Behavioral Queueing: An Agent Based Modeling Approach

Karthik Sankaranarayanan, Carlos Arturo Delgado-Alvarez, Erik R. Larsen, and Ann van Ackere

***Abstract*—Queueing research has a plethora of applications and has been an area of study spanning from telecommunications to economics. Traditionally, studies on queueing has mainly concentrated on design, performance and running of the service facility with customers arriving following a stochastic process. In this paper we take an agent based modeling approach to develop a behavioral model of a queueing system using Cellular Automata (CA). We study how adaptive expectation along with a simple information network (as defined by the CA) affects decision-making behavior among agents (customers).**

***Index Terms*—Agent based modeling, adaptive expectations, cellular automata, queueing.**

I. Introduction

Since the publication of the first paper in 1909 by the Danish engineer Agner Krarup Erlang [1] queueing theory [2]-[4] has been studied extensively in several disciplines. While there is a large number of models and wide range of applications of queueing they have one thing in common i.e. they take an aggregated view where customers arrive following an exogenous distribution, they disappear after being serviced and their experiences will have no influence on the future arrival of customers. Furthermore, most models have been linear and it has been possible to find a closed form solution and to determine the optimal capacity. However, many queueing situations are in fact not one-off experiences but repeated choices of which queue to join based on past experiences. Recently there has been a general move towards behavioral studies in operations management, and this has included empirical studies in service marketing. For instance, Law et al. [5] and Bielen and Dumoulin [6] have studied customers' repeat purchases based on past queuing experiences. A number of more recent theoretical models include feedback in order to understand the relationship between customer satisfaction and their decision to return to the service facility [7], [8]. Despite this, there has been relatively little emphasis on understanding the effects of expectations and experiences (while using the service facility) on customers' decision making and the impact of individual choice on the formation of queues.

We begin by deviating from traditional queueing research and include past experience into queueing by adopting an agent based modeling [9] paradigm wherein we model a population of agents who repeatedly make a choice of which facility to use. Agents use standard economic behavior such as adaptive expectations [10] for updating their expectations regarding sojourn time; agents then decide which facility (among several other service points) to choose for the next time period. We adopt a framework based on cellular automata, which defines the structure and the interactions between agents. Another way to view this is that we create a collective choice model with negative externalities, i.e. all agents have to take their decisions at the same time without knowing what other agents decide and the more agents who choose the same facility, the longer the sojourn time.

By adopting a disaggregated framework we try to delve into the micro dynamics of queueing. It allows us to understand and study the formation of agents' expectations and how the flow of information between agents affects expectations and in turn impacts their decision making as well as the aggregated effect of these choices in terms of the formation of queues at the service facilities.

The paper is organized as follows. After this brief introduction, we provide a model description, which is followed by the simulation setup and results. Finally we conclude the paper with comments and future work.

II. Model Description

We consider a group of customers who, each period must choose which service facility (referred to as queues) to patronize. They make their choice based on the sojourn time they expect to face at the different facilities. We model this situation using a variation of one-dimensional cellular automata (CA) [11]-[13], which has local interactions between intelligent (neighboring) adaptive agents.

The model assumes a ring structure, wherein each cell of the CA represents an agent and each agent has exactly two neighbors, one on each side. Agents base their decision on their own experiences as well as on their neighbors' experiences through local interactions, i.e. sharing of information. The parameter K defines the number of neighbors, referred to as the K-neighborhood [14], from whom there are direct information exchanges. The neighborhood represents for instance a social network encompassing colleagues, friends, people living next-door etc. As an example, if $K = 1$ and A^i represents the agents then agent A^i will interact with agents A^{i-1} and A^{i+1} while in the case of $K = 2$ agent A^i will interact with the two neighbors on each side. We call the agents intelligent because each agent (i.e. each cell in the CA) has a memory, which contains the

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expected sojourn time for each facility. We use the following notation: there are N agents, who each time period choose one of the Q queues. We use the same service rate μ for all the queues and an arrival rate λ_j. Note that λ_j is endogenous and represents the number of agents choosing queue j at that period t. Each time period, each agent updates its expectation (memory) of the sojourn time based on two sources of information: its own experience, and that of its neighbors.

