Applying Fuzzy Analytic Hierarchy Process to Evaluate and Select Product of Notebook Computers

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Abstract—**The ability, portability and mobility of a notebook computer are important factors that causes the notebook computers to be used widely. To buy a notebook computer, one should look for a product that packs together best features at an affordable price. However, highly competitive business of notebook computers makes the difficulty for buyers to determine. Therefore, using the experts' decision making in evaluating and selecting the alternative among the current products of notebook computers is the beneficial way to help the buyers choose the best one. The objective of this paper is to apply the fuzzy analytic hierarchy process (fuzzy AHP) in determining the relative importance of the decision criterion in order to eventually select the best product of notebook computers. The real numerical finding results have also been demonstrated. Both the theoretical and practical background of this paper have shown that fuzzy AHP is capable to efficiently handle the fuzziness of the data involved in the multi-criteria decision making problem of this study.**

Index Terms—**Analytic hierarchical process, decision analysis, fuzzy logic, multi-criteria decision making, notebook computer selection**

I. Introduction

Notebook computers can be considered as the important roles in human life in this era of technology because of their ability, portability, and mobility. Therefore, the selection of effective notebook computers to suit the needs of buyers is essential. Nowadays, many information sources have presented about choosing a suitable notebook computer. They have mostly presented the features, prices, and pros and cons of each product and model of notebook computers. Most of them haven't decided that which one is the best or the worst to be bought or not bought, but they have just given information and let the buyers compare and decide by themselves. In practice environment, the buyers have to face with a variety of notebook computers' information types that are difficult to determine the decision alternatives. This problem can be a multi-criteria decision making (MCDM) problem. MCDM refers to find the best alternative from all of the feasible alternatives in the presence of multiple decision criteria [1]. Therefore, using the experts' decision making in evaluating and selecting the alternative among the current products of notebook computers under several qualitative and quantitative criteria is the beneficial way to help the buyers choose the best one.

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There have been different methods on MCDM problems, i.e., Analytic Hierarchy Process (AHP) [2]-[3], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [4], Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) [5], etc. The AHP is accepted to be a powerful and flexible method for ranking decision alternatives and selecting the best ones when decision maker has multiple criteria [6]. Its main advantages are handling multiple criteria, easy to understand, and effectively handling both qualitative and quantitative data. However during the decision making, the experts may be imprecise because of the incomplete information of the considered notebook computers, the vagueness of the human thought process, and the inherent complexity and uncertainty of the decision environment.

The objective of this paper is to apply the extension of AHP, namely Fuzzy AHP [7]-[9], in order to handle the fuzziness of data involved in MCDM problem of this study. The fuzzy APH has been applied in a variety of computer science and information technology areas in literature for evaluating and selecting, e.g., the product of notebook computers [10], the mobile phone alternatives [1], the operating system [11], the computer integrated in manufacturing systems [12], the software quality of vendors [13], the best technical institutions [14], and so on. In this research, the Fuzzy AHP will be employed to determine the relative importance of the decision criteria in order to eventually select the best product of notebook computers.

The rest of this paper is organized as follows. The concept of AHP and fuzzy AHP are presented in section II and III, respectively. Section VI describes the data gathering and preprocessing. Section V shows and explains the finding results in the selection problem of notebook computers. Finally, section VI is the conclusion and discussion of this research.

II. Concept of Analytic Hierarchy Process

Analytic Hierarchy Process (AHP), proposed by Saaty [2]-[3], is a traditional powerful decision-making methodology in order to determine the priorities among different criteria, comparing alternatives for each criterion, and determining an overall ranking of the alternatives. The final outcome of the AHP is the best choice among decision alternatives. The basic procedure to carry out the AHP consists of the following steps:

1) Decomposing the decision problem into a hierarchy. The top level of the hierarchy represents the overall goal of the decision problem, the intermediate levels represent the criteria and sub-criteria affecting the decision, and the bottom level represents the possible alternatives.

2) Calculating the relative importance weights of decision criteria in each level of the hierarchy using pair-wise comparisons. In this step, the decision maker uses the fundamental scale or weight between 1 (equal importance) and 9 (extreme importance) defined by Saaty [2] to assess the priority score for each pair of criteria in the same level. That is, the pair-wise comparison matrix is constructed in which the elements a_{ij} inside the matrix can be interpreted as the degree of the precedence of the i^{th} criterion over the j^{th} criterion. Then, the average weight for each normalized criterion is computed.
3) Evaluating the decision alternatives taking into account the weights of decision criteria. The alternative scores are combined with the criterion weights to produce an overall score for each alternative.

