Tuning of Centroid with Modified Kernik Mendel (KM) Algorithm

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***Abstract*—Centroid computation of interval type-2 fuzzy logic system, is a pioneering work that provides a measure of uncertainty using foot print of uncertainty. The basics of fuzzy logic do not change from type-1 to higher type but the nature of the membership functions and the operations that depend on the membership functions inevitably change. In the paper, we first review the approaches of type-2 and interval type-2 fuzzy logic system (IT2FLS). In the paper a modified KM algorithm has been proposed to compute centroid of generalized type 2 fuzzy logic system. The modification in KM algorithm and generalization of centroid concept gives a new dimension in foot print of uncertainty.**

***Index Terms*—Type-2 fuzzy logic system, interval type-2 fuzzy logic system (IT2FLS), KM algorithm, footprint of uncertainty (FOU).**

I. Introduction

Fuzzy Logic Systems are comprised of rules and quite often, the knowledge used to build the rules not certain. Such uncertainty leads to rules whose antecedents or consequents are uncertain too and therefore, translated into uncertain membership functions [1], called footprint of uncertainty (FOU). For type-2 fuzzy systems, the antecedent or consequent membership functions are type-2 fuzzy sets and their membership grades themselves are type-1 fuzzy sets. A type-2 fuzzy set is characterized by a fuzzy membership function, where the membership grade for each element is a fuzzy set in [0,1], unlike a type-1 set where the membership grade [2] is a crisp number in [0,1]. Such membership functions are useful in circumstances where determining an exact membership function [3] is difficult. Figure 1 shows FOU of a standard Gaussian function with deliverables. In the original KM algorithm, FOU with variable base width is not calculated, so it is not applicable for centroid computation in generalized type-2 fuzzy logic system (FLS). The modified KM algorithm provides a new dimension of FOU where separate membership functions are not required for different applications.

Introduction of just one function can solve the problem with different level of FOU. Intuitively, we anticipate that geometric properties of the FOU, such as its area and the center of gravities (centroids) of its upper and lower MF, associated with the amount of uncertainty in such a T2 fuzzy system.

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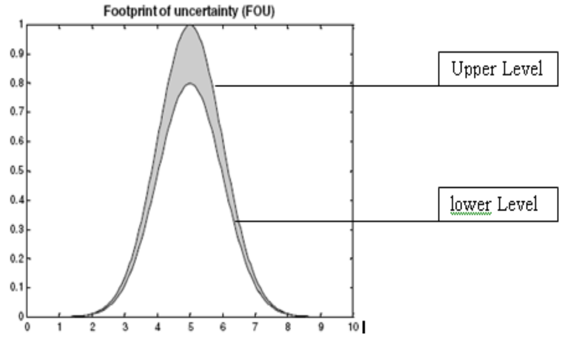

Fig. 1. Footprint of uncertainty of a gaussian membership function

II. Type-2 Fuzzy Systems

Type-2 FLS is very similar to type-1 FLS, except the defuzzifier block of a type-1 FLS [4], replaced by the output processing block, consists of type-reduction followed by defuzzification process [5], shown in Fig. 2.

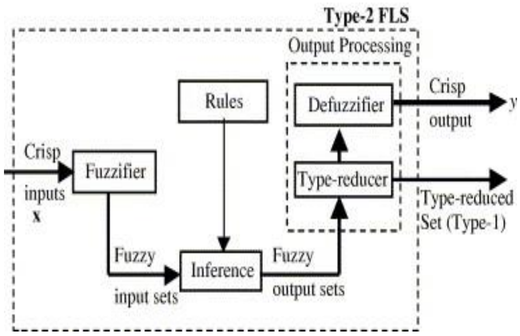

Fig. 2. The structure of a type-2 FLS

Consider a T2 FLS having p inputs $x_1 \in X_1, \ldots, x_p \in X_p$, one output $y \in Y$, and r rules, where the lth rule given below.

