Modeling Large Sliding Frictional Contact Along Non-Smooth Discontinuities in X-FEM

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***Abstract*—Modeling large frictional contact is a numerical tool to simulate abrasion resistant material. We have therefore proposed a new method to impose contact constraints in eXtended Finite Element Method (XFEM) framework. For this technique we impose penalty constraint on Node To Segment(NTS) elements. Moreover, the effect of friction has been investigated based on the Coulomb friction law. In our algorithm, new Lagrangian shape functions are proposed to solve the problems of the conventional Heaviside enrichment function. Finally, two numerical simulations for non-smooth constraints are delivered to show the efficiency of our algorithm.**

***Index Terms*—Coulomb law, large sliding contact, non-smooth constraint, X-FEM.**

I. Introduction

Nano-architecture as the new contemporary architectural style of the 21st century has revolutionized every aspects in this major even in the way architects think or how they inspire their ideas. This adaption of architecture with technology has peaked with the discovery of new conventions of different materials and has revolted the traditional way of thinking. For instance, Walt Disney concert Hall, designed by Frank Ghery in 2003 shows how the new materials has helped the architect to develop his ideas to come up with different new adaptable forms and construction system. Thus, this technology contributes to the architectural inspiration which relies on the creativity with no limits in order to be able to form new exquisite patterns of architecture. Architects therefore now can produce more controversial ideas and create novel designs for the current and future generations.

One kind of new materials which is considerably considered by architectural engineers has scratch proof and abrasion resistant quality. Nanotechnology makes it possible for substance to have this characteristic while they maintain transparent. Scratch-resistance is a desirable property for many materials and also for coatings which can be applied to wood, metal and ceramics.

To improve our program to consider this physical phenomenon, we needed a very robust algorithm for modeling large frictional contact. In the following this algorithm is discussed completely.

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Imposing constraints in eXtended Finite Element Method (XFEM) framework is one of the debating issues between all computational mechanics researchers. Not directly related to contact problems, firstly professor Ji concerned the difficulty to impose conditions on an interface with XFEM[1]. After that, this concept was investigated by many researchers from different aspects, and several costly methods have proposed to solve the problem. For instance, LBB stabilization algorithm was suggested by Professor Geniaut et al. and then used by other groups in modeling large sliding contact[2]. What's more, this problem was investigated by Professor Moes et al.[3, 4] to find the reason of oscillations of contact pressure by imposing constraints in XFEM. But, it should be mentioned that all methods presented have a costly numerical algorithm.

In this work, a new algorithm has been presented to model large deformation- large sliding frictional contact in XFEM. Node To Segment (NTS) elements have been used in order to simulate large relative displacements of interfaces. This method has a search algorithm which can efficiently model the updated status of the interfaces. Moreover, new shape functions have been used in order to alleviate the burden caused by ordinary Heaviside enrichment function. In addition, the effect of friction on the movement of two sliding bodies has been presented. Furthermore, in order to impose the non-penetration constraint, the stiffness of the normal spring has a great value based on the penalty method. In the following, first, our new shape functions have been introduced. Then, after a brief explaining of NTS elements in Finite Element (FE), the usage of this method with our shape functions will be discussed. Finally, numerical simulations result has been presented to show the accuracy of our new algorithm.

II. Generalized Finite Element Method

$$u(X) = \sum_I N_I(X)\bar{u}_I + \sum_J N_J(X)(\Psi(X) - \Psi(X_J))a_J \qquad n_I \in n_T \text{ and } n_J \in n_e \tag{1}$$

The difficulty of mesh generation in finite element method motivated the scientist to invent new method which is not dependent to this factor. Therefore, XFEM proposed which can model the discontinuous displacement field inside the element by adding several degrees of freedom to the nodes of that element[5], and changing the displacement field of the element to the following equation to consider the discontinuity. So, the standard FE approximation can be enriched with additional functions by using the notion of partition of unity. The enriched approximation in modeling of discontinuity Γ_c can be expressed in form (1).

