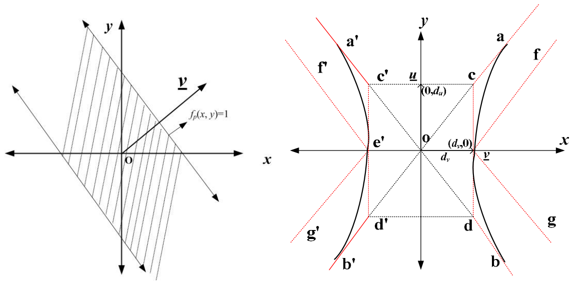


Fig. 4. (a).The iso-curve of a two-branch lineal function $fp(x, y)$. (b). The iso-curve of a two-branch super-hyperbolic function $fh(x, y)$, a pair of solid curves bounded in red dotted curves.

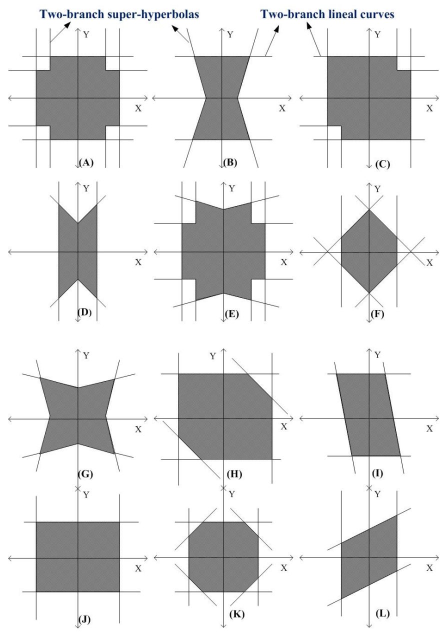


Fig. 5. Contour curves $h(x, y)$=1 or profile curves $m(x, z)$=1, which are defined by performing a super-ellipsoidal intersection blend on two-branch lines in (5) (parallel lines) and super-hyperbolas in (6) (folded lines).

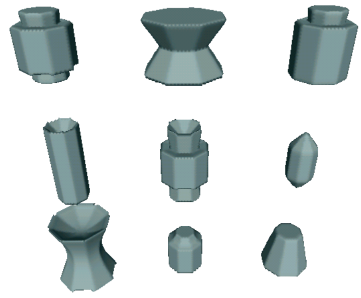


Fig. 6. Implicit spherical product surfaces $m(x, z) \otimes h(x, y)$ =1 defined by using the curves in Fig. 5(A)-(I) as $m(x, z)$ =1 and the curve in Fig. 5(K) as $h(x, y)$ =1.

When the curve in Fig. 5(K) is used as contour function $h(x,$

y) and is written by

$$h(x, y)=(f_{p1}(x, y)^{n1}+f_{p2}(x, y)^{n1}+f_{p3}(x, y)^{n1}+f_{p4}(x, y)^{n1})^{1/n1},$$

where $f_{p1}=|x/15|$, $f_{p2}=|y/15|$, $f_{p3}=|x/\sqrt{2}+y/\sqrt{2}|/15$, $f_{p4}=|-x/\sqrt{2}+y/\sqrt{2}|/15$ and n_1=20, and the curves in Figs. 5(A)-(I) are used as profile functions $m(x, z)$, then they generate spherical product surfaces $m(x, z)\otimes h(x, y)$=1 listed in Fig. 6.

V. Asymmetric Contour and Profile Curves

The iso-curves of two-branch lineal and super-hyperbolic functions in (5) and (6) is always in pairs, so $m(x, z)\otimes h(x, y)$=1 created from them always has a contour or a profile with symmetrical shapes as seen in Fig. 6. To develop $m(x, z)\otimes h(x, y)$=1 whose contour or profile has an asymmetrical shape, this section proposes:

1) Ray-linear one-branch lineal functions and
2) Ray-linear one-branch super-hyperbolic functions

whose iso-curves are a single curve, not in pairs.

A. Ray-linear One-Branch Lineal Function

A ray-linear one-branch lineal function $f_{sp}(x, y)$ with a single iso-curve as shown in Fig. 7(a) is defined by:

$$f_{sp}(x, y)=[f_p(x, y)]_+ = Max(0, f_p(x, y)) \quad (7)$$
$$f_p(x, y) = (\underline{v}\bullet[x, y]) / d_v,$$

where $\underline{v}$ is the unit normal vector of $f_{sp}(x, y)$=1 and d_v is the shortest distance from the origin to the line. Symbol $[*]_+$ stands for operation $Max(0, *)$ in this paper.