R^l :IF x_1 is $\tilde{F}_1$ and … and x_p is $\tilde{F}_P$ then y is $\tilde{G}$ l=1..r

The rule represents a type-2 relation between the input space $x_1 \times \ldots \times x_p$, and the output space Y, of the type-2 FLS, where $\tilde{F}_1^l \times \ldots \times \tilde{F}_p^l = \tilde{A}^l$ and R^l expressed using (1).

$$R^l : F_1^l \times \ldots \times F_p^l \rightarrow G^l = A^l \rightarrow G^l, l = 1,\ldots,r \tag{1}$$

R^l is described by the MF $\mu_{R^l}(x, y) = \mu_{R^l}(x_1,,, x_p, y)$ and F is the fuzzy relation where $(\tilde{F}_l^l, \tilde{F}_p^l) \in F$ In other way, a type-1 fuzzy set characterized by a Gaussian membership function (mean M and standard deviation σ_x), calculates crisp membership $m(x)$ for each input $x \in X$, defined in (2).

$$m(x)=\exp\{-½\,[(x-\mathrm{r})/\sigma_x]^2\} \qquad (2)$$

Membership of x is a fuzzy set and the domain elements of this set are "primary memberships" ($\mu1$). Membership grades [6] of $\mu1$ are "secondary memberships" of x represented by $\mu2(x, \mu1)$ and for a Gaussian function with mean $m(x)$ and standard deviation σ_m, is given in (3).

$$\mu_2(x,\mu_1) = e - \tfrac{1}{2}[(\mu_1 - m(x))/\sigma_m]^2 \qquad (3)$$

The FOU of an IT2FS is described by lower and upper MFs, $\underline{\mu}_{\tilde{A}}(x)$ and $\bar{\mu}_{\tilde{A}}(x)$, described in (4).

$$FOU(? = \bigcup_{x\in X}[\underline{\mu}_{\tilde{A}}(x), \tilde{\underline{\mu}}_{\tilde{A}}(x)] \qquad (4)$$

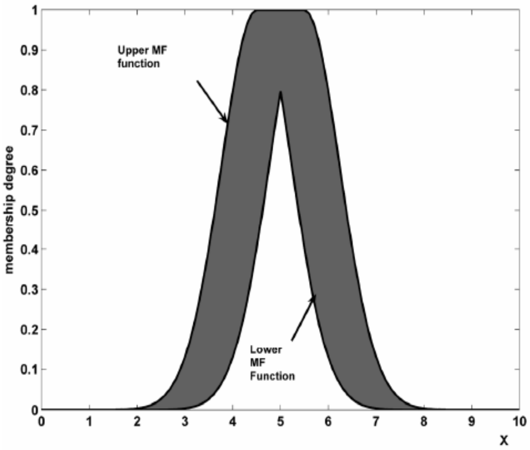

Fig. 3. Interval type-2 membership function

In an IT2FS, X and U are discrete and the domain of $\tilde{A}$ is equal to the union of all embedded type-1 FS, expressed using (5).

$$\tilde{A} = 1/FOU(\tilde{A}) = 1/\bigcup_{j=1}^{n_A} A_e^j \qquad (5)$$

where A_e^j is an embedded T1 FS ($j=1,....n_A$), defined in (6).

$$A_e^j = \bigcup_i^j f(x_i)/x_i \, where \, U_i^j \in \{\bar{\mu}_{\tilde{A}}(x_i),...\bar{\mu}_{\tilde{A}}(x_j)\} \qquad (6)$$

A. Non stationary Fuzzy System

The notation 1/FOU($\tilde{A}$) means putting a secondary grade of 1 to all the elements in the 1/FOU($\tilde{A}$). Garibaldi [7] introduced the concept of a *non-stationary type-1 FS* [8], where $\tilde{A}$ is characterized by MF $\mu_{\tilde{A}}(x,t)$, with $x \in X$, $\mu_{\tilde{A}}(x,t) \in [0,1]$ and t is a free variable. The FS is instantiated at time t, described by (7)

$$\tilde{A} = \int_{x\in X} \mu_{\tilde{A}}(x,t)/x \qquad (7)$$

Three main alternative of non-stationary *type-1FS* are constituted by variation in location, slope and noise. For example, let c denote the center value of a type-1 MF, and $c(t) = c + kf(t)$ denotes a time-varying model where $f(t)$ is a "perturbation function", which may be random.

