Stabilization for the Discrete-Time Nonlinear Systems of Lorenz System

Kreangkri Ratchagit

***Abstract*—In this paper, we study Lu's system, and we study the stability of equilibrium point of Lu's system. Then, we control the chaotic behavior of Lu's system to its equilibrium point using Adaptive Control with two controllers method.**

***Index Terms*—Lorenz system, adaptive control with two controllers.**

I. Introduction

Chaos in control systems and controlling chaos in dynamical systems have both attracted increasing attention in recent years. A chaotic system has complex dynamical behaviors that possess some special features, such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in the phase space [1], [2]. Controlling chaos has focused on the nonlinear systems such as a Lu's system [3], [4].

Lu's system was first introduced in [5] which is described by

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= -xz+cy \\ \dot{z} &= xy-bz \end{aligned} \tag{1}$$

where x, y, z are state variables, a, b, c are positive constants.

The objective of this paper gives sufficient conditions of parameters that make equilibrium point of Adaptive Control with two controllers of the Lu's system to be asymptotically stable.

II. Main Results

In this section, the chaos of system (1) is controlled to one of three equilibrium point of system. Feedback control method is applied to achieve this goal.

Let us consider the Controlled system of the system (1) which has the form

$$\begin{aligned} \dot{x} &= a(y-x)+u_1 \\ \dot{y} &= -xz+cy+u_2 \\ \dot{z} &= xy-bz+u_3 \end{aligned} \tag{2}$$

where x, y, z are state variables, a,b,c are positive constants and u_1, u_2, u_3 are external control inputs which will drag the chaotic trajectory (x_1, x_2, x_3) of the Lu's system to equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$ which is one of three steady states E_0, E_1, E_2.

In this case the control law is

$$\begin{aligned} u_1 &= -g(x-\bar{x}) + \frac{3a}{4}\bar{x}(x-\bar{x})^2, u_2 = 0, \\ u_2 &= -(k(z-\bar{z}) + b(x-\bar{x})^2 + c\bar{x}(x-\bar{x})) \end{aligned}$$

where k, g (estimates of k^*, g^*, respectively) are updated according to the following adaptive algorithm:

$$\begin{aligned} \dot{g} &= \mu(x-\bar{x})^2 \\ \dot{k} &= \rho(z-\bar{z})^2 \end{aligned}$$

where μ, ρ are adaptation gains. Then the controlled system (2) has following form:

$$\begin{aligned} \dot{x} &= a(y-x) - k_1(x-\bar{x}) - g(x-\bar{x}) + \frac{3a}{4}\bar{x}(x-\bar{x})^2 \\ \dot{y} &= -xz + cy - k_2(y-\bar{y}) \\ \dot{z} &= xy - bz - k_3(z-\bar{z}) - (k(z-\bar{z}) + b(x-\bar{x})^2 + c\bar{x}(x-\bar{x})) \\ \dot{g} &= \mu(x-\bar{x})^2 \\ \dot{k} &= \rho(z-\bar{z})^2 \end{aligned} \tag{3}$$

Theorem 3.1 For $g = g^* > c, k = k^* > 0,$ the equilibrium point $E = (\bar{x}, \bar{y}, \bar{z})$ of the system (3) is asymptotically stable.

Proof. Let us consider the Lyapunov function

$$V(\xi_1, \xi_2, \xi_3) = \frac{1}{2}\begin{bmatrix} \frac{a}{c}(x-\bar{x})^2 + b(y-\bar{y})^2 \\ + \frac{c}{\mu a}(g-g^*)^2 + \frac{1}{\rho}(k-k^*)^2 \end{bmatrix}.$$

The time derivative of V in the neighborhood $E = (\bar{x}, \bar{y}, \bar{z})$ of the system (3) is

$$\begin{aligned} \dot{V} &= \frac{a}{c}(x-\bar{x})\dot{x} + b(y-\bar{y})\dot{y} + \frac{c}{\mu a}(g-g^*)\dot{g} \\ &\quad + \frac{1}{\rho}(k-k^*)\dot{k}. \end{aligned} \tag{4}$$