The first term of above equation denotes the classical finite element approximation and the second term indicates the enrichment function considered in X-FEM. In this equation, $\bar{u}_I$ is the classical nodal displacement, a_J the nodal degrees of freedom corresponding to the enrichment functions, $\Psi(X)$ the enrichment function, and $N(X)$ the standard shape function. In equation (1), n_T is the set of all nodal points of domain, and n_e the set of nodes of elements cut by the interface, i.e. $, n_e = \{ n_J : n_J \in n_T \}$, and $, \omega_J \cap \Gamma_c \neq 0$, with $, \omega_J = supp(n_J)$, denoting the support of nodal shape function $N_J(X)$, which consists of the union of all elements with n_J as one of its vertices, or in other words the union of elements in which $N_J(X)$ is non-zero. The idea of adding a displacement field term to the continuous displacement field of the ordinary FEM could bring a lot of works in this field, and various enrichment functions used for different types of discontinuities [6,7,8]. The conventional enrichment function for contact problems is Heaviside which should model the large relative sliding of two side of the element in which the interface has been passed. However all computational simulators who are involved in working with this type of enrichment function, assert that they have problem for contact pressure on the interface even in static contact interface. Moreover, in large sliding they can obviously see that this displacement formula is not responsive for the discontinuous displacement field of the element as it still have the term of connectivity between nodes on both sides which are not beside each other anymore. In other word, in the state such as shown in Fig. 1 the discontinuous displacement field will bring stiffness between two sides which do not have any connection any more. And it cannot represent the total discontinuities of the system. Therefore, a new shape function has been proposed which has the FEM basement and does not have any problem in modeling large sliding of the system. Furthermore, these shape function will not bring the oscillation of the contact pressure and we have the accuracy near to FEM simulations. In this simulation we have two types of elements as shown in Fig. 2. Therefore, we have two types of shape functions for these different elements as it is specified in (2) and (3).

In our method in situation similar to case (2), node (1) is not enriched, instead node (3) has two enrichment shape function These shape functions can model the large relative sliding of two sides of interface very well and do not show any connectivity between two side nodes.

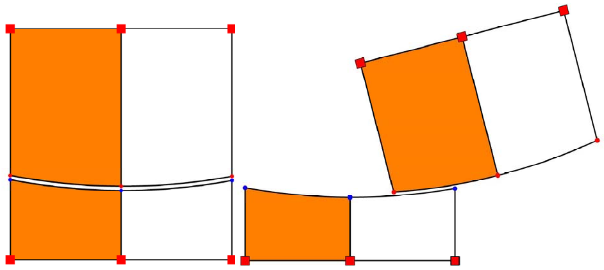


Fig. 1. Typical large sliding in XFEM

TABLE I. EQUATIONS (2),(3)

$$
\begin{aligned}
N_1 &= \frac{(1-\xi)(1-\eta)}{4}\, H(-S(x,y)) \\
N_2 &= \frac{(1+\xi)(1-\eta)}{4}\, H(-S(x,y)) \\
N_3 &= \frac{(1+\xi)(1+\eta)}{4}\, H(S(x,y)) \\
N_4 &= \frac{(1-\xi)(1+\eta)}{4}\, H\big(S(x,y)\big) \\
N_1^{enr} &= \frac{(1-\xi)(1+\eta)}{4}\, H(-S(x,y)) \\
N_2^{enr} &= \frac{(1+\xi)(1+\eta)}{4}\, H(-S(x,y)) \\
N_3^{enr} &= \frac{(1+\xi)(1-\eta)}{4}\, H(S(x,y)) \\
N_4^{enr} &= \frac{(1-\xi)(1-\eta)}{4}\, H(S(x,y))
\end{aligned}
\qquad (2)
$$

$$
\begin{aligned}
N_1 &= (1-\xi-\eta)\, H(-T(x,y)) \\
N_2 &= H(-S(x,y)) \begin{cases} \xi & T(x,y) \le 0 \\ \dfrac{(1+\xi)(1-\eta)}{4} & T(x,y) > 0 \end{cases} \\
N_3 &= \eta\, H(S(x,y)) \\
N_4 &= H\big(-S(x,y)\big) \begin{cases} \eta & T(x,y) \le 0 \\ \dfrac{(1-\xi)(1-\eta)}{4} & T(x,y) > 0 \end{cases} \\
N_2^{enr} &= \frac{(1+\xi)(1+\eta)}{4}\, H(-S(x,y)) * H(T(x,y)) \\
N_3^{enr(1)} &= \xi\, H(S(x,y)) \\
N_3^{enr(2)} &= (1-\xi-\eta) H(S(x,y)) \\
N_4^{enr} &= \frac{(1-\xi)(1+\eta)}{4}\, H(-S(x,y)) * H(T(x,y))
\end{aligned}
\qquad (3)
$$

