# Estimation of Switching Angles by Using PSO of Three Phase Voltage Source Inverter

Bijayalaxmi Patra, Brijesh Kumar, J. Yadagiri, and A. Dasgupta

Abstract—This paper reports particle swarm optimization (PSO) technique for selective harmonic elimination (SHE) in pulse width modulated inverter. A PSO based optimization technique is proposed to minimize the overall THD of the output voltage of PWM inverter and corresponding switching angles are computed. This method is applied for the bipolar switching in three phase inverter for three switching angles and five switching angles. The switching angles are computed to completely eliminate the lower order harmonics. The result of the bipolar case using three switching angles are compared with that of a recently reported work, based on PSO technique, and is observed that the proposed method is effective in eliminating the lower order harmonics completely and overall THD is reducing to a greater extent than the previously reported work.

Index Terms—Selective harmonic elimination (SHE) , particle swarm optimization(PSO) , pulse width modulation (PWM).

#### I. INTRODUCTION

The problem of eliminating harmonics in switching converters has been the focus of research for many years. For systems where high switching efficiency is of utmost importance, it is desirable to keep the switching frequency much lower. In this case, the switching angles are chosen such that a desired fundamental output is generated and specifically chosen harmonics of the fundamental are suppressed. This is referred to as harmonic elimination as the switching angles are chosen to eliminate specific harmonics. In PWM based inverter, generally the harmonic elimination problem is formulated as a set of nonlinear transcendental equations that must be solved to determine the switching angles so as to produce desired fundamental amplitude while eliminating selected harmonics [1-2]. The fundamental component is assigned to a desired output value and other selected orders of harmonics are equated to zero to form the set of transcendental equations.

The numbers of nonlinear equations in terms of unknown switching angles depend on number of harmonic components to be eliminated. The equations have to be solved for each value of modulation index using numerical minimization approach. The technique is not so simple to solve the equations as they have multiple solutions. Moreover, the solutions may have discontinuity at certain modulation indices. In the application of SHE technique, generally the PWM switching angles are calculated off line and stored as look up tables. These tables are stored in a programmable memory and using for example, microcomputer or microcontroller board which has been programmed to accept the value of modulation index and generate the corresponding switching angles. The problems of the conventional methods are the off line look up tables calculations, the selection values of PWM pattern, or using the analog and digital hardware because the solution of the equations for solving switching angles is difficult in on-line . The hardware needs a large look up tables to store the information of switching angles of SHE-PWM technique. An online computation method has been proposed to make the technique a more flexible and interactive one by using neural networks [3]. Predicted initial values allow rapid convergence of Newton iteration for the solution of non linear equation [4]. Multiple solutions to the SHE problem are found in [5] and harmonics are minimized through an objective function by optimization [6]. Another scheme for online calculation of PWM angles has been proposed by Salam [7]. A modified carrier waveform approach is suggested by Wells [8], which is based on quadratic curve fitting of trajectories of the exact PWM angles. Switching angles are also calculated by using genetic algorithm [9]. Moreover, convergence speed of the GA algorithm is low and each step of algorithm to find the switching angles has a time consuming process. Since over all procedure needs a lot of time to find the solutions in the all range of modulation indexes. In this paper the novel and popular particle swarm optimization method is applied to solve the SHE problem for three phase inverter. It is well known that PSO technique is able to find high quality solutions with fast convergence and simple implementation [10], [11]. Particle swarm optimization will allow the switching angles to be computed online ; thus avoiding the need for large memory storage. An algorithm for computing switching angles are presented in [12]. In PSO as the variables are randomized initially, the initial guess does not affect the actual solution. It avoids the multiple solutions of the equations. The problem of discontinuity of solutions at certain point is avoided.

### II. SELECTIVE HARMONIC ELIMINATION

The output voltage waveform of a PWM inverter is shown in Fig. 1. The general Fourier series of the wave form can be given as

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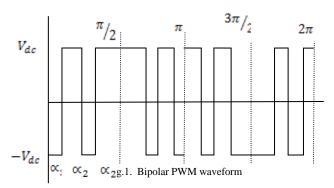
$$V(t) = \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$
 (1)

Due to quarter wave symmetry of the output voltage, the even harmonics are absent and only odd harmonics are present. The amplitude of the  $n^{\text{th}}$  harmonics expressed as

$$b_n = \left(\frac{4}{n\pi}\right) \left[1 + 2\sum_{k=1}(-1)^k \cos n\alpha_k\right]$$
(2)

The switching angles  $\alpha_1$  to  $\alpha_m$  must satisfy the condition:

$$\alpha_1 < \alpha_2 < \cdots \dots < \alpha_m < \pi/2 \tag{3}$$



Given a desired fundamental voltage the problem is to find the switching angles that satisfy the following equation:

$$\frac{4}{\pi} (1 - 2\cos \alpha_1 + 2\cos \alpha_2 - 2\cos \alpha_3) = V_1$$

$$(1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 - 2\cos 5\alpha_3) = 0 \qquad (4)$$

$$(1 - 2\cos 7\alpha_1 + 2\cos 7\alpha_2 - 2\cos 7\alpha_3) = 0$$

Here V<sub>1</sub> is the amplitude of the fundamental component. Nonlinear transcendental equations are thus formed and after solving these equations,  $\alpha_1$  through  $\alpha_3$  are computed. Triplen harmonics are eliminated in three phase balanced system and these are not considered in "(4)." It is evident that (m-1) harmonics can be eliminated with 'm' no. of switching angles.