The AHP provides a consistency rate (CR) to measure the consistency of judgment of the decision maker that will be presented in the section of fuzzy AHP.

III. Fuzzy AHP

The conventional AHP is inadequate for dealing with the imprecise or vague nature of linguistic assessment. In fuzzy AHP, common sense linguistic statements have been used in the pair-wise comparison which can be represented by the triangular fuzzy numbers [15]. Afterwards, the step of aggregating the pair-wise comparison and the synthesis of the priorities to determine the overall priorities of the decision alternatives will be done.

A. Triangular Fuzzy Numbers (TFNs)

The TFNs used in the pair-wise comparison are defined by three real numbers expressed as a triple (l, m, u) where $l \le m \le u$ for describing a fuzzy event. From a number of TFNs that have been proposed in literature, the one that seems to correspond better to the preferences scale of the crisp AHP is summarized in Table I.

TABLE I: Triangular Fuzzy Conversion Scale

Linguistic Scale	TFNs	Reciprocal TFNs
Equally important	(1, 1, 1)	(1, 1, 1)
Weakly more important	(2/3, 1, 3/2)	(2/3, 1, 3/2)
Strong more important	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strong more important	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)
Absolutely more important	(7/2, 4, 9/2)	(2/9, 1/4, 2/7)

B. Construct the Fuzzy Pair-Wise Comparison Matrix

To construct the fuzzy judgment matrix $\tilde{A}=\{\tilde{a}_{ij}\}$ of n criteria or alternatives via pair-wise comparison, the TFNs are used as follows.

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 \ \ldots & \tilde{a}_{2n} \\ \ldots & \ldots & \ldots \\ \tilde{a}_{n1} & \tilde{a}_{n2} \cdots & 1 \end{bmatrix}$$

where $\tilde{a}_{ij}$ is a fuzzy triangular number, $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, and $\tilde{a}_{ji} = 1/\tilde{a}_{ij}$. For each TFN, $\tilde{a}_{ij}$ or $M = (l, m, u)$, its membership function $\mu_{\tilde{a}}(x)$ or $\mu_M(x)$ is a continuous mapping from real number $-\infty \le x \le \infty$ to the closed interval [0, 1] and can be defined by equation (1).

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-l)/(m-l) & , l \le x \le m \\ (u-x)/(u-m) & , m \le x \le u \\ 0 & , otherwise \end{cases} \quad (1)$$

The operations on TFNs can be addition, multiplication, and inverse. Suppose M_1 and M_2 are TFNs where $M_1=(l_1, m_1, u_1)$ and $M_2=(l_2, m_2, u_2)$, then

Addition: $M_1 \oplus M_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$ (2)

Multiplication: $M_1 \otimes M_2 = (l_1 \cdot l_2, m_1 \cdot m_2, u_1 \cdot u_2)$ (3)

Inverse: $M_1^{-1} = (l_1, m_1, u_1)^{-1} \approx (1/u_1, 1/m_1, 1/l_1)$ (4)

C. Aggregate the Group Decisions

After collecting the fuzzy judgment matrices from all decision makers, these matrices can be aggregated by using the fuzzy geometric mean method of Buckley [16]-[17]. The aggregated TFN of n decision makers' judgment in a certain case $\tilde{u}_{ij} = (l_{ij}, m_{ij}, u_{ij})$ is:

$$\tilde{u}_{ij} = \left(\prod_{i=1}^{n} \tilde{a}_{ijk}\right)^{1/n} \quad (5)$$

where $\tilde{a}_{ijk}$ *is* the relative importance in form of TFN of the k^{th} decision maker's view, and n is the total number of decision makers.