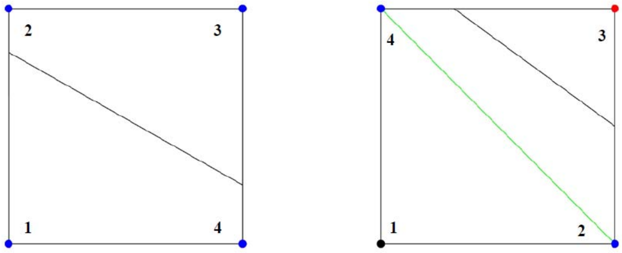


Fig. 2. Two types of enriched elements

III. Node To Segment Elements

The main factor in large relative displacement between two interfaces is having an efficient search algorithm in order to update the new status of interfaces during two bodies movement. Furthermore, it should be capable of imposing constraint in the new situation. NTS contact elements not only have an efficient search algorithm, but also it establishes the condition with a very easy method. Its formulation is based on the energy stored in the springs which is considered at the normal and tangential direction between slave node and master segment Fig3. In other words, to impose the non-penetration condition it considers a spring in the direction of normal vector of the master segment, and based on the penalty algorithm its stiffness considered as great number. Besides, it considers tangential spring to model the tangential stiffness which two interface show during movement along each other based on the friction rules. Therefore, two following terms will be added to the whole energy of the system, which are the energy stored in these springs.

$$\Pi = \frac{1}{2}\alpha_n(du_n)^2 + \frac{1}{2}\alpha_t(du_t)^2 \tag{4}$$

If we consider the shape function of the whole of the Node To Segment element as:

$$\boldsymbol{N} = \begin{bmatrix} 1 & 0 & -(1-\xi) & 0 & -\xi & 0 \\ 0 & 1 & 0 & -(1-\xi) & 0 & -\xi \end{bmatrix}$$

We can consider the normal relative movement (the relative penetration) as , $du_n = \boldsymbol{n}_m^T \boldsymbol{N}_n d\bar{\boldsymbol{u}},$ in which ,$\boldsymbol{N}_n = (\boldsymbol{n}_m \otimes \boldsymbol{n}_m)\boldsymbol{N},$ and $\boldsymbol{n}_m$ is the unit normal vector to master segment. Moreover, the tangential relative movement (the sliding) is calculated based on, $du_t = \boldsymbol{t}_m^T \boldsymbol{N}_t d\bar{\boldsymbol{u}}$, that ,$\boldsymbol{N}_t = (\boldsymbol{I} - \boldsymbol{n}_m \otimes \boldsymbol{n}_m)\boldsymbol{N},$ and t_m is the unit tangential vector along the master segment. Therefore by differentiating from the total energy of the system relative to displacement we have two more terms, ($\boldsymbol{K}_n^c = \boldsymbol{N}_n^T \alpha_n \boldsymbol{N}_n$, $\boldsymbol{K}_t^c = \boldsymbol{N}_t^T \alpha_t \boldsymbol{N}_t$), added to the ordinary finite element problem which show the normal and tangential stiffness matrix. Therefore, we can impose our constraints very easily just by adding these two terms to the whole stiffness of the system. It should be mentioned that based on the coulomb friction law the max tangential strength of the system cannot be more than the max resistance of material which is ,$(\Delta F_t)_{max} = C_f + \mu_f\,(\Delta F_n)$, and so if it increase this value the tangential strength, and the tangential relative movement will be modified based on the following formula.

$$\alpha_t = \frac{(\Delta F_t)_{max}}{\Delta u_t} \tag{5}$$

$$\Delta F_t = (\Delta F_t)_{max}\frac{\Delta \boldsymbol{u_t}}{\Delta u_t} \tag{6}$$

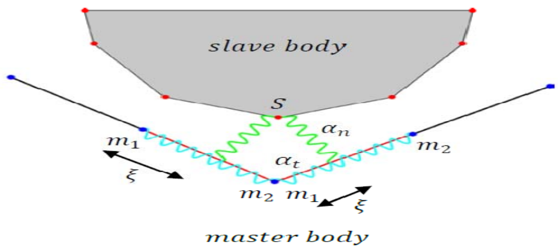


Fig. 3. Modeling of contact constraints in normal and tangential directions in concave edge