B. Ray-linear One-branch Super-hyperbolic Function

Let $\underline{v}$ and $\underline{u}$ be unit vectors in R^2, $\underline{v}\bullet\underline{u}$ be 0, d_v, d_u, and m all are greater than 0. Therefore, a ray-linear one-branch super-hyperbolic function $f_{sh}(x,y)$ with a single iso-curve as shown in Fig. 7(b) is defined by

$$f_{sh}(x,y)=[[f_v(x,y)]_+^m - f_u(x,y)^m]_+^{1/m}, \quad (8)$$
$$f_v(x, y)=(\underline{v}\bullet[x, y]) / d_v, f_u(x, y)=|\underline{u}\bullet[x, y]| / d_u,$$

where like those in (6), $\underline{v}$ and $\underline{u}$ control the orientation of the curve, d_v decides the shortest distance from the origin to the curve and m controls how much square the curve is. It can be proved that functions in (7)-(8) are ray-linear.

C. Asymmetric contour and profile curve

When used as $f_i(x, y)$ in (4), $f_{sp}(x, y)$ and $f_{sh}(x, y)$ in (7)-(8) generate asymmetric contour and profile curves whereas $f_p(x, y)$ and $f_h(x, y)$ in (5)-(6) symmetric contour and profile curves. This is shown in the following examples. In a case that: $h(x, y)$ is an intersection of four pairs of two-branch super-hyperbolas $f_h(x, y)$ in (6), whose shape is shown in Fig. 8 and which is written by

$$H(x,y)=(f_{h1}(x, y)^{n1}+f_{h2}(x, y)^{n1}+f_{h3}(x, y)^{n1}+f_{h4}(x, y)^{n1})^{1/n1},$$

where
$f_{h1}(x, y)$: $f_{v1}(x, y)=|x/12|$ and $f_{u1}(x, y)=|y/12|$,
$f_{h2}(x, y)$: $f_{v2}(x, y)=|y/12|$ and $f_{u2}(x, y)=|x/12|$,
$f_{h3}(x, y)$: $f_{v3}(x, y)=|x/\sqrt{2}+y/\sqrt{2}|/12$ and $f_{u3}(x, y)=|-x/\sqrt{2}+y/\sqrt{2}|/12$,
$f_{h4}(x, y)$: $f_{v4}(x, y)=|-x/\sqrt{2}+y/\sqrt{2}|/12$ and $f_{u4}(x, y)=|x/\sqrt{2}+y/\sqrt{2}|/12$,

And parameters m of f_{h1}, f_{h2}, f_{h3} and f_{h4} are set close to 1; $M(x, z)$ is an intersection of two pairs of parallel lines defined by $f_p(x, y)$ in (5), and it is written by

$$M(x, z)=(f_{p1}(x, z)^{1.1}+f_{p2}(x, z)^{1.1})^{1/1.1},$$

where $f_{p1}(x, z)=|x|$ and $f_{p2}(x, z)=|z/12|$;

Then, based on $h(x, y)$ and $m(x, z)$ stated above, surfaces $m(x, z)\otimes h(x, y)$= 1 where n_1 of $h(x, y)$ is set 100, 8, 2, 1.5, 1.1 and 0.7, respectively, are shown in Fig. 9. These surfaces have symmetric contours because two-branch functions are applied.

In another case that $h(x, y)$ is an intersection of four one-branch functions $f_{sp}(x, y)$ and $f_{sh}(x, y)$ in (7)-(8) and one two-branch function $f_h(x, y)$ in (6), whose shapes is shown in Fig. 10 and which is written by

$$H(x, y)=(f_{h1}(x, y)^{n1}+f_{sh2}(x, y)^{n1}+f_{sh3}(x, y)^{n1}+f_{sh4}(x, y)^{n1}+f_{sp5}(x, y)^{n1})^{1/n_1},$$

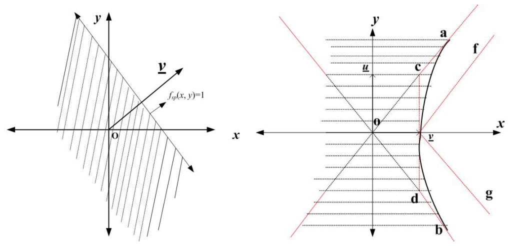
where
$f_{h1}(x, y)$: $f_{v1}(x, y)=|x/12|$ and $f_{u1}(x, y)=|y/12|$,
$f_{sh2}(x, y)$: $f_{v2}(x, y)=(y/12)$ and $f_{u2}(x, y)=|x/12|$,
$f_{sh3}(x, y)$: $f_{v3}(x, y)=(x/\sqrt{2}+y/\sqrt{2})/12$ and $f_{u3}(x, y)=|-x/\sqrt{2}+y/\sqrt{2}|/12$,
$f_{sh4}(x, y)$: $f_{v4}(x, y)=(-x/\sqrt{2}+y/\sqrt{2})/12$ and $f_{u4}(x, y)=|x/\sqrt{2}+y/\sqrt{2}|/12$,
$f_{sp5}(x, y)$: $f_{p5}(x, y)=(-y/12)$,