Based on its own experience, the agent will update its estimate of the sojourn time at its chosen service station using an exponentially weighted average (adaptive expectations) [15] with weight α given by:

$$\bar{M}_{i,j}^{t} = \alpha \bar{M}_{i,j}^{t-1} + (1-\alpha) M_{i,j}^{t-1} \tag{1}$$

where $\bar{M}_{i,j}^{t}$ - denotes agent i's expected sojourn time for queue j at time t; $\bar{M}_{i,j}^{t-1}$ - denotes agent i's expected sojourn time for queue j at time $t-1$ and $M_{i,j}^{t-1}$ - denotes agents i's actual sojourn time for queue j at time $t-1$.

For $\alpha = 0$, no weight is given to the past, which implies that the expected sojourn time equals the most recently experienced time. *A* value $\alpha = 1$ implies no updating of expectations, i.e. the expectation will never change whatever the agent's experience. Thus, the higher the value of α, the more conservative (or inert) the agent is towards new information, while a lower value means agents consider their recent experiences to be more relevant.

The second source of information comes from the experience of the agent's neighbors. The agent checks, which neighbor in its neighborhood, had the shortest sojourn time. Using the same logic as described above for its own experience, the agent will update its expectation for the queue used by this neighbor using parameter β (Equation (1) holds true also for beta; interchange α with β). In the special case where the facility chosen by the agent and that chosen by its best performing neighbor coincide, the agent only updates its expectation once, using the minimum of α and β as weight. The queue for the next period is chosen based on these updated expectations.

Finally, we need to define the sojourn time W_{ijt} at queue j, given that λ_{jt} agents selected this queue at time t. Let us consider an M/M/1 system (i.e. a one-server system with Poisson arrivals and exponential service times) in steady state. Such a system satisfies the following two equations:
Expected number of people in the system =

$$L = \rho \big/ (1-\rho) = \lambda \big/ (\mu - \lambda) \tag{2}$$

Expected sojourn time =

$$W = L \big/ \lambda = \left[\rho \big/ (1-\rho) \right] \big/ \lambda = 1 \big/ (\mu - \lambda) \tag{3}$$

where ρ denotes the utilization rate given by λ/μ.

Note that these two concepts are related to each other by the well known Little's law:

$$L = \lambda W \tag{4}$$

Unfortunately, such steady state equations are only valid for systems that reach equilibrium, and in particular only for systems where $\rho < 1$. We need a congestion measure that can be used for our transient analysis where, at peak times, customers cluster in the same queue, and the arrival rate temporarily exceeds the service rate. Consider for instance the case of toll bridge where during peak time there is clustering of commuters as the arrival rate temporarily exceeds the service rate, and it takes a while for this queue to be absorbed.

We have therefore attempted to identify a congestion measure that satisfies the behavioral characteristics of (2) to (4), but remains well defined when $\rho \geq 1$. Such a measure should satisfy the following criteria:

(i) If ρ equals zero the number of people in the facility L equals zero (2) (ii) As ρ increases there is a more than proportional increase in L (2) (iii) If the arrival rate tends to zero then the waiting time W is inversely proportional to the service rate μ (3) (iv) When the arrival rate and service rate increase proportionally, leaving ρ unchanged, the waiting time W decreases (3) (v) Little's Law is satisfied: $W = L/\lambda$ (4).

With these requirements in mind, we have defined the following measure for the average sojourn time:

$$L = \rho(\rho + 1) = \rho^2 + \rho \tag{5}$$

Using Little's law and the definition of ρ yields the following expression for the sojourn time:

$$W_{jt} = \frac{\lambda_{jt}}{\mu^2} + \frac{1}{\mu} \tag{6}$$

Regarding the decision rule, we consider the agents to be rational: each time period they choose the facility with the lowest expected sojourn time. The following describes the decision rule in an algorithmic form:

Step 1. Chose the minimum among the expected values of sojourn time.

Step 2. Is there more than one queue with the minimum value?

2.1 ***NO***: Choose the queue pertaining to the minimum value & go to *Step 6*

2.2 ***YES****:* go to *Step 3*.

Step 3. Does the expectation of the queue chosen by the agent in the previous period match the minimum value?

3.1 ***YES***: choose that queue & go to *Step 6*

3.2 ***NO***: Go to *Step 4*.

Step 4. Does the expectation of the queue chosen by the fastest neighbor in the previous period match the minimum value?

4.1 ***YES***: Choose that queue & go to *Step 6*

4.2 ***NO***: Go to *Step 5*.

Step 5. Choose the queue randomly & go to *Step 6*.

Step 6. Execute the choice of queue.

