Multi-Resolution Triangular Approximation of Terrains

Yong H. Chung, Dae S. Chang, Ka R. Yang, Ji W. Oh, Kang H. Cho, and Sang C. Park

***Abstract*—Presented in this paper is a procedure to generate multi-resolution models for a terrain height field. Although there have been many existing algorithms for simplifying geometric models, they cannot be applied to a height field, because most of them have been developed in the context of triangular meshes. An intuitive way is to apply existing algorithms after converting the height field into a triangular mesh. However, the conversion procedure requires an additional computational load as well as a large amount of memory as a general triangular mesh needs to maintain the topology information as well as the geometry information. To cope with the problems, we propose a procedure working directly on a height field. The proposed algorithm has been implemented and tested with various examples.**

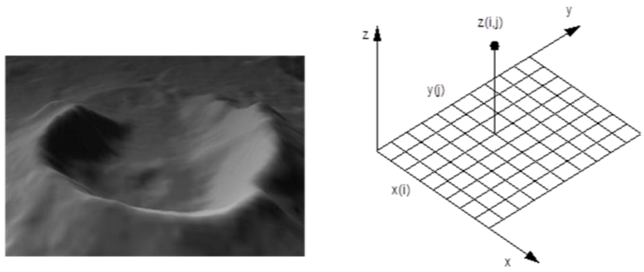
***Index Terms*—Height field, multi-resolution, quadric error metrics, simplification.**

I. Introduction

In the area of the modeling & simulation which requires synthetic environment, the terrain data is one of the backbone elements that provide a consistent simulation environment. Terrain data is usually represented as a height field which is a set of data values that are sampled at certain specified points in a planar domain. Applications of terrain data include computer graphics, interactive 3D games for entertainment, geographic information systems, and military simulators (flight simulators, ground vehicle simulators, and submarine simulator). As many of the applications are interactive with human operators, it is necessary to render terrain data rapidly and with high fidelity [1]-[5].

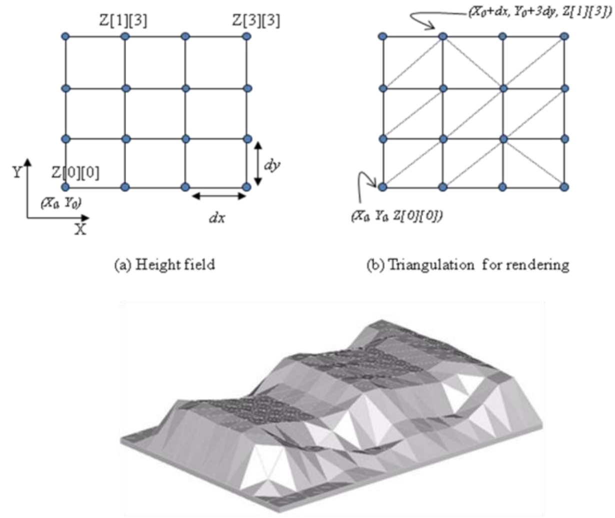
As shown in Fig. 1, a height field is a 2D-array of real numbers in which the height values of the terrain, sampled at regular grid-points, are stored. In order to render a height field, it is necessary to convert the height field into a triangular mesh. Intuitively, we can render a height field, by drawing two triangles for each cell, as shown in Fig. 2. In order to build a triangular mesh, two triangles are created from four points of a range image that are in adjacent rows and columns.

The shorter of the two diagonals between the points is determined. This is used to identify the two triplets of points that may become triangles.



(a) Terrain data (b) Height field structure

Fig. 1. Terrain data & height field.



(c) Height field rendering

Fig. 2. Height field rendering by triangulation.

Although, rendering a height field is conceptually quite simple, it may be prohibitively expensive for the terrains of any significant size. As the height fields may consist of a large number of polygons, even the most high performance visualization systems face great difficulties to visualize even the moderately sized height fields at interactive frame rates. A common solution for the visualization systems is to reduce the complexity of the height field while maintaining a high image quality with the usage of limited system resources. The investigation of multi-resolution methods to dynamically adapt rendered model complexity has continued to be a very active computer graphics research area.