#### III. SOLVING NON LINEAR EQUATIONS

### A. Newtonraphson method

"Equation (4)" consists of nonlinear equations and transcendental in nature. As a result many people have utilized numerical iterative techniques in order to solve these equations.

Unfortunately numerical iterative techniques require an initial guess in order to work. However if initial guess is not good enough a solution will not found.

- 1) And they needed large time for calculation.
- 2) More than one solution might to exist to the problem at hand.
- B. Particle swarm optimization

Particle Swarm Optimization (PSO) refers to a relatively new family of algorithms that may be used to find optimal solutions to numerical and qualitative problems. PSO was introduced by Russell Eberhart and James Kennedy in 1995 inspired by social behavior of birds flocking or fish schooling. It is easily implemented in most programming languages and has proven to be both very fast and effective when applied to a diverse set of optimization problem.

In PSO, the particles are "flown" through the problem space by following the current optimum particles. Each particle keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) that it has achieved so far. This implies that each particle has memory, which allows it to remember the best position on the feasible search space that has ever visited. This value is commonly called *pbest*. Another best value that is tracked by the particle swarm optimizer is the best value obtained so far by any particle in the neighborhood of the particle. This location is commonly called *gbest* The basic concept behind the PSO technique consists of change in the velocity (or accelerating) of each particle toward its pbest and gbest positions at each time step. This means that each particle tries to modify its current position and velocity according to the distance between its current position and *pbest*, and the distance between its current position and gbest. The position and velocity vectors of the *i*<sup>th</sup> particle of a d- dimensional search space can be represented as

$$X_i = (x_{i1}, x_{i2}, \dots, x_{id}) \quad V_i = (v_{i1}, v_{i2}, \dots, v_{id})$$

respectively. On the basis of the value of the evaluation function, the best previous position of a particle is recorded and represented as  $pbest_i = (p_{i1}, p_{i2}, \dots, p_{id})$ . If the  $g^{th}$  particle is the best among all particles in the group represented as  $gbest = pbest_g =$  $(p_{g1}, p_{g2}, \dots, p_{gd})$ . The particle tries to modify its position using the current velocity and the distance from pbest and gbest. The modified velocity and position of each particle for fitness evaluation in the next iteration are calculated using the following equations,

$$v_{id}^{(k+1)} = w \times v_{id}^{k} + c_1 \times \text{rand}() \times \text{pbest}_{id} - x_{id}^{(k)} + c_2 \times \text{rand}() \times (\text{gbest}_d - x_{id}^{(k)})$$
(5)

$$x_{id}^{(k)} = x_{id}^{(k)} + v_{id}^{(k+1)}$$
(6)

The original procedure for implementing PSO to find optimum switching angles of three phase inverter is as follows:

The equations can be solved in 8 steps. Initially the switching angles are randomly generated between 0 and  $\pi/2$  for the chosen number of population. After generating the initial population a repairing process (using pseudo code based algorithm) is carried out to satisfy system constraints. The initial searching point is set as the initial *pbest* values of the particles. The best value among all the *pbest* values is identified as *gbest*. Where  $c_1$ ,  $c_2$ = acceleration constants, w = inertia weight

rand (.) = randomly generated value between 0and 1.

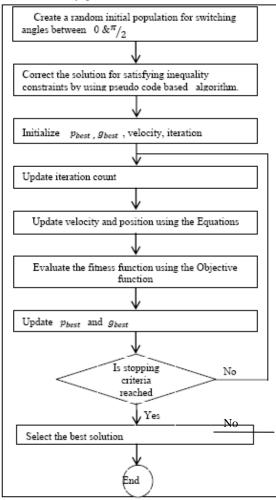


Fig. 2. Flowchart of PSO algorithm

## IV. PROPOSED PSO METHOD

The PSO algorithm is simple in concept, easy to implement and computational efficient. The THD equation is given as

$$THD = \frac{\sqrt{\sum_{n=5,7}^{31} v_n^2}}{v_1} \tag{7}$$

The THD (7), of the output voltage is taken as the objective function. The particle velocity is updated using (5). The position of the particle is then updated using (6). The merit of each individual particle in the swarm is found using a fitness function called evaluation function. The evaluation function should be such that THD is minimized while constraints are satisfied. The value of the *pbest*, *gbest* are updated after evaluation of the fitness function. If the new value is better than the previous *pbest*, the new value is set to *pbest*. Similarly, value of *gbest* is also updated if the best *pbest* is better than the stored value of *gbest*.

