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A System Identification Algorithm for Vehicle Lumped Parameter Model in Crash Analysis

Javad Marzbanrad and Mostafa Pahlavani

***Abstract*—This study has investigated a vehicle lumped parameter model (LPM) in frontal crash. There are several ways for determining spring and damper characteristics and type of problem shall be considered as system identification. This study use genetic algorithm (GA) procedure, being an effective procedure in case of optimization issues, for optimizing errors, between target data (experimental data) and calculated results (being obtained by analytical solving). In this study analyzed model in 5-DOF then compared our results with 5-DOF serial model. In this paper, the solution method of crash equations for lumped parameter is investigated in discrete analysis method and presented a general solution with the help of numerical solution.**

***Index Terms*—Vehicle, Lumped-parameter model, Genetic algorithm, Optimization, System identification.**

I. Introduction

Historically, those considerations about the manner of decorating materials and necessities about physical structure of vehicle have resulted in designing structure and its body. Generally, final design of a vehicle is the product of a long term process, being derived by several tests and supported by simple linear stiffness ways. By developing software and hardware, it is possible to use more analytical facilities, making several tools, for analytical designing modern structure of vehicle. Therefore, engineers are able to meet their growing needs and better performance of crashworthiness and safe driving. These tools include lumped parameters models (LPMs), Beam element models, hybrid models and finite elements models (FE). Although, they are different in case of complexity, but each is designed, on the basis of structural mechanics, needing into stability of mass, momentum and energy. During recent years, auto industry has confronted with most requests of customers, law makers and media, for production safe vehicles. This progress includes improving crashworthiness of structure in crashes, for example, Federal Motor Vehicle Safety Standard (FMVSS), New Car Assessment Program (NCAP), Insurance Institute for Highway Safety (IIHS), consistency test and insuring from keeping children and the short adults. In other words, it is needed to use vast picture from crashworthiness of vehicle.

In 1970, Kamal [1] presented a simple and strong model for simulation of crashworthiness in frontal crash. As, providing acceptable results, data was used by crash engineers, widely. Note, spring characteristics were experimentally determined in static damper.

Also in 1988, Magee [2] presented a model from crashing with barrier. This study used actual crashes information, for determining properties of springs, masses and breaking models. This model has been designed with considering the properties of load-moving springs, for obtaining best consistency of accelerations, its peak and crash scheduling. Cheva et al. [3] presents a one dimensional lumped mass model. This used simulations of finite elements, for determining spring properties. Also, recorded normalized acceleration in test and simulation of lumped mass have had desired consistency with each other.

Alexandra et al. [4] has presented a lumped mass model in frontal/offset crash, in national Highway Traffic Safety Administration (NHTSA), being directly extracted structural properties of vehicle from data, related into crash test. Thomas and Joseph [5] designed a model, being indicated in Figure 1. Also, Tomassoni [6], considered a model, presented in Figure 2.

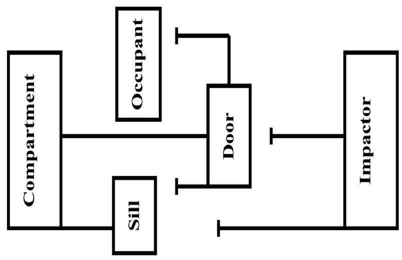

Figure 1. LPM as stated by Thomas and Joseph in side crash. [6]

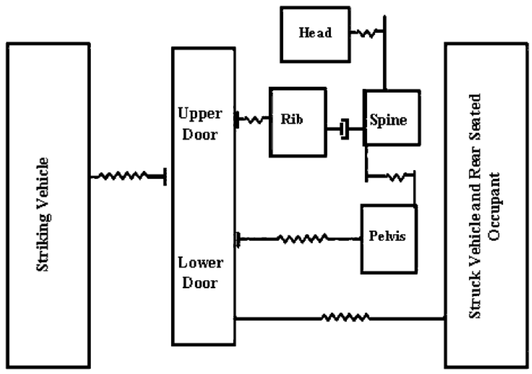

Figure 2. LPM as stated by Tomassoni in side crash. [7]

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Kim and Arora [7] studied on linear and nonlinear systems for vehicle crash. It was performed, theoretically, for the purpose of stating the importance of numerical ways in modeling and analyzing crash.