D. Compute the Value of Fuzzy Synthetic Extent

Based on the aggregated pair-wise comparison matrix, $\tilde{U}=\{\tilde{u}_{ij}\}$, the value of fuzzy synthetic extent S_i with respect to the i^{th} criterion can be computed as (6) by making use of the algebraic operations on TFNs as described in (2)–(4).

$$S_i = \sum_{j=1}^{m} \tilde{u}_{ij} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{u}_{ij}\right]^{-1} \quad (6)$$

where $\sum_{j=1}^{m} \tilde{u}_{ij} = \left(\sum_{j=1}^{m} l_j,\ \sum_{j=1}^{m} m_j,\ \sum_{j=1}^{m} u_j\right)$ and

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{u}_{ij} = \left(\sum_{i=1}^{n} l_i,\ \sum_{i=1}^{n} m_i,\ \sum_{i=1}^{n} u_i\right).$$

E. Approximate the Fuzzy Priorities

Based on the fuzzy synthetic extent values, the non-fuzzy values that represent the relative preference or weight of one criterion over others are needed. Therefore, this paper firstly uses Chang's method [2] to find the degree of possibility that $S_b \ge S_a$ as follows:

$$V(S_b \ge S_a) = \begin{cases} 1 & , if\ m_b \ge m_a \\ 0 & , if\ l_a \ge u_b \\ \dfrac{l_a - u_b}{(m_b - u_b) - (m_a - l_a)} & , otherwise \end{cases} \quad (7)$$

where d is the ordinate of the highest intersection between μ_{Sa} and μ_{Sb} as shown in Fig 1. That is, it can be expressed that $V(S_b \ge S_a) = hight(S_a \cap S_b) = \mu_{S_a}(d)$.

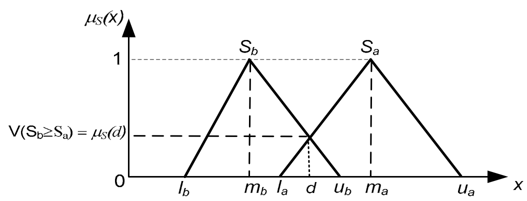

Fig. 1. The intersection between S_a and S_b and their degree of possibility

It is noted that both values of $V(S_a \geq S_b)$ and $V(S_b \geq S_a)$ are required. The degree of possibility for a TFN S_i to be greater than the number of n TFNs S_k can be given by the use of operation min proposed by Dubois and Prade [18]:

$$V(S_i \geq S_1, S_2, \ldots, S_k) = \min \; V(S_i \geq S_k), \quad = w'(S_i) \tag{8}$$

where k= 1, 2, …, n and k # i, and n is the number of criteria described previously. Each $w'(S)$ value represents the relative preference or weight, a non-fuzzy number, of one criterion over others. However, these weights have to be normalized in order to allow it to be analogous to weights defined from the AHP method. Then, the normalized weight $w(S_i)$ will be formed in terms of a weight vector as follows:

$$W = (w(S_1),\ w(S_2),\ \ldots,\ w(S_n))^T \tag{9}$$

Once the weights of criteria are evaluated, it is required to calculate the scores of alternatives with respect to each criterion and then determine the composite weights of the decision alternatives by aggregating the weights through hierarchy.

F. Consistency Test of the Comparison Matrix

To assure a certain quality level of a decision, we have to analyze the consistency of an evaluation. In order to test the value of consistency of the comparison matrix depended on n, the consistency rate (CR) have to be computed. The CR is defined in (10) as a ration between the consistency of a consistency index (CI) and the consistency of a random consistency index (RI). Its value should not exceed 0.1 for a matrix larger than 4x4. For pair-wise comparison matrix being compatible, upper-bound of CR should be like what is shown in Table II [2]-[3].

$$CR = CI \,/\, RI \tag{10}$$

TABLE II: Upper Bound for Pair-Wise Comparison Matrix to Be Compatible

n	3 x 3	4 x 4	n > 4
CR ≤	0.58	0.90	1.12

The CI is used to measure the inconsistency pair-wise comparison as shown in (11) where the eigenvalue λ_{max} can be computed by averaging all eigenvalues of the pair-wise comparison matrix (12). Table III shows values of RI in different values of n.

$$CI = (\lambda_{\max} - n)/(n-1) \tag{11}$$

$$\lambda_{\max} = \sum_{j=1}^{n} a_{ij} \frac{W_j}{W_i} = n \quad ,i, j = 1,2,\ldots,n \tag{12}$$

TABLE III: Values of Random Consistency Index (RI) per Different Number of Criteria

n	RI	n	RI
3	0.58	8	1.41
4	0.90	9	1.45
5	1.12	10	1.49
6	1.24	11	1.51
7	1.32	12	1.48

Next, the process moves on to the phase in which relative weights are derived for various decision criteria. The composite weights of the decision alternatives are then determined by aggregating the weights through the hierarchy.