Existing simplification methods can be roughly divided into two different groups; iterative removal approach [6]-[9] and iterative insertion approach. The proposed simplification algorithm in the paper employs the iterative removal approach, which starts from a full model. Although there are many proven simplification algorithms for general 3D triangular meshes, it impossible to directly apply those algorithms to the simplification of a height field. An intuitive way, is to apply those algorithms after converting the height

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field into a triangular mesh (initial triangular mesh), as shown in Fig. 2. However, the conversion procedure will require an additional computational load as well as a large amount of memory as a general triangular mesh needs to maintain the topology information as well as the geometry information. Usually, a general triangular mesh requires at least 15 times a greater memory compared to a height field for the same geometry.

The objective of this paper is to develop an algorithm through which multi-resolution triangular meshes can be directly generated from a height field without converting the height field into an initial triangular mesh. For a simplification, the paper employs the edge contraction approach based on the QEM (quadric error metrics) method [10]. The remainder of this paper is organized as follows. Section II presents the modified QEM for the simplification of a height field. Section III describes the proposed simplification procedure. The concluding remarks are presented in Section IV.

II. Modified QEM for Height Field Simplification

As mentioned earlier, the proposed procedure generates the multi-resolution models of a height field. For the approximation of a height field, this paper employs the edge contraction approach that is based on the QEM method. Since the original QEM method is developed for a general triangular mesh, we modified the QEM method to reflect the inherent attributes of a height filed.

Before further explanation on the proposed approach, it is necessary to give a brief explanation on the concept of a Q matrix (details of the QEM method can be found in [10]). The QEM method is based on the observation that each vertex is the solution of the intersection of a set of planes – namely, the planes of the triangles that meet at the vertex. It is possible to associate a set of planes (triangles) with each vertex, and also to define the error of the vertex with respect to this set as the sum of the squared distances to its planes:

$$\Delta(v) = \Delta([v_x, v_y, v_z, 1]^T) = \sum_{p \in planes(v)} (p^T v)^2 \qquad (1)$$

where, $p = [a, b, c, d]^T$ represents the plane that is defined by the equation $ax+by+cz+d = 0$ where $a^2+b^2+c^2=1$. The error metric can be rewritten in the quadratic form:

$$\Delta(v) = \sum_{p \in planes(v)} (v^T p)(p^T v) = \sum_{p \in planes(v)} v^T (pp^T) v = v^T \Big(\sum_{p \in planes(v)} K_p \Big) v = v^T (Q) v \qquad (2)$$

where, K_p is the matrix:

$$K_p = PP^T = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix} \qquad (3)$$

The fundamental error quadric, K_p can be used to find out the squared distance of any point in space to the plane p. We can sum these fundamental quadrics together, and represent an entire set of planes by a single matrix Q. For a vertex, the Q matrix can be computed by adding all the fundamental error quadric (K_p) of the triangles that share the vertex. After the computation of the Q matrices for all vertices, it is possible to perform the edge contractions in order to simplify the geometry. In order to perform this, the QEM method computes an optimal contraction target (V_c) for an edge [V_i, V_j]. During this, the contraction cost of ([V_i, V_j] → V_c) becomes $\sqrt{v^T (Q_i + Q_j) v}$. As contraction cost implies to the approximation error, the cost should be smaller than the given tolerance.

While the original algorithm of the QEM method finds an optimal position for the contraction target, the proposed approach in this paper chooses V_i or V_j for the contraction target. The restriction on the contraction target is imposed because the proposed approach should inherit the memory efficiency of a height field. In other words, new vertices are not allowed in the multi-resolution models. In order to do this, the contraction cost in our approach is defined as follows.

Contraction cost of an edge [V_i, V_j]

If ($v_i^T (Q_i + Q_j) v_i < v_j^T (Q_i + Q_j) v_j$)

cost = $\sqrt{v_i^T (Q_i + Q_j) v_i}$; // V_i: contraction target, [V_i, V_j] → V_i

else

cost = $\sqrt{v_j^T (Q_i + Q_j) v_j}$; // V_j: contraction target, [V_i, V_j] → V_j

III. Mr-Height-Map

As the contraction cost can be considered as the sum of the distances to the planes that share V_i or V_j and it is possible to contract the edge if the cost is smaller than the given tolerance. We implicitly track sets of planes by using a single matrix. Instead of computing a set union (planes(V_1) U planes(V_2)), we simply add the two quadrics (Q_1+Q_2).

