Considering Preference Parameters in Multi Response Surface Optimization Approaches

Amineh Zadbood and Kazem Noghondarian

Abstract—**Most of the studies in Response Surface Methodology commonly involve one response or quality characteristics, whereas in most industrial applications considering all responses simultaneously is required. Multiple Response Surface (MRS) Optimization Problems often deal with responses that are conflicting. In dealing with incommensurate responses, incorporating a decision maker's preference information into the problem has lots of advantages although a few researches in MRS literature has taken this into attention. This paper tends to take a detailed look at the most prominent approaches that has been suggested so far in MRS, also review and discuss the classifications with a special focus on the decision maker's preference information. In today's competitive market satisfying the customer is of high importance. The DM can be a customer and reaching a compromise with an interactive method would help the firm to succeed in having loyal customers. Results of the case study shows that applying a meta-heuristic algorithm with existing MRS approaches lead to better findings .finally future areas for research are discussed.**

Index Terms—**multiple response surface optimization; response surface methodology; decision maker; design of experiments.**

I. Introduction

Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques that has been widely used in developing, improving, and optimizing processes [1]. I n most RSM problems, the form of relationship between the response variables and the independent variables is not known. Therefore, the first step in RSM is to seek for a suitable approximation. Commonly a low-order polynomial is employed. For any curvature in the system, a polynomial of higher degrees, mostly second order is applied. Then, by the method of steepest ascent or steepest descent the mos appropriate set of input for response is determined [2]. The RSM method was first introduced by Box and Wilson in 1951, although it was not considered for optimizing multiple responses variables until many years later [3].

Many industrial and manufacturing problems commonly deal with several quality characteristics to be optimized at the same time. For simultaneous consideration of multiple responses, first building appropriate response surface model for each response is required. Then we attempt to find a set of operating conditions that optimizes all responses or at least keep them in desired ranges [4]. There are 3 stages in these problems: 1) collecting data 2) building the model 3) optimization.

A multiple response surface problem is formally formulated as the following:

$$\text{Optimize } \widehat{y_1}(x),, \dots, \widehat{y_k}(x) \} \quad \text{s.t.} \quad x \in \psi \quad (1)$$

$$i = 1, \dots, k$$

$\widehat{y_i}(x)$ is the *i*th estimated response model and input variables are in the experimental region.

In the multi objective decision making problems there are several techniques has been used solving optimization (MOO) literature (Hwang et al. 1979) [5]. In fact the timing of the decision maker's preference information is important. In the first category prior information of the Decision Maker (DM) is needed when the problem solving has not started yet. Most of the studies in MRS problems required prior information from the DM. The second category is comprised of techniques that in which an interaction with the DM is required. And for the last category a posteriori information from the DM is required.

In this paper, we review and discuss various approaches in MRS and pay special attention to the DM role in the method. Section 2 reviews the MRS approaches. Section 3 is about Decision Maker's preference Information in the MRS and classifies some of the most prominent articles into a table. Section 3 discusses a case study and compares the result. Finally section 4 is the conclusions.

II. MRS Approaches Review

Many Methods have been introduced for multiple response optimization. Pignatiello (2004) categorized the existing methods in three categories [6]. Then Tajbakhsh and Noorossana (2005) classified them in four basic categories [7] .Here we focus on the most prominent approaches of the newest classification. In the next section according to the most recent studies we classify the approaches considering the decision maker's preference information in the problem solving process.

A. Overlaying Contour Plots

Myers and Montgomery suggested that an approach for optimizing several responses is to overlay the contour plots for each response. The experimenter can visually examine the contour plot to discover the appropriate operating conditions [1]. It must be emphasized that it is mainly used

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when there are few process variables. For more than 3 design variables this approach becomes awkward.

In this approach there is no need to the DM's information. And Contour plots play the main role.

