# Discrete Model for the Movement of Industrial Manipulator Used in Hot Rolling Process

Iulia Clitan, Vlad Muresan, Mihail Abrudean, and Valentin I. Sita

*Abstract*—The presented paper describes the process of determining a discrete model for the horizontal movement of an industrial manipulator. The manipulator is a specialized crane used to unload billets from inside a rotary hearth furnace, during the process of manufacturing tube pipes by hot rolling. The continuous transfer function model is already known, due to previous research, thus this paper focuses on digitizing this transfer function by means of zero-order hold transform, by applying simple substitution and Tustin substitution.

The pulse transfer functions yielded from digitization are compared graphically and by computing the root mean square deviation between the continuous signal and the discrete ones. The best fitted discrete transfer function model, for the horizontal movement, is converted into a dependency law between the input and the output (that can be easily implemented on a numerical device), and into a discrete structure with delay blocks.

*Index Terms*—Digitizing methods, discrete model, horizontal movement, industrial manipulator, pulse transfer function.

#### I. INTRODUCTION

Several types of manipulators, robotic handlers with different degrees of freedom or specialized cranes, are widely used in different branches of industry, due to their robustness and capability to handle heavy loads [1]–[4]. The case study presented in this paper deals with an industrial manipulator used to extract previously heated billets from inside a rotary hearth furnace [5]. The described equipment is part of the technological flow of a hot rolling process that manufactures seamless tube pipes [6]. Throughout the production chain, there are several types of robotic handlers in use, due to the high temperature at which billets need to be heated (around 1300°C) and due to their weight.

Previous research focused on continuous modeling of the robotic handler movement in a horizontal plane [6]–[10], however, the desire to implement the model on a numerical device arise the need to obtain a discrete mathematical model for this system.

Thus, this paper deals with the conversion of the continuous mathematical model into a discrete one and implementing the resulting positioning model in a discrete form (as a discrete dependency law between the input and the output and as a discrete structure with unit delay blocks).

## II. INDUSTRIAL MANIPULATOR'S DESCRIPTION

The industrial manipulator presented in this paper is an electro-hydraulically driven robot, having three degrees of freedom. Fig. 1 shows a side view of the industrial manipulator and one can observe the components, such as: the end effector (the pliers) and the mobile unloading arm. In order to protect the economical partner some technical details will not be disclosed.

The movements of advance or lowering/lifting the manipulator's arm take place at predetermined lengths that are known for each loading batch; thus these movements require simple control. A more complex and precise control strategy is required however for the horizontal movement of positioning the end effector over the billet, prior to the lowering of the arm movement. This movement is able due to the rotation of the arm around the static point (see Fig. 1), resulting a horizontal movement of about 300 millimeters. This distance, along with the pliers opening is enough to ensure a precise billet grip.

Such horizontal motion is required due to the repositioning of the billets toward the furnace hearth with respect to the loading position, caused for example by rotation inertia. The distance between billets is small, to ensure maximal load inside the furnace, and since there is no admittance of heating adjacent billets the control for horizontal movement must be precise, thus a control strategy and a mathematical model is necessary.

Based on a set of experimental data a continuous model, as the  $G_{sys}(s)$  transfer function, was previously obtained for the horizontal positioning process of this industrial manipulator, [6]-[10]:

$$G_{sys}(s) = \frac{18}{(3s+1)(0.25s+1)} \tag{1}$$



Fig. 1 Side view of the industrial manipulator and its main components [10].

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## III. DIGITIZING THE TRANSFER FUNCTION MODEL

The conversion of the continuous transfer function model to a pulse transfer function one is done by means of the *z*-transform approximations.

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Since the digital to analog converters can be represented by a zero-order hold (ZOH) circuit, this is one of the methods we used in order to obtain the pulse transfer function for the horizontal movement. This implies the usage of the equation given bellow in (2), where the system must be expanded into partial fractions and further, the *z*-transform is determined based on the correspondence table between the *s*-domain functions and the *z*-domain ones [11]. The discrete transfer function that results is noted as  $G_{ZOH}(z)$ .

$$ZOH(z) = (1 - z^{-1})Z\left\{\frac{G_{sys}(s)}{s}\right\} = (1 - z^{-1})Z\left\{\frac{18}{s} - \frac{19.636}{s + \frac{1}{3}} + \frac{1.636}{s + 4}\right\} (2)$$
$$G_{ZOH}(z) = (1 - z^{-1})\left\{\frac{18}{1 - z^{-1}} - \frac{19.636}{1 - e^{-\frac{T_s}{3}}z^{-1}} + \frac{1.636}{1 - e^{-4T_s}z^{-1}}\right\} (3)$$

Another digitizing method used is simple substitution, based on substituting the following expression for s in the linear transfer function ( $T_s$  represents the sampling time):

$$s = \frac{1 - z^{-1}}{T_S}$$
(4)

From simple substitution  $G_{simple}(z)$  is obtained, as the discrete transfer function for the horizontal movement of the industrial manipulator.

$$G_{simple}(z) = \frac{18}{\left(1 + \frac{3.25}{T_S} + \frac{0.75}{T_S^2}\right) - \left(\frac{1.5}{T_S^2} + \frac{3.25}{T_S}\right) z^{-1} + \frac{0.75}{T_S^2} z^{-2}}$$
(5)

The third method we used is a more exact substitution, the Tustin substitution, based on the same principle as the simple substitution with the expression given in (6) and the yielded pulse transfer function given in (7).

$$s = \frac{2(1-z^{-1})}{T_s(1+z^{-1})} \tag{6}$$

$$G_{Tustin}(z) = \frac{18T_S^2(1+2z^{-1}+z^{-2})}{T_S^2+6.5T_S+3+(2T_S^2-6)z^{-1}+(T_S^2-6.5T_S+3)z^{-2}}$$
(7)

## IV. DISCRETE MODELS VALIDATION

The validation of the above discrete transfer function is done via computer simulation, using MATLAB, and by computing the root mean square error between the response of the continuous model and all the discrete ones, at sampling instances.