IV. Data Gathering and Preprocessing

This section presents the way to collect the data of notebook computers, the construction of hierarchical structure to be analyzed, and the steps of generating the fuzzy pair-wise comparison matrix.

A. Collecting Data of Notebook Computers

At this step, we prepare the questionnaire about the preference of the product of notebook computers to ask ten experts who have computer skill more than five years. In this study, the identification of the criteria set for selecting the product of notebook computers has been performed by a combination of common features of notebook computers offered in different brochures, magazines, and websites.

The questionnaire consists of two sections. The first section lists the features of notebook computers and allows the experts to give the preference with rating scales: highest, high, medium, low, and lowest. The second section provides the list of ten well-known products of notebook computers, i.e., Acer, Asus, Dell, BenQ, Fugitzu, HP, Levono, Sumsung, Sony, and Toshiba. Ten up-to-date models of each product offered during the year 2010-2011 are also provided. Therefore, there are a hundred notebook computers the experts have to give the subjective judgment based on the focused features of notebook computers. The experts were requested to evaluate whether or not to buy these notebook computers by considering their features. Eight features and ten products of notebook computers are considered as criteria and alternatives of the hierarchy respectively.

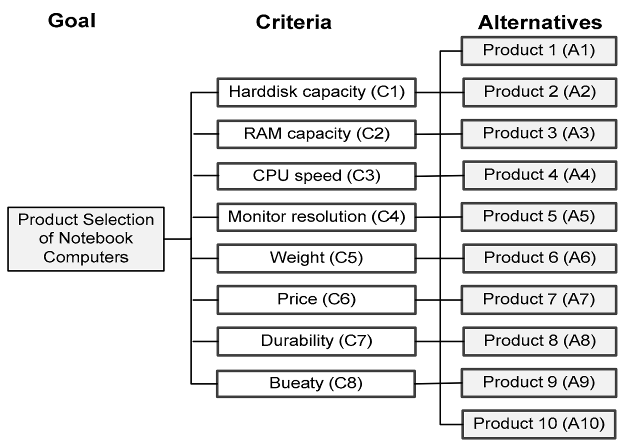

Fig. 2. Hierarchical structure of selecting products of notebook computers

B. Generating the Pair-Wise Comparison Matrices

The next phase after the questionaires are answered by experts is to establish the fuzzy pair-wise comparision matrices. The data preprocessing steps are as follows:

Step 1: From the experts' answers of the first section in the questionaire, the preference of features will be compared. The compared results can be five different scales. Therefore, we transform these scales to the triangular fuzzy numbers.

Step 2: For each feature value of each product, we compute the ratio of the brought notebook computers with respect to ten models of each product.

Step 3: Compare the ratio results of each product obtained from step 2 with other products. The different or distance values will be transformed into the linguistic scale, that is, we will get the triangular fuzzy numbers (TFNs).

Step 4: Construct the fuzzy pair-wise comparison matrices based on the transformed TFNs.

According to these steps, ten fuzzy pair-wise comparison matrices will be constructed according to ten experts. These ten constructed matrices will be subsequently used to determine the product of notebook computers. Table VI illustrates only the original fuzzy pair-wise comparison matrix of the first expert evaluation.

TABLE VI: THE PAIR-WISE COMPARISON MATRIX OF THE FIRST EXPERT EVALUATION IN CRITERIA LEVEL

	C1	C2	C3	C4	C5	C6	C7	C8
C1	(1, 1, 1)	(0.67, 1, 1.50)	(0.67, 1, 1.50)	(0.67, 1, 1.50)	(1, 1, 1)	(0.67, 1, 1.50)	(0.67, 1, 1.50)	(0.67, 1, 1.50)
C2	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1.50, 2, 2.50)
C3	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1.50, 2, 2.50)
C4	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1.50, 2, 2.50)
C5	(1, 1, 1)	(0.67, 1, 1.50)	(0.40, 0.76, 1.50)	(0.40, 0.76, 1.50)	(1, 1, 1)	(0.67, 1, 1.50)	(0.67, 1, 1.50)	(0.67, 1, 1.50)
C6	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1.50, 2, 2.50)
C7	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(0.67, 1, 1.50)	(1, 1, 1)	(1, 1, 1)	(1.50, 2, 2.50)
C8	(0.67, 1, 1.50)	(0.40, 0.50, 0.67)	(0.40, 0.50, 0.67)	(0.40, 0.50, 0.67)	(0.67, 1, 1.50)	(0.40, 0.50, 0.67)	(0.40, 0.50, 0.67)	(1, 1, 1)

V. FINDING RESULTS IN THE SELECTION PROBLEM OF PRODUCT OF NOTEBOOK COMPUTERS

This section presents the finding results in the selection problem of product of notebook computers including the aggregated fuzzy pair-wise matrix, the computed fuzzy synthetic extent values, the approximated fuzzy priorities for criteria, and the approximated fuzzy priorities for alternatives.