Except finite element models, being time consuming and difficult, hybrid models of finite element and Lumped mass models are paid attention by analysts and designer, such as Hollowell research, in 1986 [8, 9]. In 1986, Ni and Song [10] described 3 methods for simulating vehicle structures in crashes.

In 2008, Deb and Srinivas [11] focused on lumped mass model in side crash, presenting a simple and comprehensive model. Their studies were performed, on the basis of attracted energies comparisons, in inside impact of cart and vehicle and lumped mass model with obtained results, from finite element simulations.

This study presents a simple and comprehensive model with linear spring and damper, for modeling a frontal crash. Also, it shall consider differences of deceleration's peak and deceleration on occupant. It is necessary to use damper, because of measuring amount of vehicle structure damping, being made by existing injecting foams.

II. Equation and Analyzing Solution Algorithm

Jonsén et al. [12] had a research about hybrid model of LPMs and FEMs. In this section, Jonsén's method in LPMs' equations solving is presented then a similar method will be approved.

Figure 3 shows a n-degree of freedom LPM and Equation (1) described in continuous time by a second-order differential equation:

$$M\ddot{X} + C\dot{X} + KX = bf \tag{1}$$

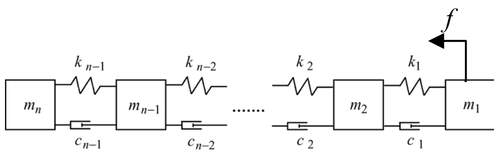

Figure 3. One-dimensional mass spring damper model.

where m, c and k represent $n \times n$ mass, damping and stiffness matrices, respectively. The displacement x is an $n \times 1$ vector, and x and x are both vectors of the same size with velocities and accelerations. The matrix b is the $n \times r$ input matrix and f is an $r \times 1$ vector of input excitations. By introducing the state vector X as $[x\dot{x}]^T$, the system can be written in the first-order matrix form as:

$$\begin{aligned} \dot{X} &= AX + Bf \\ Y &= HX + Df \end{aligned} \tag{2}$$

where $A_{N\times N}$,$B_{N\times r}$,$H_{M\times N}$ are the time-invariant continuous time system matrices, where $N = 2n$ and M denotes the number of outputs Y. Multiplying first line of Equation (2) with e^{-At} and integrating yields:

$$X(t) = e^{At}X(0) + \int_0^t e^{A(t-\tau)}Bf(\tau)d\tau \tag{3}$$

Equation (3) is the analytical solution for the continuous model in Equation (2). In the digitizing of Equation (3), the assumption of zero-order hold, i.e. f is assumed to be constant during each time step, T. In the Equations (4), the discrete formulation is presented.

$$\begin{aligned} &X[k] = X(kT) \\ &X[k] = e^{AkT}X(0) + \int_0^{kT} e^{A(kT-\tau)}Bf(\tau)d\tau \\ &X[k+1] = e^{A(k+1)T}X(0) + \int_0^{(k+1)T} e^{A((k+1)T-\tau)}Bf(\tau)d\tau \\ &\Rightarrow X[k+1] = e^{AT}\left[e^{AkT}X(0) + \int_0^{kT} e^{A(kT-\tau)}Bf(\tau)d\tau\right] \\ &\qquad + \int_0^{(k+1)T} e^{A(kT+T-\tau)}Bf(\tau)d\tau \end{aligned} \tag{4}$$

The expression enclosed by the brackets in Equation (4) can be recognized as $X[k]$. The second term can be simplified by introducing υ as:

$$\upsilon = kT + T - \tau \tag{5}$$

and assuming that the excitation vector f is constant over the integration interval results in:

$$X[k+1] = e^{AT}X[k] + \left(\int_0^T e^{A\upsilon}d\upsilon\right)Bf(k) \tag{6}$$