And m of f_{h1}, f_{sh2}, f_{sh3} and f_{sh4} are set close to 1; $m(x, z)$ is an intersection of two pairs of parallel lines defined by $f_p(x,y)$ in (5),

$$m(x,z)=(f_{p1}(x, z)^{1.1}+f_{p2}(x, z)^{1.1})^{1/1.1},$$

where $f_{p1}(x, z)=|x|$ and $f_{p2}(x, z)=|z/12|$;

Then, based on $h(x, y)$ and $m(x, z)$ stated above, surfaces $m(x, z)\otimes h(x, y)$=1 where n_1 of $h(x, y)$ is set 20, 4, 2, 1.5, 1.1 and 0.7, respectively, are shown in Fig. 11. These surfaces indicate that they have asymmetric contours because one-branch functions are applied.



(a) (b)

Fig. 7. (a). The shape of the iso-curve of a one-branch lineal function. (b). The shape of the iso-curve of one-branch super-hyperbolic functions, a single solid line bounded between red dotted curves

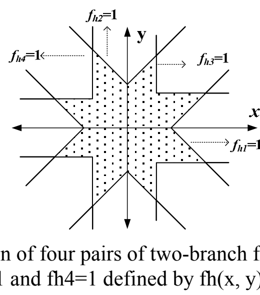


Fig. 8. The intersection of four pairs of two-branch functions, fh1=1, fh2=1, fh3=1 and fh4=1 defined by fh(x, y) in (6).

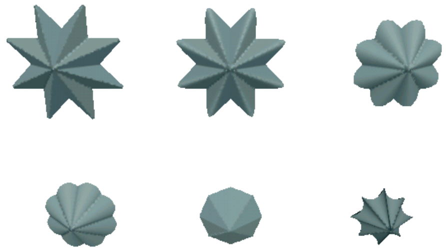


Fig. 9. The shapes of $m(x, z) \otimes h(x, y)=1$ where the curve in Fig. 8 defines contour curve $h(x, y)=1$ and n1 of $h(x, y)$ is set 100, 8, 2, 1.5, 1.1, and 0.7, respectively, for the objects from top left to bottom right.

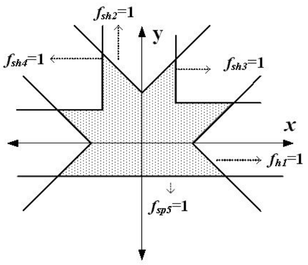


Fig. 10. The intersection of four one-branch functions, fsh2=1, fsh3=1, fsh4=1 and fsp5=1, and one two-branch function fh1=1.

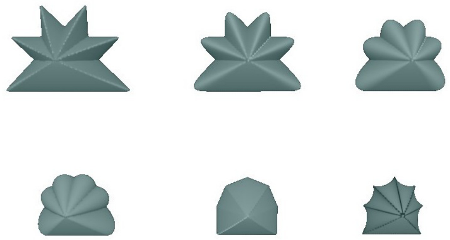


Fig. 11. The surfaces $m(x, z) \otimes h(x, y)=1$ where the curve in Fig. 10 defines the contour curve $h(x, y)=1$ and n1 of $h(x, y)$ are set 20, 4, 2, 1.5, 1.1 and 0.7, respectively, for the objects from top left to bottom right.

VI. Conclusion

To create new primitive defining functions with shapes more diverse than or different from existing defining functions, spherical product functions have been proposed to define implicit spherical product surfaces in this paper. A spherical product function is the composition of a contour function and a profile function, whose level curves determine the shapes of the contour and the profile of the implicit spherical product surface.

Besides, this paper has proposed a theorem that if both contour and profile functions are ray-linear, then the implicit spherical product surface has a parametric representation, which means it has both the advantages of implicit and parametric surfaces. According to the theorem, this paper also helps create new ray-linear contour functions and profile functions with more diverse shapes by the following:

1) This paper has proposed ray-linear two-branch lineal and super-hyperbolic functions, whose iso-curve is a pair of symmetric curves.
2) This paper has proposed ray-linear one-branch lineal and super-hyperbolic functions, whose iso-curve is a single curve.
3) This paper has also shown that ray-linear contour functions and profile functions can be created by performing ray-linear super-ellipsoidal intersection blend on ray-linear lineal and super-hyperbolic functions stated above.

Thus, an implicit spherical product surface can have symmetric contour and profile if ray-linear two-branch lineal and super-hyperbolic functions are used to create new contour and profile functions, and it can has asymmetric contour or profile if ray-linear one-branch lineal and super-hyperbolic functions are used. Especially, it can be parameterized, too.

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