Step 7. Update the expected sojourn times as explained above & go to *Step 1.*

To summarize, the flow sequence of the model is as follows: at time $t = 0$, agents are allocated random memories for each queue, they identify the minimum expected time among these random allocated sojourn times and select that queue, they experience a sojourn time for the selected facility and learn about their neighbors' experience. They update their expectation based on this new information and use these updated expectations to select the queue with the shortest expected sojourn time for period 1. These new choices lead to new experiences, and the updating and decision process is repeated.

III. Simulation Setup and Results

We cannot provide a closed form solution for the model described above, due to its nonlinearities. This is the reason that we are turning to simulation to analyze the behavior. It is also well known in this type of models that even though one knows the individuals' decision rule (as we do) it is virtually impossible to predict the macro outcome of the interactions. However, we have a clear benchmark for our simulations. The social optimum and the Nash equilibrium coincide and correspond to the case where there is an equal distribution of agents across the three facilities. The corresponding average sojourn time is 1.80

It is important to ensure that the one dimensional CA has a large enough number of cells to make behavior independent of the number of cells. Sensitivity analysis has indicated that for our example 120 agents is adequate. The cells are wrapped around so that each cell has two neighbors. In this paper we will only deal with models that take these two nearest neighbors into account, i.e. $K = 1$, due to limitation of space. The initial value of the memory for each agent is allocated by drawing random numbers from a uniform distribution around the equilibrium value. The model consists of three equal service facilities, each with a service rate of $\mu = 5$. The model can easily be configured with service facilities of different service rates, but we will not discuss this here due to space limitations. The model is typically run for 50 time periods for different values of α and β. For the implementation we use Matlab, a numerical computing environment used in engineering and science.

The four panels in Fig. 1 capture the evolution of the agents' choices of service facility over 50 time periods (one iteration) for 4 different pairs of (α, β) values. The horizontal axis represents time and the vertical axis the 120 agents. The 3 colors (white, blue and black) depict the three service facilities. The agents' choice each time period is indicated by color, i.e. the same color in consecutive time periods means that the agent used the same service facility in both periods. The first observation applies to the four panels: the agents initially explore the different queues or in most cases at least two of the three choices. The exact sequence of these explorations depends on the (randomly allocated) initial expected sojourn times. After this transition period of around five time periods we can observe the emergence of more coherent patterns. In Fig. 1 (a) the parameters (α, β) = (0.5,0.5) i.e. agents update their expectations with equal weight to their previous expectation and the experienced sojourn time. In this case agents distribute themselves more or less equally among the three service facilities after a transition period of about 10 time units. During the latter part of this transition period (times 5 to 10) the vast majority of the agents find themselves in the same queue (e.g. the blue queue at time 5). When many agents select the same queue, they face a very long sojourn time, and thus are reluctant to return to this queue. This happens to the black queue in period 9. Agents very gradually return to this queue between periods 10 to 20. At this time the system pretty much stabilizes, only agents who are located on the boarder of two homogenous groups keep changing queue.

In Fig. 1 (b) agents give very little weight to the experience of their best performing neighbor: (α, β) = (0.3,0.9). After the initial transition we observe a relatively stable period which lasts until time = 28. The automaton suddenly becomes unstable due to too many agents choosing the white service facility that period. Over the next 10 periods we observe people switching in droves to a given queue. An interesting pattern materializes between times 33 and 36: at time 33 a vast majority chooses the white service facility, increasing the expected sojourn time, so all agents move away from this queue. At time 36, they all end up at the black facility. Consequently they experience a huge sojourn time, which has such a major impact on their expectations that no agent choose the black facility after this event, i.e. the black facility is “forgotten” by the agents.

Fig. 1 (c) represents the case where (α, β) = (0.0,0.0), i.e. the extreme case where agents place no weight on past expectations; their expected sojourn time for a service facility equals their (or their fastest neighbor’s) most recent experience. The massive move to the black facility in period 3 results in this choice being discarded by all the agents in the next period, where almost all agents select the white facility. Consequently, in period 5 most people find themselves at the blue facility, except for three agents venturing back to the black one. These three agents results in the creation of a diffusion process, with agents gradually moving back to the black road. But it is not until period 14 that one of the agents’ moves back to the blue queue. From there onwards, the diffusion process continues across the three facilities until a more or less equal distribution among the facilities is achieved. The observed behavior with large area of the same choice of facility results from the agents' lack of memory.