A. The Aggregated Fuzzy Pair-wise Matrix

After the experts' fuzzy pair-wise comparison matrices are constructed, the aggregated fuzzy pair-wise comparison matrix is computed according to (5) as shown in Table V. The sums of horizontal and vertical directions from Table V are illustrated in Table VI. The sum of row or column sums will be used to compute the fuzzy synthetic extent values.

TABLE V: THE AGGREGATED FUZZY PAIR-WISE COMPARISON IN CRITERIA LEVEL

	C1	C2	C3	C4	C5	C6	C7	C8
C1	(1, 1, 1)	(0.65, 0.87, 1.18)	(0.59, 0.76, 1)	(0.78, 1, 1.28)	(1.11, 1.25, 1.39)	(0.92, 1.15, 1.41)	(0.71, 0.87, 1.08)	(0.78, 1, 1.28)
C2	(0.85, 1.15, 1.53)	(1, 1, 1)	(0.92, 1, 1.08)	(0.85, 1, 1.18)	(0.85, 1.15, 1.53)	(0.92, 1.15, 1.41)	(0.85, 1, 1.18)	(1.41, 1.64, 1.85)
C3	(1, 1.32, 1.70)	(0.92, 1, 1.08)	(1, 1, 1)	(0.78, 1, 1.28)	(1, 1.32, 1.7)	(1, 1.15, 1.3)	(0.92, 1, 1.08)	(1, 1.32, 1.7)
C4	(0.78, 1, 1.28)	(0.65, 0.87 1.18)	(0.78, 1, 1.28)	(1, 1, 1)	(1.02, 1.25, 1.51)	(0.72, 1, 1.38)	(0.60, 0.70, 0.83)	(0.83, 1, 1.2)
C5	(0.72, 0.80, 0.90)	(0.77, 0.87, 1)	(0.59, 0.76, 1)	(0.66, 0.80, 0.98)	(1, 1, 1)	(1, 1, 1.50)	(0.67, 0.70, 1.50)	(0.29, 1, 2.50)
C6	(0.72, 0.80, 1.08)	(0.60, 0.70, 0.83)	(0.54, 0.61, 0.71)	(0.77, 0.87, 1)	(0.72, 1, 1.38)	(1, 1, 1)	(0.71, 0.87, 1.08)	(0.85, 1.15, 2.53)
C7	(0.92, 1.15, 1.41)	(0.67, 1, 1.50)	(0.67, 1, 1.50)	(0.92, 1, 1.08)	(1.2, 1.43, 1.67)	(0.92, 1.15, 1.41)	(1, 1, 1)	(1.18, 1.52, 1.88)
C8	(0.78, 1, 1.28)	(0.29, 0.70, 1.50)	(0.29, 0.61, 1)	(0.59, 0.76, 1)	(0.83, 1, 1.20)	(0.65, 0.87, 1.18)	(0.53, 0.66, 0.85)	(1, 1, 1)

TABLE VI: THE SUMS OF HORIZONTAL AND VERTICAL DIRECTIONS

Criteria	Row Sums	Column Sums
C1	(6.55, 7.89, 9.61)	(6.77, 8.29, 10.20)
C2	(7.46, 8.88, 10.58)	(6.23, 7.31, 8.72)
C3	(7.89, 9.43, 11.19)	(5.98, 6.99, 8.33)
C4	(7.36, 8.71, 10.32)	(6.36, 7.43, 8.79)
C5	(5.77, 6.93, 8.47)	(7.74, 9.93, 11.40)
C6	(6.23, 7.50, 9.17)	(7.14, 8.61, 10.40)
C7	(7.78, 9.24, 10.91)	(6.10, 7.10, 8.39)
C8	(5.53, 6.60, 8.04)	(8.26, 10.1, 12.11)
Sum of row or column sums		(54.57, 65.19, 78.31)