IV. NTS Contact Elements in XFEM Framework

As it was shown in our paper [9] completely, to use the wo nodes which are at extremes of the edge. So, the stiffness of the slave master pairs will be added if they were active to each other. This stiffness is similar to the Finite Element Method(FEM) stiffness matrix of NTS elements as based on our new shape functions each added degree of freedom to the system has a meaning of these slave and master nodes. Therefore after finding slave and master nodes, we conduct a search algorithm to find active pairs, and then after we will add the stiffness of these pairs to the total stiffness matrix of the system.

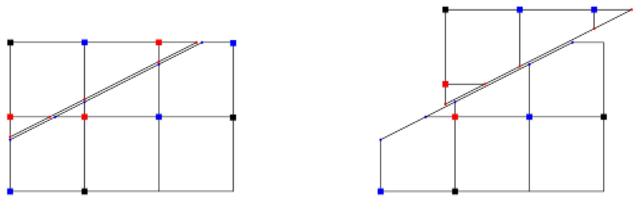
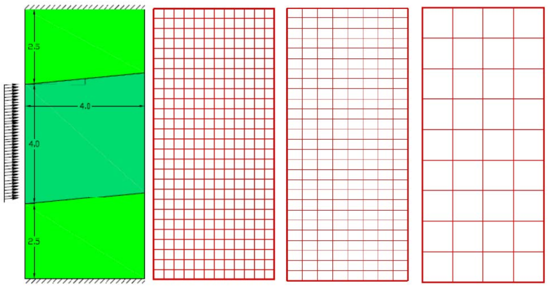


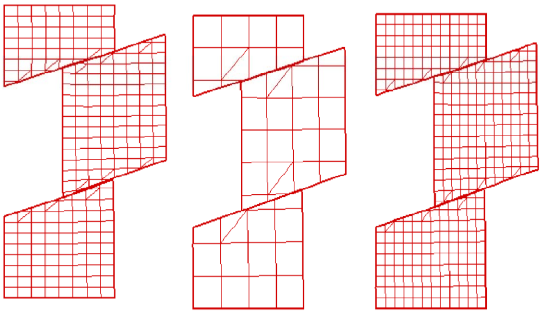
Fig. 4. The large sliding along discontinuity

V. Sliding Along Inclined and Curved Interfaces

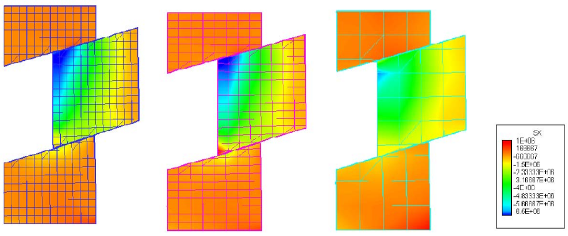
The main objective of this study was modeling large frictional contact along inclined and curved interfaces. In the following, the results of these two simulations are delivered. The problem statement for the first example is shown in Fig. 5(a). Middle plate is subjected to large sliding while the inclined displacement of 2.0 cm is applied at the left hand side of it. Three X-FEM meshes are employed to model the sliding, as shown in Fig.5(a). The deformed configuration of X-FEM meshes are presented in Fig. 5(b) for three different meshes. In Fig. 5(c)-5(e), the distribution of σ_x, σ_y, and τ_{xy}contours are shown for various X-FEM meshes. Good agreement can be observed among various X-FEM meshes. This example clearly illustrates the performance of proposed X-FEM technique in modeling of large deformation – large sliding contact problems with inclined interfaces. The force-displacement diagram shows the different force needed for various coefficients of friction.Fig.6. Finally, as depicted in the contact pressure diagram, we have not confronted with oscillation of stress along the interface as shown in Fig. 7. This can be considered as the main virtue of our new shape functions. The next example has been chosen to show the power of our algorithm in modeling non-linear contact interfaces. The results of the XFEM meshes have been compared to the FEM outcome. The problem statement for this example is shown in the Fig.8(a). The upper block will have a large sliding relative to lower one when it subject to inclined pre-displacement of 2cm at the left hand side. Various contours show a good compatibility between XFEM and FEM results Fig.9(a)-(c). Furthermore, force-displacement diagram has been shown in the last figure, which shows good agreement between XFEM and FEM results.