As mentioned above, the proposed approach employs an iterative contraction of edges (vertex pairs). An edge contraction, which is denoted as [V_i, V_j] → V_i (or V_j), connects all the incident edges of V_j to V_i, and it deletes the vertex V_j. If V_j is merged into V_i then, we refer V_i and V_j to '*a predator vertex*' and '*a prey vertex*', respectively. Subsequently, any edges or faces that become degenerated are removed. Normally, an edge contraction removes an edge and two triangles that share the edge. Fig. 3-(a) shows a height field with 8 triangles. As the topology is regular (each cell consists of two triangles), it is not necessary to maintain complicated data structures like a general triangular mesh does. However, edge contractions cause an irregular topology that is shown in Fig. 3-(b). Although the irregular topology can be implemented easily on a general triangular mesh, we want to find out a more efficient way to represent the irregular topology by utilizing the inherent attributes of a height field.

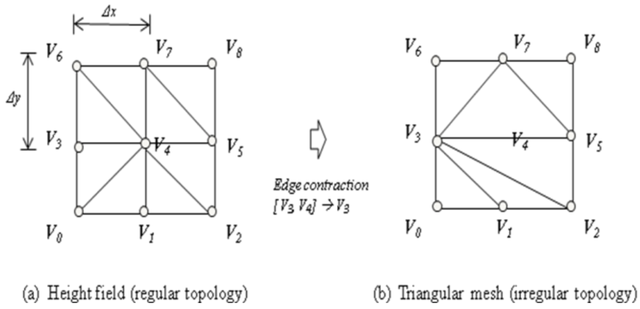

Fig. 3. Edge contraction on a height field.

A decimation procedure can be considered as a sequence of edge contractions, and each edge contraction has its own approximation error and so, it is called as a contraction *'cost'*. Let's assume that we are maintaining a full decimation procedure (edge contractions & their costs) of a height field for a tolerance (εm). At this time, a decimated model for a certain tolerance ε (ε < εm) can be constructed by applying edge contractions that have costs less than ε among the full decimation procedure. In this manner, we can intuitively generate the multi-resolution models of a height field for any tolerances that within a given tolerance range (less the maximum tolerance, ε_m), as long as we maintain the decimation procedure, a sequence of edge contractions. One of the main ideas of this paper is to develop a new data structure called as a *'MR-height-map'* (to be described later). This is developed to effectively maintain the decimation procedure of a height field. The MR-height-map can be considered as a combination of a conventional height field and the decimation procedure, an ordered sequence of edge contractions. While a height field is an array of real numbers, a MR-height-map is an array of containers and each of it can store a real number as well as the decimation related information.

The overall scheme of the proposed algorithm consists of four main steps: 1) Identification of two triplets for each cell of a given height field 2) The computation of the Q matrices for all vertices in the height field; 3) Repeated performances of the edge contractions for the given maximum tolerance (ε_m), and the renewal of the MR-height-map to reflect the topology changes; and 4) Extraction of a simplified triangular mesh for a certain tolerance (ε) by applying edge contractions that has costs less than ε among the edge contractions. Fig. 4 shows an example of the decimation procedure that consists of three edge contractions. Fig. 4-(a) shows a height field which consists of 9 vertices. As mentioned earlier, the shorter of the two diagonals between the points is determined, and this is used to identify the two triplets of points that may become triangles. For a single cell, there can be two types of triangulation (u-type & d-type), as shown in Fig. 4-(a). The topology of the initial state is very regular. However, edge contractions cause the irregular topology as shown in Fig. 4-(b), (c) and (d). The easiest way to represent the irregular topology is to employ a general triangular mesh that consumes a large amount of memory. Usually, a general triangular mesh requires at least 15 times a greater memory compared to the height field.

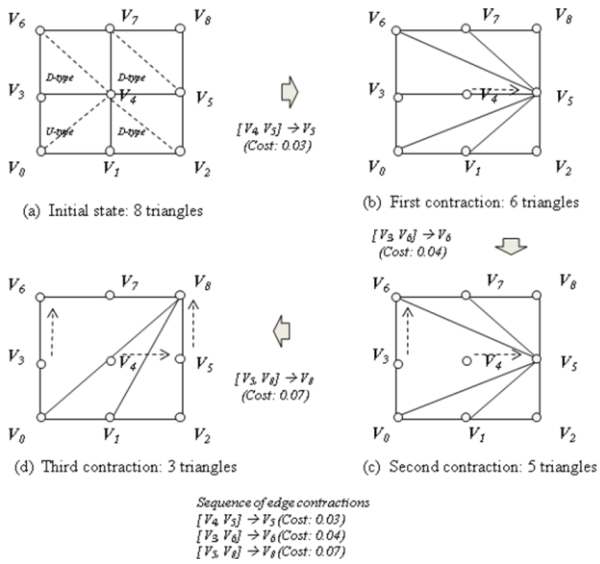

Fig. 4. Sequence of edge contractions.