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By substituting (3) in (4)

$$\dot{V} = \frac{a}{c}(x-\bar{x})\left[a(y-x)-k_1(x-\bar{x})-g(x-\bar{x})+\frac{3a}{4}\bar{x}(x-\bar{x})^2\right]$$
$$+b(y-\bar{y})[-xz+cy-k_2(y-\bar{y})]+\frac{c}{\mu a}(g-g^*)(x-\bar{x})^2$$
$$+\frac{1}{\rho}(k-k^*)(z-\bar{z})^2.$$

Let $\eta_1=(x-\bar{x}),\eta_2=(y-\bar{y}),\eta_3=(z-\bar{z})$. Since $(\bar{x},\bar{y},\bar{z})$ is an equilibrium point of the uncontrolled system (1), $\dot{V}$ becomes

$$\dot{V} = \frac{a}{c}\eta_1\left[a((\eta_2+\bar{y})-(\eta_1+\bar{x}))-k_1\eta_1-g\eta_1+\frac{3a}{4}\bar{x}\eta_1^2\right]$$
$$+b\eta_2\left[-(\eta_1+\bar{x})(\eta_3+\bar{z})+c(\eta_2+\bar{y})-k_2\eta_2\right]+\frac{c}{\mu a}(g-g^*)\eta_1^2$$
$$+\frac{1}{\rho}(k-k^*)\eta_3^2$$
$$=-a(\eta_2-\eta_1)^2-\frac{a}{c}\eta_1^2(g^*-c)-2b\eta_1^4-k^*\eta_3^2.$$

It is clear that for positive parameters a,b,c,μ,ρ, if we choose $g=g^*>c,k=k^*>0$, then $\dot{V}$ is negative semidefinite. Since V is positive definite and $\dot{V}$ is negative semi definite, $\eta_1,\eta_2,\eta_3,g,k\in L_\infty$. From $\dot{V}(t)\le 0$, we can easily show that the square of η_1,η_2,η_3 are integrable with respect to t, namely, $\eta_1,\eta_2,\eta_3\in L_2$. From (3), for any initial conditions, we have $\dot{\eta}_1,\dot{\eta}_2,\dot{\eta}_3\in L_\infty$. By the well-known Barbalat's Lemma, we conclude that $\eta_1,\eta_2,\eta_3\to(0,0,0)$ as $t\to+\infty$. Therefore, in the closed-loop system, the equilibrium point $E=(\bar{x},\bar{y},\bar{z})$ of the system (3) is asymptotically stable.

III. NUMERICAL EXAMPLES

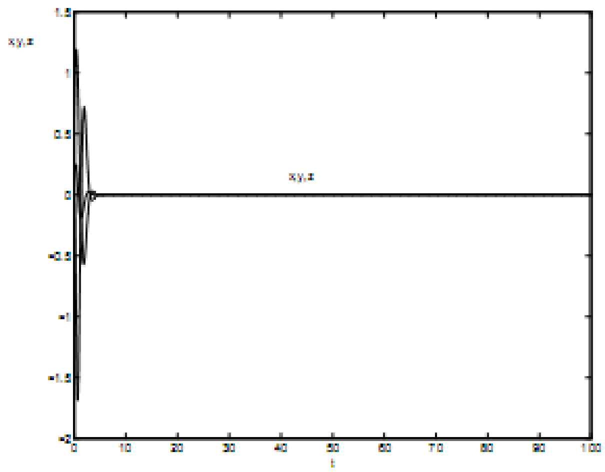

Fig. 1. The time response of the states x,y,z of the controlled system (3), where $\bar{x}=0,\bar{y}=0,\bar{z}=0$.

Numerical experiments are carried out to investigate controlled systems by using Fourth-order Runge-Kutta method with time step 0.01 [6]. The parameters a, b, c,μ,ρ are chosen as $a=15,b=10,c=0.5,\mu=1,\ \rho=0.5$ to ensure the existence of chaos in the absence of control. The initial states are taken as $x=0.5,\ y=0,z=0$. The initial values of parameters μ,ρ are 0 in this simulation. Fig. 1 shows time response for the states x,y,z of the controlled system (1.1) after applying adaptive Control with two controllers.

IV. CONCLUSIONS

In this paper, we give sufficient conditions for stability of equilibrium points of adaptive Control with two controllers which control the chaotic behavior of Lu's system to its equilibrium points. Numerical Simulations are also given to verify results we obtained.

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