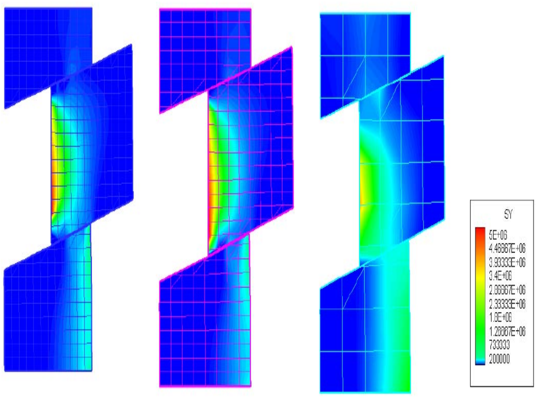
(a)



(b)

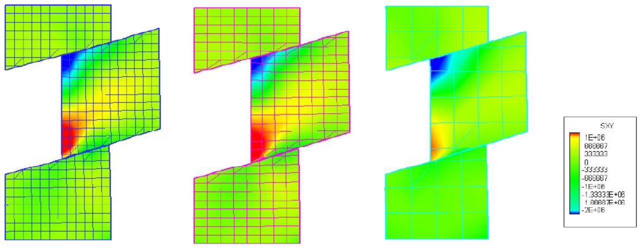


(c)



(d)

In numerical examples which are given, good agreement between different sizes of mesh was observed. The force Displacement diagram does not show any substantial difference due to various friction coefficients. The reason is that we do not have any compression loading on contact interface. If we got result with the same geometry, but by putting compression loading on upper and lower cube, we could see more difference in force-displacement diagram.



(e)

Fig. 5. a) Geometry and three XFEM meshes. b) Deformed configuration. c) σ_x contour. d) σ_y contour. e) σ_{xy} contour.

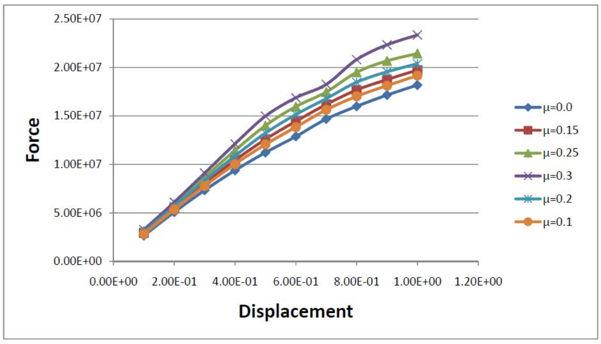


Fig. 6. Force-Displacement diagram

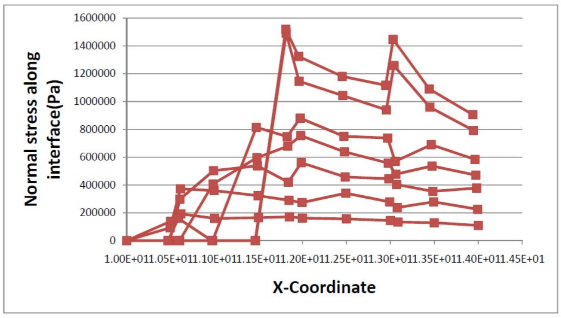
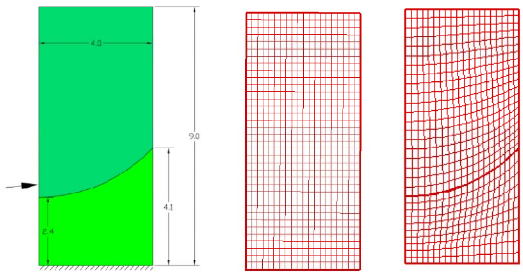
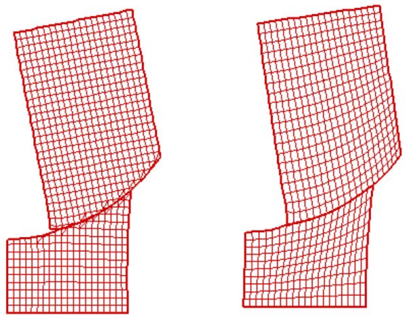


Fig. 7. The distribution of contact pressure along the contact surface at different sliding

