Fuzzy Optimization in Cost, Time and Quality Trade-off in Software Projects with Quality Obtained by Fuzzy Rule Base

Roya M. Ahari and S. T. A. Niaki

Abstract—**Project managers are often facing the challenge of optimal allocation of resources to different project tasks involving different and usually conflicting objectives. The aim of this research is to simultaneously optimize the time, the cost, and the quality of a project by analyzing their trade-offs. To achieve this goal, the quality of a project task is first defined as a function of its time and cost. Then, a fuzzy rule-base is used to determine these functions more adequately. Next, the total project cost, the total project time, and the overall project quality are simultaneously optimized by a linear fuzzy multi-objective optimization model to decide the preferred mode of executing a task. An example is given at the end to show the applicability and the validity of the proposed methodology.**

Index Terms—**Multi-objective optimization, project time, cost and quality, fuzzy rule-base, software projects.**

I. Introduction and Literature Review

Being a project manager requires having an accurate plan of allocating resources to the tasks involved in a project. Different resource allocations to the tasks affect time, cost, and quality of the project directly. In order to consider different alternatives of executing a task, one assumes each task has different modes of execution with respect to its time, cost, and quality. This assumption is especially suitable for the projects that are accomplished several times repeatedly and every experience can be one mode. At a glance, it seems all activities of a project can be performed in a few reasonable modes. However, since most projects contain several activities, selecting the execution modes of the activities is a complicated process.

In project management environment, the time-cost-quality trade-off problem is a multi-objective optimization problem, which mainly focuses on selecting modes with their corresponding time, cost and quality for a task to minimize project completion time and cost, while project quality is maximized. In the pioneer work, a framework was proposed to study the trade-off among time, cost and quality using three inter-related linear programming models [1].

In recent years, multi-objective optimization models are used to transform the traditional two-dimensional time-cost trade-off analysis to an advanced three-dimensional time, cost, and quality trade-off analysis [2]. Moreover, a survey developed three inter-related integer programming models such that each optimizes one of the given three entities by assigning desired bounds on the other two [3]. Further, another presented separate mathematical models for time, cost, and quality, along with a multi-objective optimization model for the time-cost-quality trade-off problem by setting up synthesizing weighted single-objective models [4].

Many researchers tried to apply some evolutionary optimization algorithms characterized by different global search capabilities to solve the trade-off problem. These algorithms are mainly genetic algorithm (GA) [5], ant colony optimization (ACO) [6], Pareto optimal front (POF) [7] and electromagnetic scatter search (ESS) [8], and particle swarm optimization algorithm (PSO) [9]. In a case study, authors first compared the standard particle swarm optimization (SPSO) with the differential evolution (DE) algorithm. Then, they concluded the most satisfied decision results could be obtained by applying the hierarchical subpopulation particle swarm optimization algorithm (HSPSO) to solve the time-cost-quality trade-off problem. The extension of their research divided costs into direct and indirect [4].

As applications of the models proposed by Babu for the trade-off problem, a cement-factory construction project in Thailand is selected to evaluate the applicability of the methodology [10]. Further, a research considers a multi-objective optimization model with an algorithm to provide the capability of quantifying quality in construction [2].

Since different project trade-off preferences exist in industries, it can be proposed a set of axioms in project risk management that reflects the relationships among key project variables and employed Bayesian networks (BN) to model the trade-off problem [11]. Based on the types of projects and their resources surveyed, it is possible to add some constrains on the resources of an actual construction schedule and employed a GA to solve the model [12].

To define different modes of performing an activity one should define a relation function between its quality, time, and cost. While most of previous studies assumed that the quality received by fulfilling an activity is independent of the its time and cost and it is measured based on the managers' judgment, in this paper, the independence assumption between quality in one side and time and cost on the other side is relaxed. Moreover, since there is no predefined rule in the literature to calculate the quality of an activity, in this paper a flexible framework is provided to determine this parameter. To do this, linguistic terms are first defined in Section II to describe time, cost, and quality of the activities. Then, a complete set of if-then rules are determined in this section to help a practitioner to calculate the quality based on the time and cost. In Section III, a linear multi-objective fuzzy optimization model is next

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developed to determine weights that are assigned to functional relationships between the quality, time, and costs of activities. The fuzziness of the linguistic terms and solving the multi-objective optimization model are the bases in figuring out the relation between quality, time, and cost. Finally, a numerical example is provided in Section 4 to demonstrate the applicability of the proposed methodology.

It should be noted that while many firms use some indices such as number of failures and number of inspections to evaluate an activity, most software companies have no tendency to record data in order to determine these indices. However, all of them affect the cost and the time of a software-programming project and hence in this paper we focus on these two parameters as well. We assume that all resources in a software-programming project are renewable (such as human resources). Further, while some researchers proposed methods to provide group of answers rather than one optimum solution to their multi-objective optimization problem, since it is more suitable for managers to have a practical and specific answer, in this paper a single near-optimum solution is provided.