B. Constrained Optimization Problem

A popular approach is to formulate and solve the problem as a constrained optimization problem [1] . Kim classified it as a priority based approach [8]. The priority based approach ,that is similar to method bounded objective in the Multi objective decision making problems, chooses the response with the highest importance as the objective function and the rest of the functions are considered as constraints, Although it is not always much straightforward.

The idea was first suggested by Myers and Carter (1973) [9]. 2 responses are assumed and referred as a "primary response" and a "constraint response". The goal is to find conditions on a set of design variables which maximize the primary response function subject to the constraint response function. Biles (1975) Considered multiple process responses and extended the myers and carter idea [10]. The priorority based approach was studied in the later years like Delcastillo and Montgomery (1993) [11].

C. Desirability Function Approach

In this approach an estimated response such as $\hat{y}_i(x)$ is transformed to a scaled free value d_i that is called desirability.It changes from 0 to 1.The overall desirability (D) is also in the [0,1] interval is obtained by combining all desirabilities (d_i) [12]. Derringer and Suich (1980) extended the idea and presented a method to construct an overall desirability[13].

Desirability function for the larger-the-better case is as following:

$$d_i = \begin{cases} 0 & \hat{y}_i(x) \le y_i^{min} \\ \left[\frac{\hat{y}_i(x) - y_i^{min}}{y_i^{max} - y_i^{min}}\right]^t & y_i^{min} \le \hat{y}_i(x) \le y_i^{max} \\ 1 & \hat{y}_i(x) \ge y_i^{max} \end{cases} \quad (2)$$

In the above equation The min and max indexes on the y denote the lower and upper limits accepted for $\hat{y}_i(x)$ respectively.

The most important advantage of this approach is that the Decision maker's preference information an be easily applied in the model.In addition it is esat to use and popular among the authors.

D. Loss Function Approach

Pignatiello (1993) first suggested a squared error loss function as follows [14]:

$$L(y(x)) = (y(x) - \Phi)'C(y(x) - \Phi) \quad (3)$$

Where

y(x)= response vector

Φ= target vector

C= cost matrix

The cost matrix determines the relative importance of the response variables.

Extended studies about this approach can be found in Vinning (1998),Tsui(1999),Riberio et al.(2000) [15] , [16], [17].

E. Process Capability Approach

Process capability index is used to evaluate whether a process is able to meet current specification limits [18]. Hsiang and Taguchi presented the index C_{pm} as follows [19]:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \quad (4)$$

USL and LSL are specification limits. The mean, variance and target are other elements of the above equation.

In addition Chan et al.,in an independent work proposed this index later.

This index can be applied in multi response optimization. Plante considered the maximization of process capability as a criterion for multi response optimization[20].For a detailed study about this approach the reader can refer to [18],[20].

F. Distance Function Approach

This approach was proposed by Khuri and Conlon [21]. The distance function is [21] :

$$distance[\hat{y}(x),T] = [\hat{y}(x - T)' \textstyle\sum_{\hat{y}(x)}^{-1} [\hat{y}(x) - T]] \quad (5)$$

T represents the target value. $\hat{y}\ (x)$ is the predicted response and $\sum_{\hat{y}(x)}$ is the variance-covariance matrix of the predicted responses. The optimal settings are achieved if the distance function gets minimized.