B. The Computed Fuzzy Synthetic Extent Values

The fuzzy synthetic extent value S_i with respect to the i^{th} criterion can be computed with (6). The example of calculating this value for the criterion *C1* is shown below. For other criteria, their fuzzy synthetic extent values are shown in Table VII.

$$S_{C1} = (6.55,\ 7.89,\ 9.61) \otimes (54.57,\ 65.19,\ 78.31)^{-1}$$
$$= (0.0836,\ 0.1211,\ 0.1762)$$

TABLE VII: THE FUZZY SYNTHETIC EXTENT OF EACH CRITERION

Criteria	Fuzzy Synthetic Extent Value (S_i)
C1	(0.0836, 0.1211, 0.1762)
C2	(0.0952, 0.1362, 0.1938)
C3	(0.1008, 0.1448, 0.2052)
C4	(0.0940, 0.1338, 0.1891)
C5	(0.0737, 0.1063, 0.1553)
C6	(0.0796, 0.1151, 0.1680)
C7	(0.0993, 0.1418, 0.1999)
C8	(0.0706, 0.1012, 0.1474)

C. The Approximated Fuzzy Priorities for Criteria

From the fuzzy synthetic extent values, the non-fuzzy values that represent the relative preferences or weights of one criterion over other criteria will be approximated. Each of them is the degree of possibility computed as follows (show only the weight of *C1* criterion over others):

$$V(S_{C1} \geq S_{C2}) = (0.0952 - 0.1762)/((0.1211 - 0.1762) - (0.1362 - 0.0952))$$
$$= 0.8429$$

$$V(S_{C1} \geq S_{C3}) = (0.1008 - 0.1762)/((0.1211 - 0.1762) - (0.1447 - 0.1008))$$
$$= 0.7618$$

$$V(S_{C1} \geq S_{C4}) = (0.0940 - 0.1762)/((0.1211 - 0.1762) - (0.1338 - 0.0940))$$
$$= 0.8672$$

$$V(S_{C1} \geq S_{C5}) = 1$$

$$V(S_{C1} \geq S_{C6}) = 1$$

$$V(S_{C1} \geq S_{C7}) = (0.0993 - 0.1762)/((0.1211 - 0.1762) - (0.1418 - 0.0993))$$
$$= 0.7878$$

$$V(S_{C1} \geq S_{C8}) = 1$$

Hence, the relative weight of the criterion *C1* is:

$$V(S_{C1} \geq S_{C2},..S_{C8}) = \min(0.8428,\ 0.7618,\ 0.8672,\ 1,\ 1,\ 0.7878,\ 1)$$
$$= 0.7618 = w'(S_{C1})$$

The relative weights of other criteria ($w'(S_{C2})$ until $w'(S_{C8})$) are computed and illustrated in Table VIII. These relative weights have to be normalized in order to allow them to be analogous to weights defined from the AHP method. The normalized weight $w(S_i)$ is shown in Table VIII.

TABLE VIII: THE NORMALIZED WEIGHT VALUES OF EACH CRITERION

Criteria	Relative Weight ($w'(S_i)$)	Normalized Weight ($w(S_i)$)
C1	0.7618	0.1202
C2	0.9162	0.1446
C3	1	**0.1578**
C4	0.8892	0.1403
C5	0.5872	0.0927
C6	0.6943	0.1095
C7	0.9718	0.1533
C8	0.5173	0.0816
Sum of $W'(S_i)$	6.3380	

From the weights of criteria, the criterion *C3* (*CPU speed*) has the highest weight followed by the criterion *C7 (durability)*. It can be interpreted that the experts give precedence to the *CPU speed* over every criteria in the problem. The *durability* is the second preference criterion and so on. The consistency rate we got is 0.0028 which is less than 0.1, therefore, the matrix can be considered to be consistent.

Now we know the decision criterion that is most important to select the product of notebook computers. Other criteria may affect the product selection of notebook computers also. The next step is to select the most importance product of notebook computers.

D. The Approximated Fuzzy Priorities for Alternatives

Similarity, then, the transformation procedures for comparison between criteria based on each alternative will be calculated. The relative weights of criteria based on each alternative are shown in Table XI. Finally, the final results of normalized weights from this table with respect to the overall criteria weights are computed and shown in Table X.