II. Fuzzy Rule Base

In this section, a fuzzy rule base is developed based on expert opinions to evaluate the quality of the project tasks assuming all the resources are renewable. Since firms usually have more precise statistics on the time and cost of a task, five discrete levels for both time and cost and three levels for quality are chosen as the levels of the linguistic terms. The five levels of linguistic terms for cost and time are defined very low (VL), low (L), medium (M), high (H), and very high (VH). The three different levels for the quality parameter are defined L, M, and H. The membership function diagrams of the defined linguistic terms for quality and cost are shown in Fig. 1 and Fig. 2. Membership function of time is similar to the cost one.

Moreover, Table I shows the designed rule base of the experts' opinions. To extract these rules covering all the relations among time, cost, and quality, in a time-consuming process some project manager's ideas are gathered on the quality of an activity obtained under different combinations of fuzzy time and cost. As an example, consider rule number 4 in Table I. In this rule if a cost spent to an activity is low (relative to a normal cost) with a time of very low, then the consequence quality of performing the activity will be low. These rules along with the membership functions are then used to figure out the value of the task quality based on predefined amounts of cost and time. Fig. 3 demonstrates relationships among cost (C), time (T), and quality (Q) of a project using the MATLAB software (Anfis module).

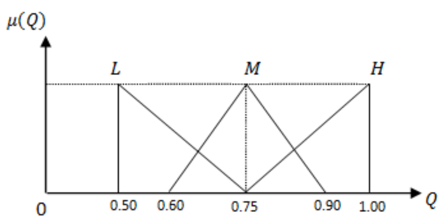

Fig. 1. Membership diagram (μ) of linguistic terms in quality (Q).

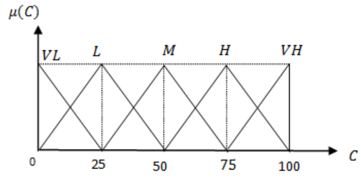

Fig. 2. Membership diagram (μ) of linguistic terms in cost (C).

TABLE I: The Rule Bases

	If cost & time		Then quality
Rule	cost	Time	Quality
1	VL	L	L
2	VL	M	L
3	VL	H	L
4	L	VL	L
5	L	M	L
6	L	VH	M
7	M	VL	L
8	M	M	M
9	M	H	M
10	M	VH	H
11	H	VL	L
12	H	L	M
13	H	M	M
14	H	H	H
15	H	VH	H
16	VH	VL	M
17	VH	L	H
18	VH	M	H
19	VH	H	H
20	VH	VH	H

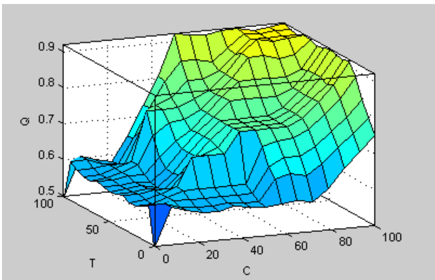

Fig. 3. The relationships among time (T), cost (C), and quality (Q).

This module uses the rule bases and definition of membership functions to achieve the surface of Fig. 3. This figure shows that if one has numerical values of cost and time in the scale of 0-100 (while 50 defines the standard cost and time), he can obtain the quality in the scale of 0-1. Having an if-then rule and initial values for cost and time we can find the quality amount. Fig. 4 and Fig. 5 shows the Mamdani's method just about rule numbered 4. When we have more than one rule, the software calculates average of them. If-then rule defines effective linguistic term and initial values shows membership function values. Mamdani's method selects minimum of the membership functions (μ1 and μ2) for applying on "then" linguistic term. To calculating quality value we should get y1 and y2 averaged.

III. Modelling

For having figured out the relation among cost, time and quality of each activity, in this section we are looking for a model to optimize three predefined objectives. The three objective functions of the model proposed by Ref. [7] are simply defined in this research as the average quality of all tasks f_1, the total project's execution time f_2, and the summation of al tasks' cost f_3.

Using the fuzzy optimization concept, one should first find out m_i and M_i ; i=1,2,3 using the following models.

$$m_i = min\ f_i(x) \quad (1)$$

s.t model constraints $x \in S$

$$M_i = max\ f_i(x) \quad (2)$$

s.t model constraints $x \in S$

Model constraints were shown by (7), and (8). The membership functions for all objectives are obtained as

$$\mu_1(x) = \frac{f_1(x) - m_1}{M_1 - m_1} \quad (3)$$

$$\mu_1(x) = \frac{f_1(x) - m_1}{M_1 - m_1} \quad (4)$$

$$\mu_3(x) = \frac{M_3 - f_3(x)}{M_3 - m_3} \quad (5)$$

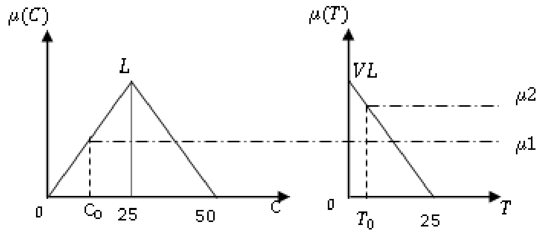

Fig. 4. Mamdani's method about rule number 4 ("if" part)