The first step in determining the pulse transfer functions is to select the sampling time, since the transfer function coefficients depend on this value. Such a period is usually chosen as ten times smaller than the dominant time constant of the continuous system [11]. Thus, for this system the sampling period is chosen equal to 0.3 seconds. The pulse transfer functions that yielded for the horizontal movement of the industrial manipulator, with the given sampling time, are given below.

$$G_{ZOH}(z) = \frac{0.7251z^{-1} + 0.4719z^{-2}}{1 - 1.206z^{-1} + 0.2725z^{-2}}$$
(8)

$$G_{simple}(z) = \frac{18}{20.17 - 27.5z^{-1} + 8.333z^{-2}}$$
(9)

$$G_{Tustin}(z) = \frac{0.3214 + 0.6429z^{-1} + 0.3214z^{-2}}{1 - 1.155z^{-1} + 0.2262z^{-2}}$$
(10)

Fig. 2 plots the evolution of the continuous system's step response, versus the discrete step response of all three transfer functions, at a unitary step input signal. Since all discrete evolutions follow well the continuous signal, the computation of the root mean square error using (11) is mandatory in order to determine the best fitted discrete model for the concerned system.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - y_i^*)^2}$$
(11)

where *N* represents the number of sampling instances, at the  $T_s$  sampling time,  $y_i$  the continuous signal value and  $y_i^*$  the discrete signal value.

From computation we obtained the following values for the root mean square errors:

$$RMSE_{ZOH} = 0.0167$$

$$RMSE_{simple} = 0.3987$$

$$RMSE_{Tustin} = 0.2523$$
(12)

Analyzing the above results, one can note that the best fitted discrete model is the pulse transfer function obtained using *ZOH* as the digitizing method.



Thus in Fig. 3 the authors plotted the unitary step response of the continuous model versus the unitary step response of the pulse transfer function from (8).

# V. DISCRETE MODEL REPRESENTATION

## A. Discrete Dependency Law

The ZOH pulse transfer function given in (8) resulted as the discrete model for the horizontal movement. By cross multiplication we get:

$$Y(z) - 1.206z^{-1}Y(z) + 0.2725z^{-2}Y(z) =$$
  
= 0.7251z^{-1}U(z) + 0.4719z^{-2}U(z) (13)

where Y(z) represents the output signal (the position of the manipulator's end effector in millimeters), and U(z) represents the input signal (the offset given to the hydraulic cylinder).

The difference equation is given below, this denotes the fact that the output at each instant can be computed by an iterative procedure, and it depends on past values of the output signal and on past values of the input signal.

$$y_k = 1.206 y_{k-1} - 0.2725 y_{k-2} + 0.7251 u_{k-1} + 0.4719 u_{k-2}$$
 (14)

where  $y_k$  represents the current value of the output signal,  $y_{k-1}$  represents the output signal delayed with one sample time  $T_s$ ,  $y_{k-2}$  represents the output signal delayed two times,  $u_{k-1}$  represents the input signal delayed once and  $u_{k-2}$  represents the input signal delayed twice.

## B. Discrete Structure with Delay Blocks

Starting from the difference equation a block structure is derived. This structure is implemented in MATLAB Simulink by converting the discrete dependency law as an input – output structure with unit delay blocks. The resulted structure is depicted in Fig. 4, as the discrete dependency input – output model. This dependency model is tested against the (8) pulse transfer function graphically (see Fig. 5). Since the overlapping of the two signals can be observed from Fig. 5 analysis, the discrete structure's representation is being validated.



Fig. 3. The unitary step response of the continuous model (1) versus the unitary step response of the ZOH pulse transfer function (8).



Fig. 4. The horizontal movement model as discrete dependency model with unit delay blocks.



Fig. 5 Graphical comparison between the discrete transfer function model and the discrete dependency model outputs.

#### VI. CONCLUSION

The research presented in this paper deals with the digitization of a continuous transfer function model, of horizontal positioning an industrial manipulator. The described equipment is part of a hot rolling chain that manufactures seamless tube pipes. The discrete model is necessary if needed to be implemented on a digital device (it is easy to convert the z-transform to an algorithm from which a computer program can be determined), and it will be further used to design an advance control structure for the horizontal positioning system.

Three different digitization methods have been used by the authors: the zero-order hold, the simple substitution and the Tustin substitution, each method yielded a different pulse transfer function.

It was shown that the pulse transfer functions coefficients, and the fitting of the discrete step signal over the continuous one, is directly dependent on the sampling time value. The sampling time must be as small as possible for a proper reconstruction of the continuous signal, and as high as possible in order to ensure sufficient computation time for the digital device. Usually the value is selected as ten times smaller than the dominant time constant of the system to be digitized. This principle was also applied by the authors in order to obtain the coefficients for each discrete function.

The pulse transfer functions were compared graphically by plotting the unitary step responses, discrete and continuous, and by computing the root mean square error, between the continuous and the discrete step responses. The best fit for this system resulted as the zero-order hold discrete transfer function. The transfer function was rewritten as a discrete dependency law between the input and the output, and converted into a discrete structure with unit delay blocks. Thus, obtaining an input-output dependency model that generates the same signal as a discrete transfer function block would.

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