TABLE XI: WEIGHTS OF CRITERIA BASED ON EACH ALTERNATIVE

	C1	C2	C3	C4	C5	C6	C7	C8
A1	0.085	0.105	0.096	0.104	0.130	0.109	0.116	0.130
A2	0.127	0.116	0.103	0.104	0.123	0.134	0.118	0.121
A3	0.072	0.055	0.039	0.065	0.005	0.097	0.074	0.037
A4	0.162	0.244	0.311	0.191	0.290	0.139	0.129	0.263
A5	0.113	0.179	0.225	0.133	0.130	0.110	0.118	0.183
A6	0.082	0.055	0.066	0.059	0.057	0.083	0.078	0.058
A7	0.107	0.062	0.073	0.118	0.046	0.089	0.084	0.057
A8	0.063	0.054	0.042	0.046	0.011	0.071	0.099	0.039
A9	0.108	0.052	0.027	0.106	0.127	0.069	0.095	0.080
A10	0.082	0.077	0.018	0.073	0.080	0.098	0.089	0.033

TABLE X: THE WEIGHTS OF CRITERIA BASED ON EACH ALTERNATIVE WITH RESPECT TO THE OVERALL WEIGHTS OF CRITERIA

	C1	C2	**C3**	C4	C5	C6	C7	C8
A1	0.010	0.013	0.012	0.013	0.016	0.013	0.014	0.016
A2	0.015	0.014	0.012	0.013	0.015	0.016	0.014	0.015
A3	0.009	0.007	0.005	0.008	0.001	0.012	0.009	0.004
A4	**0.019**	**0.029**	**0.037**	**0.023**	**0.035**	**0.017**	**0.016**	**0.032**
A5	0.014	0.022	**0.027**	0.016	0.016	0.013	0.014	0.022
A6	0.010	0.007	0.008	0.007	0.007	0.010	0.009	0.007
A7	0.013	0.007	0.009	0.014	0.006	0.010	0.010	0.007
A8	0.008	0.007	0.005	0.006	0.001	0.009	0.012	0.005
A9	0.013	0.006	0.003	0.013	0.015	0.008	0.011	0.010
A10	0.010	0.009	0.002	0.009	0.010	0.012	0.011	0.004

From Table X, the priority of each alternative was done by considering criteria. Based on the weights of criterion *C3* (*CPU speed*), the product *A4* is the highest and the product *A5* is the second. Weights of all criteria of the product *A4* also be the highest compared with other products. The graph in Fig. 3 illustrates the percentage of weight sums of each alternative.

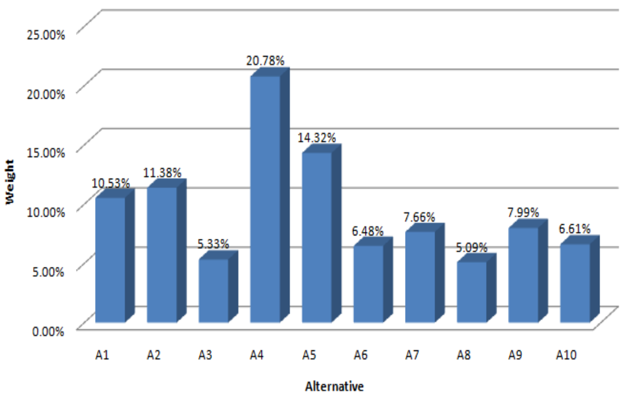

Fig. 3. The graph of weight percentage of each alternative

From Fig. 3, the product *A4* also has the highest alternative weight. It reveals that the product *A4* is the most preferable alternative over the others with *A5* the second. Therefore for selecting a product of notebook computers from many experts, a variety of products and multiple criteria with fuzzy AHP method, the product *A4* is the best one.

VI. CONCLUSION

This research used the fuzzy AHP to solve the problem of evaluating and selecting a product of notebook computers among the others. It is utilized due to its ability for taking into account both the qualitative and quantitative measures. Eight decision criteria have been used for assessing ten different products of notebook computers. In this research, the triangular fuzzy numbers are utilized in establishing the pair-wise comparisons of criteria and alternatives through linguistic scales. Further, group-based fuzzy analytical hierarchy process was used in generating criteria weights for the evaluation of products of notebook computers. By using fuzzy AHP, the qualitative judgment can be qualified to make comparison more perception and reduce assessment bias in pair wise comparison process. This finding result will help the buyers to select the best product of notebook computers. However, buying notebook computer is actually based on the buyers because of their budgets in hand and the different satisfaction in designed style of product.

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