Equation (6) represents the exact solution to the digitizing problem. The matrix exponential in Equation (6) is defined as:

$$e^{AT} = \sum_{s=0}^{\infty}\frac{1}{s!}(AT)^s \tag{7}$$

and it can be approximated by a first-order Taylor series around $T = 0$ as:

$$e^{AT} = I + AT \tag{8}$$

Substituting Equation (8) into Equation (6) yields:

$$X[k+1] \approx (I + AT)X[k] + \left(IT + AT^2/2\right)Bf(k) \tag{9}$$

For approving similar kind of algorithm and solution, it may be compared the analytical solution of system equation of motion in 2 degree of freedom and corresponding parameters optimizations with its results.

By assuming system as stated in figure 4, the model equation of motion shall be equal to Equation (10).

$$\begin{cases} m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2 + k_3)x_1 - (k_2 + k_3)x_2 = 0 \\ m_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 - (k_2 + k_3)x_1 + (k_2 + k_3)x_2 = 0 \end{cases} \tag{10}$$

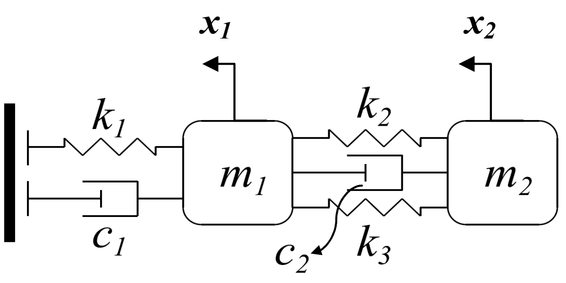

Figure 4. Tow degree of freedom model.

The initial conditions are $x_1 = x_2 = 0$ and $\dot{x}_1 = \dot{x}_2 = 14$. Equation (10) may be written in the matrix form as follows:

$$M\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\end{Bmatrix} + C\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\end{Bmatrix} + K\begin{Bmatrix}x_1\\ x_2\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix} \quad (11)$$

where:

$$M = \begin{bmatrix} m_1 & 0\\ 0 & m_2\end{bmatrix},\ C = \begin{bmatrix} c_1+c_2 & -c_2\\ -c_2 & c_2\end{bmatrix},\ K = \begin{bmatrix} k_1+k_2+k_3 & -k_2-k_3\\ -k_2-k_3 & k_2+k_3\end{bmatrix} \quad (12)$$

State space matrixes are as following:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -(K_1+K_2+K_3)/m_1 & (K_2+K_3)/m_1 & -(c_1+c_2)/m_1 & +c_2/m_2\\ (K_2+K_3)/m_2 & -(K_2+K_3)/m_2 & +c_2/m_2 & -c_2/m_2\end{bmatrix} \quad (13)$$

$$B = \begin{bmatrix}0\\0\\0\\0\end{bmatrix},\quad H = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\quad D = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

It is also can rewrite the equations as below to generalize the formation:

$$A = \begin{bmatrix} [0]_{n\times n} & [I]_{n\times n}\\ -[M]^{-1}[K] & -[M]^{-1}[C]\end{bmatrix},\ B = [0]_{2n\times 1},\ H = [I]_{2n\times 2n},\ D = [0]_{2n\times 1} \quad (14)$$

The following values were used in this part for simulation: $m_1=800kg$, $m_2=80kg$, $c_1=10000N.s/m$, $c_2=1100N.s/m$, $k_1=1000N.m$, $k_2=160N/m$ and $k_3=2700N/m$.

The main reason of considering k_1 and k_3 in parallel way is testing the state of optimization in 2 cases. The first, we have $k_2+k_3=2860$; it means k_2 and k_3 have any amounts in above mentioned condition, on the other hand, it is anticipated that $k_2=k_3=1430$. In this case, decision variables are c_1, c_2, k_1, k_2 and k_3. Also, active parameters such as $m_1=800kg$ and $m_2=80kg$, were considered as vehicle and occupant masses, respectively.