In order to cope up with this problem, we store the topology information in the MR-height-map, instead of a general triangular mesh. From the MR-height-map, we can extract a simplified triangular mesh for a certain tolerance (ε) by applying edge contractions that has lower cost compared to ε among the sequence of edge contractions. If we want to have a simplified mesh for a 0.05 tolerance (ε =0.05) for the example shown in Fig. 4-(a), the result will be Fig. 4-(c).

While the first and the second edge contractions can be applied, the third edge contraction cannot be applied as its approximating cost (0.07) is larger than the given tolerance (0.05). The details of the proposed procedure will be addressed in the next section.

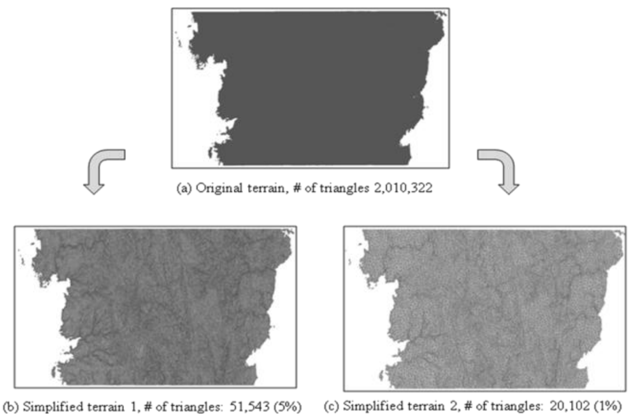

Fig. 5. Terrain decimation, example A.

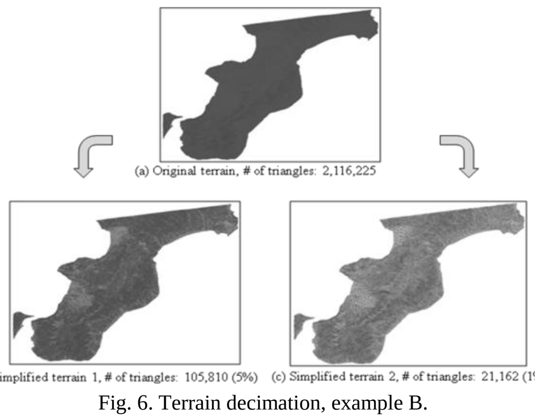

Fig. 6. Terrain decimation, example B.

The prototype of the proposed template based modeling approach has been implemented and tested with several examples. The C++ language in a Visual Studio environment was used, with OpenGL for the graphical rendering. Fig. 5, Fig. 6 show simplified triangular meshes that were extracted with different tolerances.

IV. Conclusions

For an efficient handling of a terrain data, decimation methods of a height field are motivated whenever a representation at low resolution is adequate to the needs of the applications. Although there are many proven simplification algorithms for general 3D triangular meshes, it impossible to directly apply those algorithms to the simplification of a height field. An intuitive way is to apply those algorithms after converting the height field into a triangular mesh. However, the conversion procedure will require a large amount of memory. To relieve the difficulty, we propose a new algorithm working directly on a height map without converting the height field into an initial triangular mesh. The proposed algorithm is based on a data structure called as a '*MR-height-map (multi-resolution-height-map)*'. The MR-height-map is able to generate the multi-resolution models for height fields. The key idea of the proposed methodology is to apply the edge contraction approach to a height field. A decimation procedure can be considered as a sequence of edge contractions. Each edge contraction has its own approximation error called as a contraction *'cost'*. A decimated model for a certain tolerance ε can be constructed by applying the edge contractions that has lower costs compared to ε among the full decimation procedure. The

MR-height-map can be considered as a combination of a conventional height field and the decimation procedure, an ordered sequence of edge contractions. The proposed method provide two major benefits compared to the conventional QEM method; 1) Saving more than 40% of memory, and 2) Supporting multi-resolution models for a height map rather than just a single simplified model.

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