When $f(t)$ is a known deterministic function [9], then $\mu_{\tilde{A}}(x,t)$ can be lower and upper bounded, and a direct connection exists between Ã and an IT2FS. On the other hand, when $f(t)$ is random, then Ã is a *random fuzzy set,* very different from the fuzzy random variables [10], which can be treated as nonlinear transformations of random processes. If the distribution function is computed for the random $\mu_{\tilde{A}}(x,t)$, then lower and upper probability bounds can be established for each value of a primary variable x.

III. Centre of Sets of Type-2 Fuzzy System

Each consequent of type-2 fuzzy set $\tilde{G}^l$, is first replaced by its centroid, $C_{\tilde{G}^l}$,(a type-1 fuzzy set) and then a weighted average of these centroids is computed. The weight associated with the l^{th} centroid is the firing set corresponding to the l^{th} rule, described as $E_l(x') \equiv \prod_{i=1}^{p} \mu_{\tilde{F}^l}(x_i')$. Until very recently, the only way to compute the center-of-sets of type reduced(TR) set, $Y_{cos}(\mathbf{x}')$, was developed using the following procedure. For each $x = x'$:

1) Discretize the output space Y into a suitable number of points, and compute the centroid $C_{\tilde{G}^l}$ of each consequent set on the discretized output space. These consequent centroid sets [11] can be computed ahead of time and stored.

2) Compute type-1 firing set, $E_l(x') = \prod_{i=l}^{p} \tilde{\mu}_i^p (x_i')$

3) Discretize the domain of each type-1 FS, $C_{\tilde{G}^l}$ into a suitable number of points, say N_l

4) Discretize the domain of each type-1 FS, $E_l(\mathbf{x}')$ into a suitable number of points, say M_l , (l = 1, … , N).

Each consequent of type-2 fuzzy set $\tilde{G}^l$, is first replaced by its centroid, $C_{\tilde{G}^l}$,(a type-1 fuzzy set) and then a weighted average [12] of these centroids is computed. The weight associated with the l^{th} centroid is the firing set corresponding to the l^{th} rule, described as $E_l(x') \equiv \prod_{i=1}^{p} \mu_{\tilde{F}^l}(x_i')$.

5) Enumerate all the possible combinations (d_1, … , d_N, $e_1(\mathbf{x}')$, … , $e_M(\mathbf{x}')$) such that $d_l \in C_{\tilde{G}^l}$ and $e_l(\mathbf{x}') \in E_l(\mathbf{x}')$. The total number of combinations C is given in (8)

$$C = \prod_{l=1}^{M} M_l N_l \,. \qquad (8)$$

6) Compute the centroid $(\sum_{l=1}^{M} d_l e_l(x') / \sum_{l=1}^{M} e_l(x'))$ of each of the enumerated combinations and assign it a membership grade equal to the *t*-norm, shown in (9)

$$T_{l=1}^{M} \mu_{C_{\underset{G}{\sim}l}}(d_l) \times T_{l=1}^{M} \mu_{E_i}(x')(e_l(x')) \tag{9}$$

Mathematically,

$$Y_{cos}(x') = \{(\xi_k, T_{l=1}^{M} \mu_{C_{\underset{G}{\sim}l}}(d_l) \times T_{l=1}^{M} \mu_{E_i}(x')(e_l(x')))_k\} \prod_{l=1}^{M} M_l N_l$$

where

$$\xi_k = \left(\frac{\sum_{l=1}^{M} d_l e_l(x')}{\sum_{l=1}^{M} e_l(x')} \right)$$

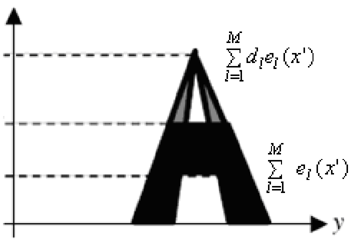

Fig. 4. FOU of type-2 FS using *t*-norm

The peak point of Fig. 4 indicates $\sum_{l=1}^{M} d_l e_l(x')$ and the middle line is the starting of the reference FOU and the last dotted line represents $\sum_{l=1}^{M} e_l(x')$. $MC_{\underset{G}{\sim}l}$ and $E_l(\mathbf{x}')$, are the middle line where M is the number of rules within the function. If two or more combinations of $(d_1, \dots, d_M, e_1(\mathbf{x}'), \dots, e_M(\mathbf{x}'))$ have the same centroid, the one with the largest value equals to $T_{l=1}^{M} \mu_{C_{\underset{G}{\sim}l}}(d_l) * T_{l=1}^{M} \mu_{E_i}(x')(e_l(x'))$ is considered.