### V. COMPUTATIONAL RESULT

To eliminate  $5^{th}$  and  $7^{th}$  order harmonics for a three phase inverter, equations set (4) must be solved. Variation of

switching angles for three switching angles in iterative method and PSO methods are shown in "Fig 3","Fig 4" respectively. Voltage THD with varying modulation index is shown in "Fig 5" while three switching angles of the proposed switching patterns are used. The result obtained in PSO is better compared to Newton Raphson method.

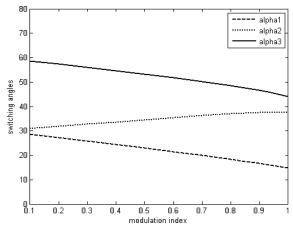


Fig. 3. Bipolar switching angles verses modulation index for three switching angles in Newton raphson technique

A comparison of the unweighted THD computed by the proposed PSO technique up to 31<sup>st</sup> harmonics, with that of the lowest values computed by the previously reported PSO technique [12] is shown in "Fig. 5". From the comparison, it is found that the proposed method is contributing lower % THD in all range of modulation index.

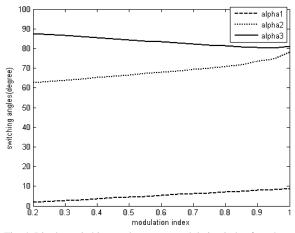


Fig. 4. Bipolar switching angles verses modulation index for three switching angles in PSO technique

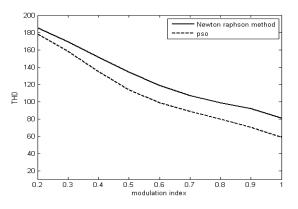


Fig. 5a. Comparison of THD between Newton Raphson and PSO method.

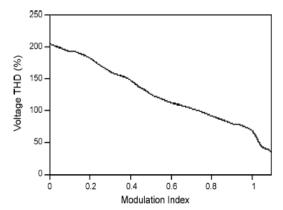
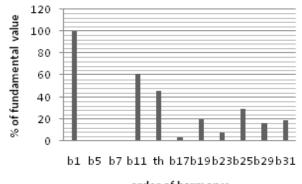


Fig. 5b. THD verses modulation index result of the previously reported PSO technique.



order of harmoncs

Fig.6. Percent of fundamental verses order of harmonics

Harmonic order in percentage of the fundamental value for modulation index of 0.75 is shown in the "Fig. 6". The  $5^{th}$  and  $7^{th}$  harmonics are completely eliminated.

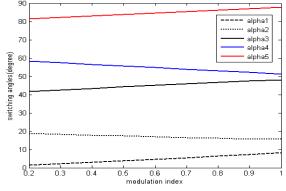


Fig. 7. Angle verses modulation index for five switching angles

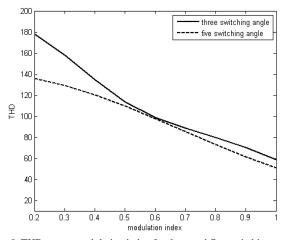


Fig. 8. THD verses modulation index for three and five switching angles

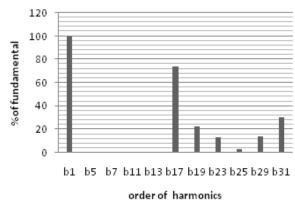


Fig. 9. Percent of fundamental verses order of harmonics

Calculation is also done for the five switching angles where the five equations in terms of five unknown angles are calculated. So with the five switch angles  $5^{\text{th}}$ , $7^{\text{th}}$ ,  $11^{\text{th}}$ , $13^{\text{th}}$ , harmonics are eliminated. Graph of bipolar switching angles verses modulation index for five switching angles is shown in "Fig. 7" and graph of THD verses modulation index for three and five switching angles is shown in "Fig. 8".In case of five switching angles the THD improved than three switching angles. The harmonic order in percentage of the fundamental value for modulation index of 0.75 is shown in "Fig. 9". The 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup> and 13<sup>th</sup> harmonics are completely eliminated.

### VI. CONCLUSION

Comparing the result in this paper it can be concluded that PSO can simply find the optimum switching angles. PSO algorithm is simple in concept, easy to implement and computational efficient. The proposed technique has been applied to Three phase voltage source inverter control schemes. The estimation of up to 5 switching angles per quarter cycle has been performed, while minimizing a preselected number of harmonics and, at the same time, controlling the fundamental component.

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