Finally, in Fig. 1 (d) we consider the other extreme: α, β) = (0.9,0.9), i.e. agents give much more weight to past expectations than to recent experience. The system thus has a “long” memory. This results in much more conservative behavior: agents need significant evidence before they change facility. There are even a few agents who stick to the same facility during the whole simulation period. Due to the relatively slow updating we also do not observe large homogenous agent groups (diffusions as in Fig. 1 (c)) but do notice a similarity with Fig. 1 (a) where agents who are located on the border between two homogenous agent groups keep switching facility.

Fig. 2 provides more aggregated results for the simulations discussed in Fig. 1: each quadrant shows the minimum, average and maximum sojourn time each time period. In steady state a few agents keep changing facility, often oscillating between two facilities indicated in the discussion of Fig. 1. Consequently the average keeps fluctuating, as we can observe in Fig. 2. Fig. 2(a) and 2(c) show qualitatively very similar behavior: initially the average waiting time is quite high, but it declines fast and ends up close to or at the Nash equilibrium (1.80 for 2(a) and 1.83 for 2 (c)). Furthermore, in these two cases the variation around the average value is relative small.

In Fig. 2 (b) we observe a period of stability from around time 5 to 25. This is followed by a period of significant fluctuations due to the behavior described in Fig. 1 (b), before the system settles down again. However, while the first period of stability resulted in an average sojourn time close to the Nash value of 1.80, the second period results in an average which is 40% higher: a value of 2.6. This is the consequence of agents no longer using the black facility. The sojourn time remains very stable thereafter, and the minimum and maximum values are close to the average.

Finally, in Fig. 2 (d), the “long memory” case, we observe a very different behavior. The initial variations in average waiting time are much less, with a peak below 2.5 compared to over 4 in the cases. We observe that the minimum and maximum values take much longer time to converge to the average waiting time than in the other cases. The slow rate at which expectations are updated result in more stability, but also in a much longer delay in approaching the Nash equilibrium.

IV. Conclusion

A simple, self-organizing disaggregated queuing system with local interaction and locally rational agents has been presented in this paper. We have shown how a simple spatial model can create a variety of different behaviors and, depending on the parameters in the agents’ expectation formation, lead in certain instances the socially optimal or Nash solution, while in other cases the resulting steady state is far away from optimal. While such a model is a poor representation of technical systems such as computer or other network system, it is applicable to repetitive situations where agents decide based on their expectations, which result from previously experienced sojourn at the different service facilities. If we want to understand the behavior of people in many situations ranging from the choice of supermarket, to car repair shop or restaurants, we need to get a better insight into how people's decisions leads to the formation of queues. The link between micro and macro behavior has long been considered an important issue in many disciplines, ranging from sociology to physics [11], [16] and so far there is generally limited understanding of how we can connect the levels of analysis. We consider this as a starting point to understand the connection between micro and macro behavior in queuing research and to supplement traditional queueing and not as a replacement to the traditional approach.

This is clearly just the beginning of a long journey; we are working on extending this framework in many directions. Much of this work is currently in preparation including extending the model by looking at the effect of increasing the flow of information (i.e. changing the K-neighborhood), considering alternative forms of interaction between the agents, allowing for service facilities of different sizes, etc. Furthermore, as a part of the future work we are adopting an experimental approach where human subject take over the role of agents in the model. This should allow us to at least partially verify the model and the agents decision heuristics.