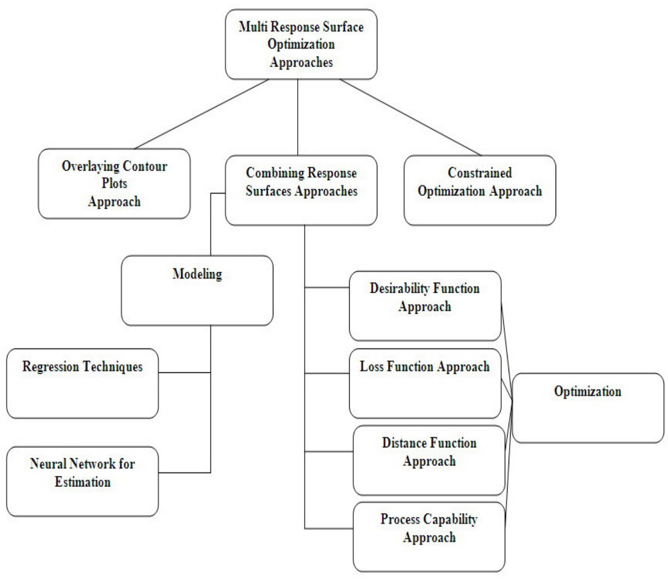

Figure 1 : MRS optimization approaches classification

All the discussed approaches are depicted in fig.1. need to be highly robust in order to avoid getting stuck at a local optimal solution. The benefit of GA is just able to obtain the global optimal solution rather fairly. Also GA does not require the specific mathematical analysis of optimization problems [22].

Pignatiello et al (2004) for the first time used both Desirability approach and Genetic Algorithm in multi response optimization[6]. Pasandideh and Niaki (2006) applied the genetic algorithm with desirability function framework for a multi response simulation optimization problem [2].

Neural network(NN) is is a class of adaptive systems consisting of a number of simple processing elements,called neurons.Neurons are interconnected to each other in a feed forward way. A significant contribution of NN is the ability to learn to perform operations, not only for inputs like the training data, but also for new data that may be incomplete or noisy [22].

Noorossana et al (2005) applied an Artificial Neural Network Approach to Multiple Response Optimization. They have chosen the data to be qualitative and has applied fuzzy theory for them.results of the paper shows great stability [23] .Bashiri et al.(2009) used neural network based on a desirability function .they used a feed forward back propagation with two hidden layers.the number of neurons in the hidden layers are determined using MSE criterion for testing data and training [24].

III. Decision Maker's Preference Parameters In The MRS Optimization

In this section the most recent works in various MRS approaches are classified into 4 categories : no information from the DM,prior information from the DM,interaction with DM,posterior information from the DM.The Preference parameters from the DM can be specification limits, including LSL and USL. Also target value or the shape of the function. Another option is assigning weights to different responses.

A. No Information from the DM

Despite the effectiveness of a DM information preference, several studies in MRS are solved with no information from the DM. Romano et al (2004) deals with a robust parameter design problem. they present a general framework for the multivariate problem when data are collected from a combined array [25].No information is obtained from a DM. Ko et al (2005) proposed a new loss function-based method for multi response optimization [26].although the proposed loss function method shows more reasonable result ,There is no information in the problem by a DM. Noorossana and Ardakani (2008) considered a robust parameter design problem with 2 objectives. An Lp metric method was applied because it does not need much information from the DM[27]. Pasandideh and Niaki (2006) integrates desirability function and simulation approach with a Genetic algorithm. The same as other works in this group no information is required from a DM [2].

B. Decision Maker's Prior Information

Some research carried out so far in MRS can be categorized here. All the DM's preference information are gathered before solving the problem. Sadjadi and khaledi (2008) studied some multi objective decision making approach for non linear multiple response surface problems [28].a Lexicographic method is used and before problem solving the DM's preference information are asked.

C. Interaction with the DM

In dealing with conflicting responses, incorporating a decision maker's preference information into the problem has lots of advantages although a few research in MRS literature has taken this into attention. According to Jeong and Kim (2009) The DM's preference information is denoted through preference parameters. The shape,bound and target of a desirability function or the cost matrix in loss function method are examples of preference parameters[29]. Some studies that have tried to use interactive methods to solve MRS problem are as follows : Montgomery and Bettencourt (1977) , Mollaghasemi and Evans (1994) , Park and Kim (2005) [30],[31],[32] .

In an interaction makes DM,feels more comfortable with the process. Therefore applying an interactive method appropriately leads to a good compromise between the problem solver and a decision maker. Among the research in this field some works are more significant.

Jeong and Kim (2005) notice that as STEM is effective in extracting a decision maker's preference information for a satisfactory compromise, they proposed a modified STEM called D-STEM [33].By some examples it is concluded that the new method is more effective.