IV. Centroid of Type-2 Fuzzy Systems

Centroid TR: let $\tilde{B}$ is another type-2 FS, combined using union operation, as described in (10)

$$\bigcup_{l=1}^{M} \tilde{B}^{L} \equiv \tilde{B} \tag{10}$$

where $\mu_{\underset{B}{\sim}}(y) = \coprod_{l=1}^{M} \mu_{\underset{B}{\sim}l}(y), \forall y \in Y$

$\mu_{\underset{B}{\sim}l}(y)$ is the given secondary MF for the l^{th} rule. As another application of the Representation Theorem [13], the centroid of TR set, $Y_c(x)$, is simply the union of the centroids of all the embedded type-2 FS of $\tilde{B}$ [7]. $Y_c(x)$ is computed using the following procedure. For each $x = x'$:

1) Compute $\mu_{\underset{B}{\sim}}(y)$. where $\mu_{\underset{B}{\sim}l}(y)$ (l = 1, … , m) is already computed for all y $\in$ Y.
2) Discretize the *y*-domain into n+*1* points $y_1, \dots, y_{n+1}$.
3) Discretize each J_{yi}(x') (the primary memberships of $\mu_{\underset{B}{\sim}}(y)$ at y_i) into a suitable number of points, say M_i ($i = 1, \dots, n$). Let $\theta_i(x') \in J_{yi}(x')$.
4) Enumerate total no of embedded T1 sets of $\tilde{B}$; is $\coprod_{i=1}^{N} M_i$.
5) Compute the centroid of each enumerated embedded type-1 set and assign it a membership grade equal to the *t*-norm of the secondary grades corresponding to that enumerated embedded type-1 set [14], described in (11)

$$Y_c(X') = \{(\xi_k (T_{i=1}^{N} f_{y_i}(\theta_i(x')))_k)\} \coprod_{i=1}^{N} M_i \tag{11}$$

where $\xi_k = \left(\frac{\sum_{i=1}^{N} y_i \theta_i(x')}{\sum_{i=1}^{N} \theta_i(x')} \right)_k$

Note that if two or more embedded type-1 fuzzy sets have the same centroid, we keep the one with the largest value of $T_{i=1}^{N} f_{y_i}(\theta_i(x'))$. This procedure is not very practical because it requires $\coprod_{i=1}^{N} M_i$ centroid calculations, and this number in general becomes astronomical. Much more practical method has been developed for computing $Y_c(x)$, based on the fuzzy weighted average (FWA). This method uses α-cuts, an α-cut decomposition theorem and the KM algorithm that was originally developed for computing the centroid of an interval type-2 FS. Another recent approach for computing $Y_c(x)$, based on randomly sampled embedded type-2 FS gives rise to a significant reduction in the time or resources needed to perform type-reduction. Greenwood [15] has examples (for four different primary MFs and different discretizations) that demonstrate the number of embedded sets which are randomly selected only marginally affects the defuzzified value. Excellent results have been obtained for as few as 10 randomly selected embedded type-2 FS. Such a small number of randomly chosen embedded type2 FS can lead to such good results is surprising that awaits a theoretical explanation.

A. Centroid Computation for Generalized T2FS

Using the Representation Theorem for an IT2FS $\tilde{A}$, centroid $C_{\tilde{A}}$ is defined in (12), as the collection of the centroids of its embedded IT2FS [7]. To compute the centroids of all the n_A number of embedded type-1 FS contained within FOU($\tilde{A}$). A smallest and largest element, $C_l(\tilde{A}) \equiv c_l \, and \, C_r(\tilde{A}) \equiv c_r$, are required. A number exist because the centroid of each of the embedded type-1 FS is a finite number. A membership grade one is associated with each of these numbers, because the secondary grades of an IT2FS are all equal to one.