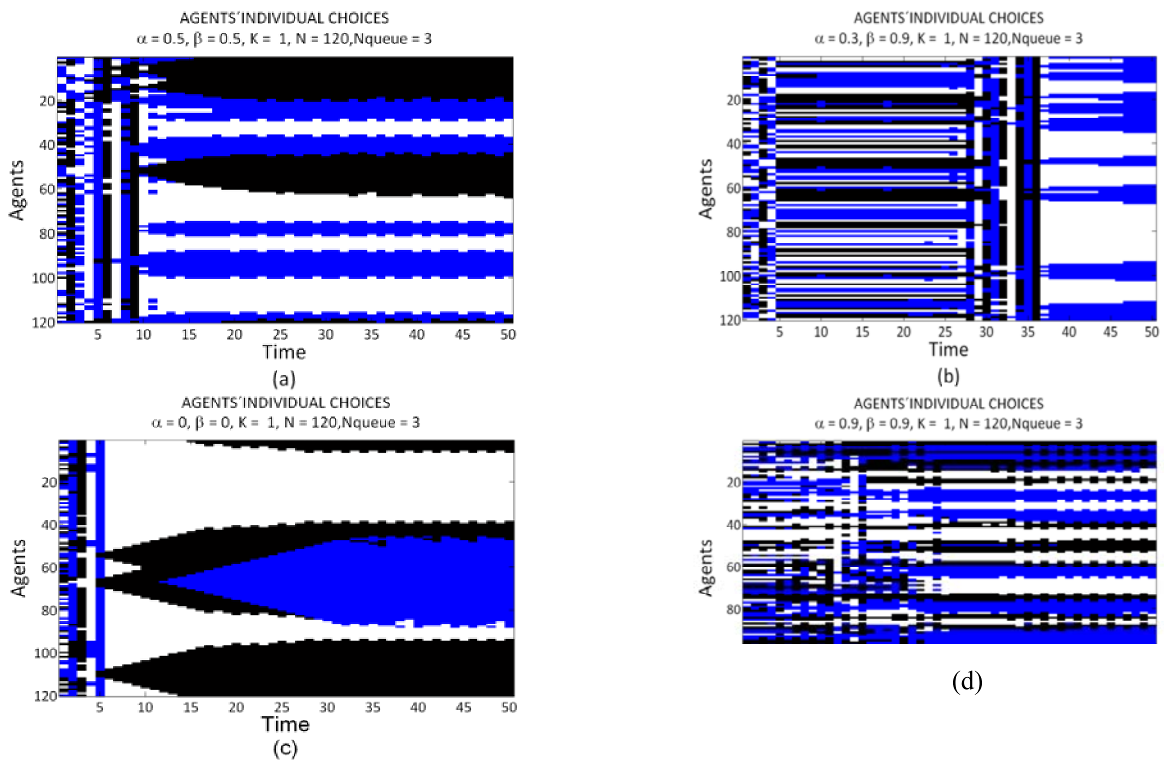

Fig. 1. Spatial-temporal behavioral evolution of agents’ choice of service facility. Each color represents one of the service facilities

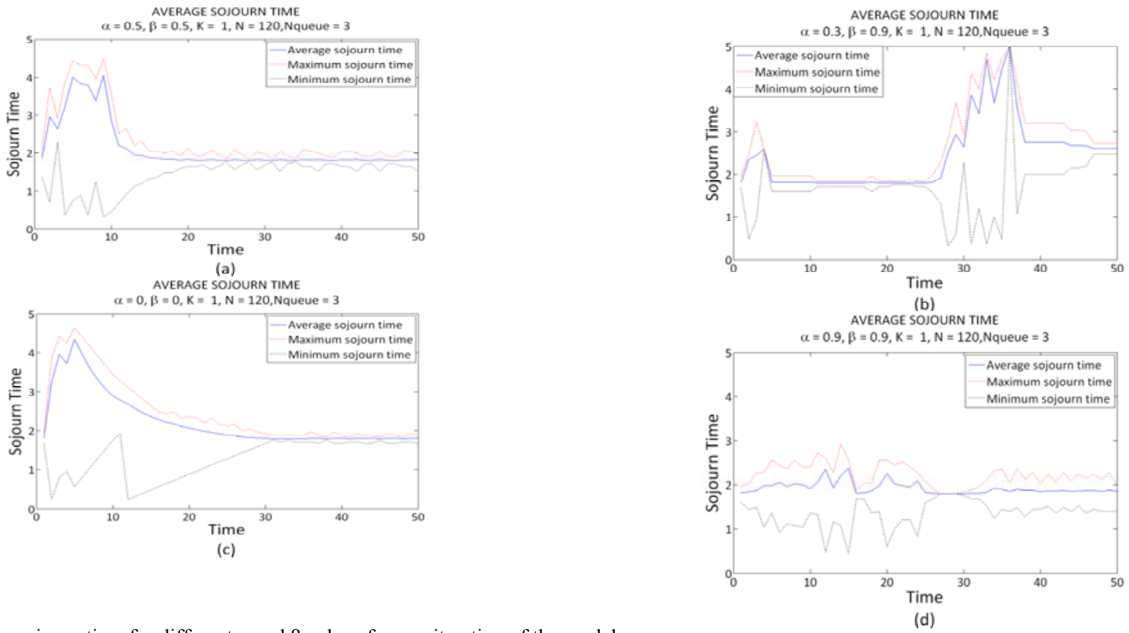

Fig. 2. Average sojourn time for different α and β values for one iteration of the model.

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