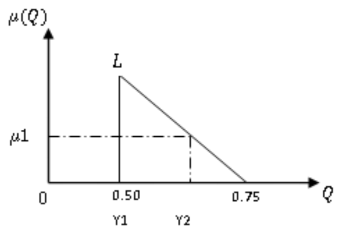

Fig. 5. Mamdani's method about rule number 4("then" part)

The model has two different types of constraints. While the basic constraints are related to the precedence relationship between project tasks (7), the other constraints are based on the fact that one and only one execution mode can be assigned to each task (8),

$$\max \min(\mu_1, \mu_2, \mu_3) \quad (6)$$

$$\text{s.t} \quad \textstyle\sum_{r \epsilon M_{ij}} x_{ijr} = 1\ ; \quad \forall\ ij \epsilon A \quad (7)$$

$$x_j - x_i \geq \sum_{r \epsilon M_{ij}} t_{ijr}\, x_{ijr}\ ; \forall\ ij \epsilon A, i, j \epsilon V \quad (8)$$

$$x_0 = 0 \quad (9)$$

$$x_i \geq 0 \quad ; \quad \forall \quad i \epsilon V, integer \quad (10)$$

$$x_{ijr} = 0{,}1; \quad \forall \quad i, j \epsilon A\ , r \epsilon M_{ij} \quad (11)$$

where A is the set of activities, V is the set of nodes in the project network, M_{ij} is the set of all execution modes of activities named ij, x_{ijr} is one if activity ij is executed in its r^{th} mode and zero otherwise, t_{ijr} is the required time for the r^{th} execution mode of activity ij, and x_i is the starting time of i^{th} node.

IV. An Example

In order to demonstrate the applicability of the proposed methodology, a numerical example on a five-task software-programming project network is considered. Activities numbered in this diagram are initiating (1), human resourcing (2), rules and flowcharts (3), coding (4), and documenting (5). Fig. 6 shows the network of the project.

The related cost and time of different modes of all activities are first converted in the range of 0-100. Then, the qualities associated with different combinations of time and cost is calculated using the proposed methodology.

Table II gives times, costs, and qualities of different task modes of the project. The execution modes for a task in Table II indicate task executer groups who do the task.

Solving the model with the Cplex solver of the GAMS software, the following results are obtained as$x_{012} = x_{023} = x_{131} = x_{211} = x_{242} = x_{351} = x_{431} = 1$, while the other variables are zero.

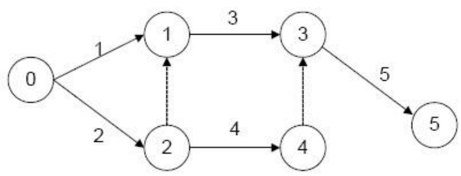

Fig. 6. The example project network.

TABLE II: THE EXAMPLE PARAMETERS

Activity Number	mode	cost	time	quality
1	1	60	100	91
	2	65	80	83
	3	90	70	86
2	1	45	90	80
	2	55	90	83
	3	80	65	83
	4	100	45	91
3	1	80	70	87
	2	100	100	92
4	1	35	100	72
	2	75	75	92
	3	95	100	91
	4	100	80	91
5	1	65	75	83
	2	50	50	75
	3	75	60	80

TABLE III: THE OBJECTIVE FUNCTIONS' SATISFACTION PERCENTAGES

Function	Satisfaction factor	Real value
f_1	0.596	85598
f_2	0.559	1099905
f_3	0.513	370035

This means that activities no. 13, 21, 35, and 43 should be executed in their first mode, activities no. 01 and 24 should be fulfill in their second mode, and activity no. 02 should be performed in its third mode. As a result, the satisfaction percentages of the three objective functions are given in Table III.

V. CONCLUSIONS

A framework was proposed in this research to determine qualities associated with different cost and time combinations of performing project tasks. The framework is based on developing a model to find out suitable modes of all tasks to optimize the project time, cost and quality respect to task's ones. Similar to other research works, different modes of execution were predefined for the tasks. The fuzzy concept was employed based on two purposes: (1) in determining quality parameter based on the cost and the time of the tasks, and (2) in solving a multi-objective model.

Our framework of estimating quality value of a task is a new and applicable way. After this step, we can use all previous approach to trade off among three mentioned inter related parameters. As an extension for future works, one can use multi-index decision rules in fuzzy concept to choose the best mode.

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