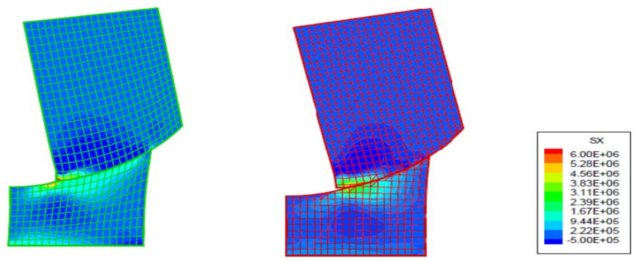


(a)

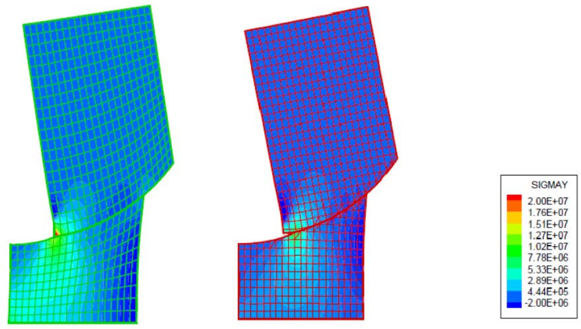


(b)

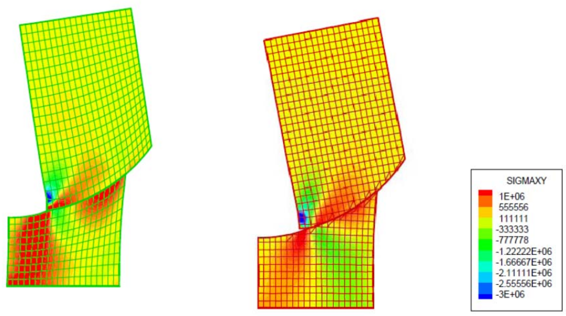
Fig. 8. a) Geometry and FEM and XFEM meshes. b) Deformed configuration



(a)



(b)



(c)

Fig. 9. a) σ_x contour. d) σ_y contour. e) σ_{xy} contour

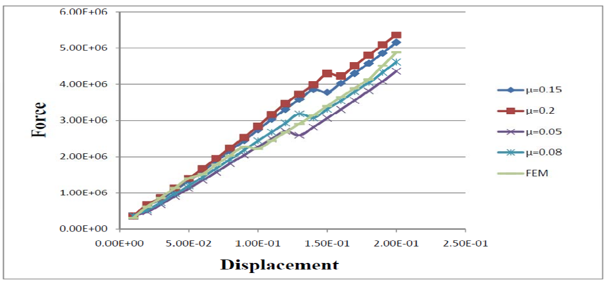


Fig.10 Force-Displacement diagram

VI. CONCLUSIONS

The main objective of this study was modeling large frictional contact in order to simulate material against abrasion. We therefore solved the problem of shape function of the XFEM as it could not describe the displacement field especially in large deformation. Furthermore, to impose the non-penetration condition of contact we have used the node to segment elements combined with penalty method. To consider friction between interfaces, we have used the coulomb friction law to revise our maximum strength of the interface. A numerical simulation has been shown to investigate the accuracy of our algorithm on two kinds of interfaces. Good agreement was observed in these simulations which show the accomplishment of our algorithm in modeling large sliding frictional contact problems. With this algorithm, we are able to model different material to see which one is more resistant to friction and can be considered as a good abrasion resistant material. For developing our algorithm, we are intended to add this technique to fracture modeling algorithm. In that case, we can see the effect of frictional contact after two sides of the crack separate and start to slide along each other. Therefore, we will be able to calculate the amount of energy dissipated due to fracture mechanics and also frictional contact. Finally, we can deliver numerical analysis of the toughness of the specimen and decide that which material show more toughness under loading.

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He is interested in application of numerical simulation to improve the performance of architectural designing. He started with abrasion resistance materials. Having a collaboration with applied mechanics scientists, he could develop a numerical tool for modelling large frictional contact which is the originate of the abrasion. He has published 2 journal papers and 3 conference papers on his research by now.

The paper of Mr. Taheri has been chosen as the best paper in the 2nd international conference on Mechanical, Industrial, and Manufacturing Technologies(MIMT 2011) which was held in Singapore.