The second, according to being parallel of springs and similarity of k_3 as a coefficient of k_2, it must be considered the rate of $k_2=16.875k_3$ of solution condition. In this case, decision variables are c_1, c_2, k_1 and k_2. m_1 and m_2 are as same as last case that was discussed about it.

The function attempts to find the constrained minimum of a scalar function of several variables. A typical problem can be formulated as:

$$\min f(\theta) \quad (15)$$

where *"θ"* denotes the unknown design variables, which, in this case are the masses, damping and stiffness constants in the model. The cost function *f(θ)* is referred to objective function, which is to be optimized. In this study, the cost function is the Root Mean Square (RMS) of differences between the measured and calculated deceleration for the load cases. The Genetic Algorithm is used for optimization of cost function. The aim is to minimize the cost function value. The cost function is defined as:

$$Z = RMS\,(ea\,/\,mean\,(abs\,(a_{\exp}))) \quad (16)$$

here "*mean*" is average of data and "*ea*" can be represented as follows:

$$ea = a_i - a_{\exp} \quad (17)$$

that "*ea*" is the deceleration error that is calculated by difference between i^{th} mass deceleration and target deceleration (a_{exp}) which obtained from experimental tests. The results of optimization after 2000 iteration with random initial population are shown in Tables 1 and 2.

TABLE I. OPTIMIZATION RESULTS OF 2-DOF MODEL IN STATE $K_2+K_3=2860$.

Method	c_1 (N.s/m)	c_2 (N.s/m)	k_1 (N/m)	k_2 (N/m)	k_3 (N/m)
Analytical[a]	10000	1100	1000	160	2700
Optimization	10004.9	1099.8	1033.4	1438.8	1418.5
Error %	0.05	-0.01	3.34	799.26	-47.48
		k_2+k_3 Error %		-0.11	

a. Parameters are assumed as active.

TABLE II. OPTIMIZATION RESULTS OF 2-DOF MODEL IN STATE $K_2=16.785K_3$.

Method	c_1 (N.s/m)	c_2 (N.s/m)	k_1 (N/m)	k_2 (N/m)
Analytical[a]	10000	1100	1000	160
Optimization	10011.91	1098.79	992.91	159.63
Error %	0.12	-0.11	-0.71	-0.23

a. Parameters are assumed as active.

The previous algorithm for optimization method is used here to calculate the spring and damper coefficient. Time history of deceleration is shown in figure 5.

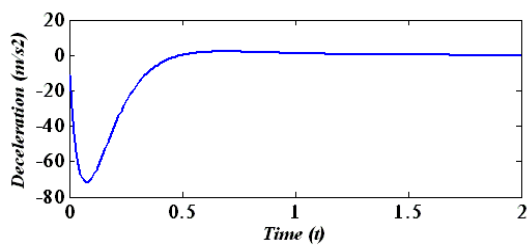

Figure 5. Deceleration on m_2 in 2 degree of freedom model obtained from analytical solution.

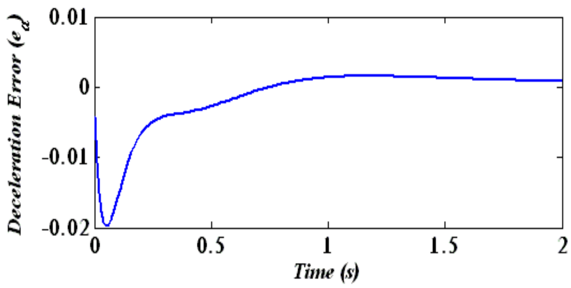

Figure 6. Error of deceleration on m_2 in 2 degree of freedom model obtained from optimization under $k_2+k_3=2860$.

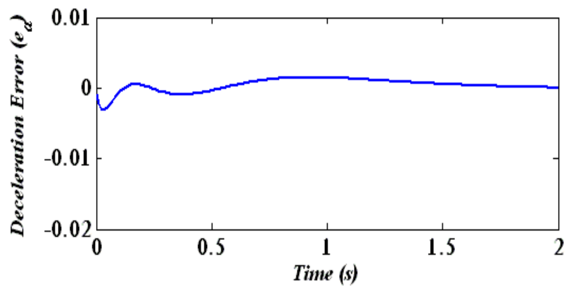

Figure 7. Error of deceleration on m_2 in 2 degree of freedom model resulted from optimization under $k_2=16/875k_3$.