Kazemzadeh et al (2008) demonstrate various types of DMs relating to MRS problems. The types are as follows: customer, experimenter, producer and manager. They also propose a general framework for Multiresponse optimization problems based on goal programming [34].

Jeong and Kim (2009) proposed an interactive version of the desirability function approach .Their method provides various channels through that the DM is able to articulate his/her preference information [29].

D. Decision Maker's Posterior Information

Lee et al. proposed a posteriori preference information approach to dual response surface optimization in 2010 [36].

In table 1 the above discussion is summarized. Recent works in MRS optimization are considered with special focus on the DM's role in the problem solving.

Table 2 is drived from works in the literature that have used the most popular MRS approaches. The problems are studied to see whether the DM's preference information is used to obtain a compromise result or not. Obviously more focus has been on using desirability function for an interactive approach to solve MRS problem.

TABLE2.DM INFORMATION IN POPULAR MRS APPROACHES

Types of DM information	**Most popular MRS approaches**		
	Constrained Optimization	***Desirability Function***	***Loss Function***
None			✓
Priori	✓		
Interactive		✓	
Posteriori		✓	

TABLE1.RECENT WORKS CLASSIFICATION CONSIDERING THE DM

Recent works in MRS	Types of DM Information			
	None	***Priori***	***Interactive***	***Posteriori***
Romano 2004	✓			
Ko 2005	✓			
Park 2005			✓	
Jeong2005			✓	
Pasandideh 2006	✓			
Sadjadi 2008		✓		
Noorossana 2008	✓			
Kazemzadeh 2008		✓		
Jeong 2009			✓	
Lee 2010				✓

TABLE3.CASE STUDY RESULT

Values	Preference Information into the problem	
	Priori Information	***Interaction with the DM***
x_1	580	580
x_2	12	13
x_3	250	250
y_1	90	93
y_2	62	64

IV. CASE STUDY

Khoo and Chen (2001) enhanced the response surface methodology with Genetic algorithm [35].They Solved a case study as follows : there are 2 responses called mean pull strength and the minimum strength that are required to be monitored. The goal is to maximize both the mean pull strength and the minimum strength simultaneously. The response surface equations established with design of experiments are as follows:

$$y_1 = 73.89 + 12.91\,x_1 + 7.11x_2 + 2.56x_3 \\ -1.96x_1^2 - 1.01x_2^2 + 0.022x_3^2 + 0.36x_1x_2 \\ -0.068x_1x_3 - 0.52x_2x_3 \quad (4)$$

$$y_2 = 45.06 + 14.11\,x_1 + 6.56x_2 \\ +2.17x_3 - 1.96x_1^2 - 1.02x_2^2 + 0.14x_3^2 \\ +1.08x_1x_2 - 0.83x_1x_3 - 0.52x_2x_3 \quad (5)$$

where output variables are mean pull strength and minimum strength respectively. Both objective functions are going to be maximized. Input variables are bonding temperature, bonding force and bonding time respectively.

x_1 varies from 500 to 580.
x_2 varies from 11 to 13.
x_3 varies from 210 to 250.

Results are produced by Matlab .The interactive method that has the desirability function framework performs much better than methods with no information from the DM.

V. CONCLUSION

In this article existing approaches in MRS are reviewd and discussed, considering a decision maker's preference information. In dealing with conflicting responses, incorporating a decision maker's preference information into the problem has lots of advantages mainly because of the fact that In today's high competitive market satisfying the customer is of high importance. However, a few research in MRS literature has taken this into attention.

DM can be a customer and reaching a compromise with having loyal customers. Obtaining posteriori information from the DM is also an area with potential scope to be studied about. Results of the case study shows that applying an interactive method with existing MRS an interactive method would help the firm to succeed in approaches lead to better results. Future studies can focus on applying metaheuristic algorithms in the optimization step.

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