$$C_{\tilde{A}} = 1/\{C_1,....C_r\} \tag{12}$$

where

$$c_l = \min_{\forall\theta_2 \in [\underline{\mu}\tilde{A}(x_j),\bar{\mu}\tilde{A}(x_j)]} \left(\sum_{i=1}^{n} x_i\theta_i \Big/ \sum_{i=1}^{n} \theta_i\right)$$

$$c_r = \max_{\forall\theta_j \in [\underline{\mu}\tilde{A}(x_j),\bar{\mu}\tilde{A}(x_j)]} \left(\sum_{i=1}^{n} x_i\theta_i \Big/ \sum_{i=1}^{n} \theta_i\right.$$

and $x_1 < x_2 < ...x_N$ The latter is true because x_i are sampled values of the primary variable where x_1 denotes the left-hand (smallest) sampled value and x_N denotes the right-hand (largest) sampled value.

The centroid of an IT2FS cannot be computed in closed form.

B. Revised KM Algorithm

Revised KM Algorithm for $C_l(\tilde{A})$:

1) Initialize θ_i by setting $\theta_i = [\mu_{\underline{\tilde{A}}}(x_i) + \bar{\mu}_{\tilde{A}}(x_i)/2]$, $i = 1, \ldots, n$, and
2) compute $c' = c(\theta_1,...,\theta_n) = \sum_{i=1}^{n} x_i\theta_i / \sum_{i=1}^{n} \theta_i$ and Find k $(1 \le k\text{-}1 \le N-1)$ such that $x_k \le c' \le x_{k+1}$.
3) Set $\theta_i = \bar{\mu}_{\tilde{A}}(x_i)$ when $i \le k$, and $\theta_i = \mu_{\underline{\tilde{A}}}(x_i)$ when $i \ge k+1$, and then compute $c_l(k)$ and c'.
4) Check if $c_l(k) = c'$. If yes, stop and set $c_l(k+1) = c_l$ and call K_L. else go to Step 5).
5) If $c' = c_l(k+1)$ then go to Step 2) else step 4).

Revised KM Algorithm for $Cr(\tilde{A})$:

Steps 1) and 2) will be the same.

3) Set $\mu_{\underline{\tilde{A}}}(x_i)$ and $\theta_i = \bar{\mu}_{\tilde{A}}(x_i)$ when $i \le k+1$, and $\theta_i = \bar{\mu}_{\tilde{A}}(x_i)$ when $i+1 \geqslant k$, and compute

$$C_r(k) = \frac{\sum_{i=1}^{k} x_i\, \mu_{\tilde{A}}(x_i) + \sum_{i=k+1}^{k} x_i\, \mu_{\underline{\tilde{A}}}(x_i)}{\sum_{i=1}^{k} \mu_{\tilde{A}}(x_i) + \sum_{i=k+1}^{k} x_i\, \mu_{\underline{\tilde{A}}}(x_i)}$$

4) Check if $c_r(k) = c_l(k+1)$. If yes, stop and set $c_r(k) = c_l(k+1)$ and call *k as* k_R. else go to Step 5).

5) If $c' = c_r(k+1)$ then go to Step 2) else step 4).

Mendel and Liu have proven that the KM algorithm is monotonically convergent. But the revised algorithm result shows it is monotonically increasing in nature but convergent with the footprint, which very fast becomes narrow in nature. Both properties are highly desirable for iterative algorithms.

C. Generalized Centriod and the Bound

Fuzzy weighted average (FWA) computation is shown in (13)

$$y(z_1,....z_n,,,w_1,....w_n) = \frac{\sum_{i=1}^{N} Z_l W_1}{\sum_{i=1}^{N} W1} \tag{13}$$

If each z_l is replaced by an interval set $z_i \in [a_i, b_i]$ and each w_l by an interval set $w_i \in [c_i, d_i]$, then it is called a *generalized centroid* (GC) where GC = $[y_l, y_r]$. The GC is used to perform center-of-sets TR, and compute the FWA. Since z_i appears in the numerator, it is easy to show that

$$y_l = \min_{\forall w_i \in [c_i,d_i]} \left(\sum_{i=1}^{N} a_i w_i \Big/ \sum_{i=1}^{N} w_i\right)$$

$$y_r = \max_{\forall w_i \in [c_i,d_i]} \left(\sum_{i=1}^{N} b_i w_i \Big/ \sum_{i=1}^{N} w_i\right)$$

and

Finally y_m is computed using the modified KM algorithm, defined in (14)

$$y_m = (y_l, y_r) = [\min_{\forall w_i \in [c_i,d_i]} (\sum_{i=1}^{N} a_i w_i / \sum_{i=1}^{N} w_i), \max_{\forall w_i \in [c_i,d_i]} (\sum_{i=1}^{N} b_i w_i / \sum_{i=1}^{N} w_i)] \tag{14}$$