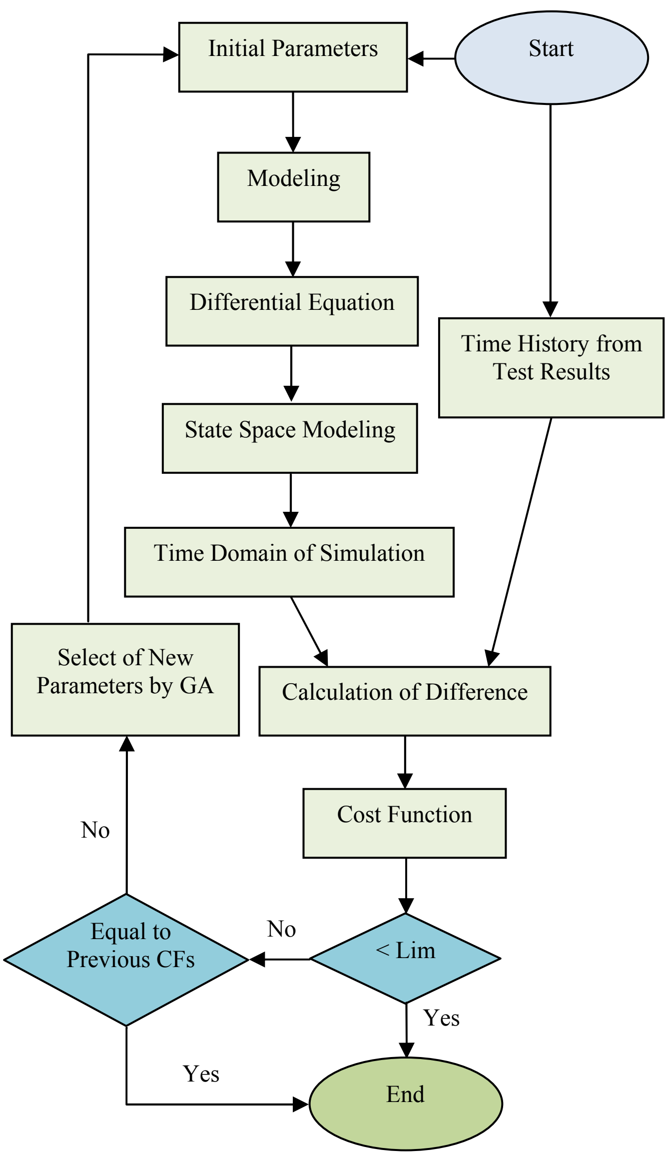

Figure 8. Algorithm of problem solving.

Time history of error values shows the differences between deceleration of m_2 in analytical and optimization solution which is occurred after 2000 iteration. It is illustrated in Figures 6 and 7. As one can see in these Figures, in both cases ($k_2+k_3=constant$ and $k_2=16.875k_3$) deceleration error of m_2 under the worse conditions is less than $0.02m/s^2$ which is considered as 0.28% error equals to max deceleration of $71.44m/s^2$.

The obtained optimized values indicate that they are acceptably close to accurate parameters after enough iteration, so it can solve complex models. We should consider that amount of corresponding between desired and target deceleration depends on degree of freedom. So, solution algorithm will be designed in such way which will be stopped after 500 iterations with equal value of cost function that is assumed 0.1 as its limit. Figure 8 shows procedure of solving problem.

III. 5-DOF Modeling and Results

TABLE III. Mass proportion for 5-DOF serial model.

Serial Model Mass No.	Lumped Components	Mass (kg)
m_1	Radiator	50
m_2	Suspension and Front Rails	100
m_3	Engine and Shotguns	300
m_4	Fire Wall and Part of Body on Its Back	820
m_5	Occupant	80

TABLE IV. Mass proportion for 5-DOF LH model.

LH Model Mass No.	Lumped Components	Mass (kg)
m_1	Engine and Radiator	300
m_2	Suspension and Front Rails	120
m_3	Engine Cradle and Shotguns	150
m_4	Fire Wall and Part of Body on Its Back	700
m_5	Occupant	80

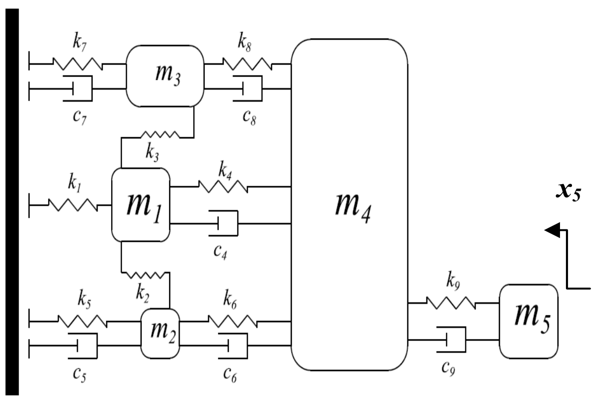

Figure 9. 5-DOF LH model.

In this section, a 5-DOF model is analyzed as a vehicle in crash and the spring and damper specifications are determined by using the optimization algorithm as indicated in Figure 8. In completion of Kamal's model, we analyzed this model in 5 degree of freedom which shown in Figure 9 and then compared our results with 5-DOF serial model as Figure 10. Tables 3 and 4 are show proportions of lumped parameters.

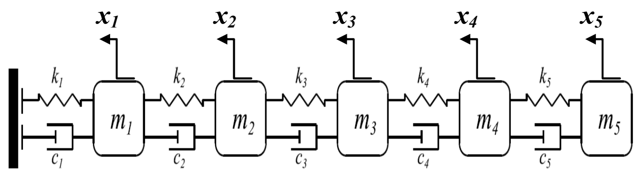

Figure 10. 5-DOF serial model

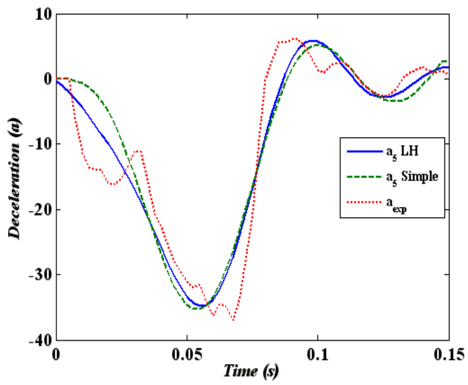

Figure 11. Optimized (a_5) and experimental (a_{exp}) results for deceleration in 5-DOF LH and simple models.

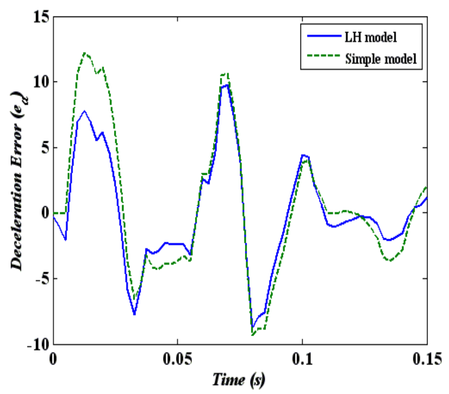

Figure 12. Different between optimized (a_5) and experimental (a_{exp}) for deceleration in 5-DOF LH and simple models.

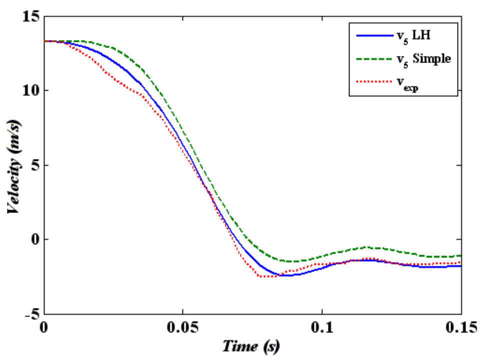

Figure 13. Optimized (v_5) and experimental (v_{exp}) results for velocity in 5-DOF and simple models.