For c_l, x_i is replaced by a_i, and $\mu_{\underline{\tilde{A}}}(x_i)$ and $\bar{\mu}_{\tilde{A}}(x_i)$ are replaced by c_i and d_i, respectively. y_r can be computed using the KM algorithm for c_r in which x_i is replaced by b_i, and again $\mu_{\underline{\tilde{A}}}(x_i)$ and $\bar{\mu}_{\tilde{A}}(x_i)$ are replaced by c_i and d_i, respectively. We have seen that an IT2FS is characterized by its FOU, which in turn is characterized by its upper and lower MF. Intuitively, we anticipate that geometric properties about FOU, such as its area and the center of gravities (centroids) of its upper and lower MF, are associated with the amount of uncertainty of a type-2 FS. Recently, Mendel and Wu [12] demonstrated that this intuition is correct. They quantified uncertainty bounds for the centroid of both symmetric and asymmetric IT2FS with respect to such geometric properties. Using these results, it is possible to formulate and solve *forward problems*, i.e. to go from parametric IT2FS models to data with associated uncertainty bounds. Here we only state results for a symmetrical FOU. The geometric properties that are used, for a FOU, symmetric about m, are A_{UMF}, the area under the upper MF; A_{LMF}, the area under the lower MF; A_{FOU}, the area of the FOU, using (15)

$$A_{FOU} = A_{UMF} - A_{LMF} = \frac{1}{2}\int_{m}^{\infty} x[\bar{\mu}(x) - \underline{\mu}(x)]dx \qquad (15)$$

and, $C_{HFOU}(\tilde{A})$, the centroid of half of $FOU(\tilde{A})$, where

$$C_{HFOU}(\tilde{A}) = \frac{\int_{m}^{\infty} x[\bar{\mu}(x) - \underline{\mu}(x)]dx}{\int_{m}^{\infty} [\bar{\mu}(x) - \underline{\mu}(x)]dx} = \frac{\int_{m}^{\infty} x[\bar{\mu}(x) - \underline{\mu}(x)]dx}{\frac{1}{4}A_{FOU}}$$

V. Conclusion

The aim of the paper is to make a general approach of computing centroid of type-2 FS using revised KM algorithm, which makes FOU more convergent, shown in Fig. 5. The centroid of half FOU is more vital than the total FOU for applications.

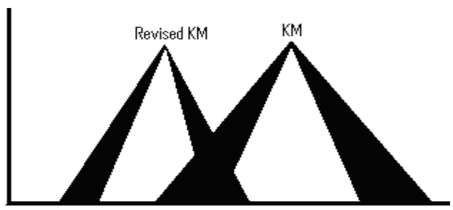

Fig. 5. FOU of KM algorithm vs revised KM algorithm

In the bound of limit, Fig. 6 shows how the value has been changed in the revised KM algorithm and symbolizes that FOU is narrow in nature.

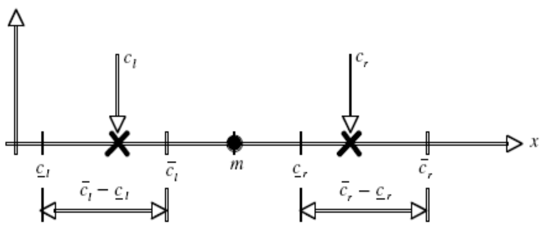

Fig. 6. End-points (X) of the centroid [c_l, c_r] of $\tilde{A}$

Here c_l and c_r are the upper bound and the lower bounds of the centroid. In the generalization approach, the centroid function in (15) is derived from type-2 function. For a general type-2 FS, the new third dimension of its secondary-grades provides additional knowledge about the FS. When all of these grades are the same, as in an IT2FS, then they convey no new information, and so the third dimension is superfluous.

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