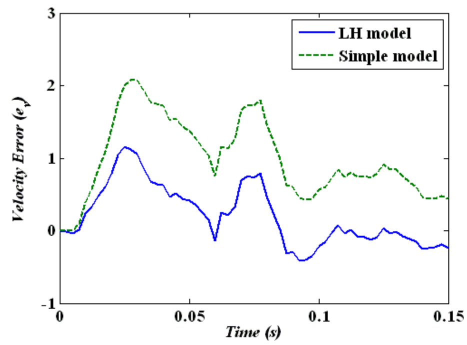

Figure 14. Different between optimized (v_5) and experimental (v_{exp}) for velocity in 5-DOF LH and simple models.

In this research, the occupant deceleration of a Dodge Neon vehicle test is used as the goal data to be criteria for optimization. The obtained results, compared and shown in Figure 11 to 14.

TABLE V. COMPARISON BETWEEN DECELERATION ERRORS OF FOUR MODELS.

Error / Model	Final Root Mean Square of Deceleration Error (g)	Maximum Deceleration Error (g)
5 DOF LH	4.1624	9.77
5 DOF Serial	5.3902	12.24

TABLE VI. COMPARISON BETWEEN VELOCITY ERRORS OF FOUR MODELS

Error / Model	Final Root Mean Square of Velocity Error (m/s)	Maximum Velocity Error (m/s)
5 DOF LH	0.4621	1.154
5 DOF Serial	1.1346	2.072

TABLE VII. VALUE OF PARAMETERS OF BOTH MODELS

Parameter	LH Model	Serial Model
c_1	-	19919388.21
c_2	-	19917598.56
c_3	-	0.18304
c_4	0.8981	19777.1246
c_5	33114.21	810.8301
c_6	1.7284	-
c_7	6764.6574	-
c_8	8648277.61	-
c_9	1595.52	-
k_1	48.8660	1341925.08
k_2	915522.58	1333105.21
k_3	1206875.43	1117990.69
k_4	1178694.84	1.91805
k_5	36.7265	572047.24
k_6	136.4661	-
k_7	38.6678	-
k_8	4761249.39	-
k_9	389232.42	-

IV. Comparison Between Two Models

In Table 5, we present final Root Mean Square of deceleration error, and maximum deceleration error in tow models and also presented final Root Mean Square of velocity error and maximum velocity error in Table 6.

Figure 11 dates reveals those responses of occupant deceleration in both 5-DOF LH and Serial models are same and follows from experimental test data reasonably. In Figure 12 dates, we founded that error of LH model deceleration is lower than of Serial model.

Figures 13 and 14 reveal time history of velocity and velocity error in both 5-DOF models. Parameters value of both models presented in Table 7.

V. Conclusion

In this research we have investigated LPMs. Solution method and algorithm are evaluated by 5 degree of freedom serial model and results of deceleration and velocity are presented and then compared to experimental data and 5-DOF model. Results of comparison show that type of spring arrangement in model has significant effect and we have concluded that we should not only address deceleration changes behavior but also total error of deceleration and velocity and maximum error and velocity behaviors to emphasized model. The following notes considered in this regard:

1) Number of degree of freedom should be determined accurately, so it shouldn't be too much equation that yields convergence equations confront with error nor will few to deceleration behavior and errors be insufficient.

2) In these models way of spring's arrangement is very important so we should analyze several models by the same degree of freedom to achieve desired results.

3) Whereas force-displacement behavior of components is not available in optimization, algorithm could set one or more parameter too little, so it don't make our solution wrong because some of parameters may be extra and becoming too little don't make mistake in our solving.

4) As mentioned, calculated behavior data are not match with experimental data, so number of iteration equals number of error for stopping of iteration is more logic than error is being zero in solution algorithm.

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