Flight Envelope Expansion Via Piezoelectric Actuation Receptance Method and Time-delayed Feedback Control

Daniela Enciu, Ioan Ursu, George Tecuceanu, Dragos Daniel Ion Guta, Andrei Halanay, and Mihai Tudose

Abstract—In aerodynamics, the phenomenon of flutter suppression represents a great challenge. Since it is a complex and difficult process, it requires an innovative approach. In this paper, a V-shaped piezoelectric actuator whose role is to widen the aircraft flight envelope by raising the speed limit at which flutter occurs is presented. The demonstrator is in fact an intelligent model of wing, which is itself a control system, with sensors, piezo actuator and an implemented control law. The control law is obtained through the receptance method of eigenvalues assignment using the measured transfer function. The content of the paper refers to technical solutions for wing model design and to experimental results in subsonic wind tunnel. Another contribution of the paper concern the consideration of a time-delayed feedback control.

Index Terms—Piezoelectric actuator, flutter, flight envelope, time-delay, feedback control, receptance method

I. INTRODUCTION

The purpose of this paper is to present a solution to counteract the flutter, based on the use of a V-shaped piezoelectric actuator. The flutter is a self-sustaining unstable oscillation that increases quickly in intensity. It is a complex and difficult process to study. In the case of planes, as speed increases, there is a threshold beyond which structural vibrations can no longer be damped, and they begin to increase in amplitude by accumulating energy in the structure. Once the flutter is reached, the plane is destabilized and it can no longer be controlled.

At the dawn of aviation, there were several flight disasters caused by unstable vibrations [1], [2]. Gradually, flight flutter tests were introduced [1], [3]. At present, all aircraft get their approval to fly after passing a series of tests, including a test for establishing the flight envelope, in which a safety margin is considered. However, no flight regime is really immune to flutter [1].

The countering of dangerous phenomena such as flutter and buffeting, for example, was made first by passive techniques: increasing structural rigidity, mass balancing, changing geometry. This led to increase both weight and cost, while reducing overall performance. At some point, the use of primary flight controls and associated actuators was considered efficient to combat flutter [4]-[7]. Thus began the active control era [8]. However, as demonstrated in [9], there are many drawbacks when using primary flight controls for auxiliary purposes.

The present paper addresses the problem of control synthesis for weakly damped aeroelastic wing structures by means of the primary flight controls servos. In order to outline a general approach, the aeroelastic model consists of a quasi-steady formulation of the aerodynamic lift and moment of a typical section with flap, which is connected with a servoactuator [10].

II. PIEZOELECTRIC ACTUATOR AND WING MODEL DESIGN

The actuator has a special design, that of two piezo stacks disposed in a V-shape, which included it in the class of an ultra-fast actuators developing a high bandwidth. The kinematic schema and the 3-D CATIA view is given in Fig. 1. The piezo stacks are arranged along the segments P1P3 and P2P5, respectively. When the stack P1P3 is activated by increasing supply voltage V which determines its extension to move to the right and slightly below the articulated point P3, the stack P2P5 withdraw to the left and slightly upward of the articulated point P1 to not oppose resistance to movement down of the articulated point P3 in the slider crank mechanism.

The piezo stacks NAC2022-H98-AO1 were bought from NOLIAC and have the following basic properties: height 98 mm, stroke 148.8 μm, capacittance 19010 nF, maximal force developed 4200 N, maximum operating temperature 150 °C, material NCE51F. The advantage of the piezo stacks considered efficient to combat flutter [4]-[7]. Thus began the active control era [8]. However, as demonstrated in [9], there are many drawbacks when using primary flight controls for auxiliary purposes.

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actuators is their small size, large bandwidth and high energy density. The disadvantage is linked to their lower strokes. It is important to note that, for the antiflutter aero-elastic control to be effective, the deflection of the control surface must be at least 5-6 degrees, to the frequency range of at least 25 to 30 Hz, as it is stated in the paper [11].

The demonstrator is represented by a wing made from a spar (longeron) covered by an aerodynamic layer (profile NACA 0012). The wing has a primary flight control surface, an aileron, at one end. At the other end of the longeron there is a flange whose role is to fix the wing in the subsonic tunnel. The spar is a rectangular tube (1200×120×25) with 1 mm thickness and provided with notches to control its stiffness. The elements defining the aerodynamic surface are made from wood and resin ROHACELL 71S. The wing structure and the position of the actuator on the wing are given in Fig. 2.

![Fig. 2. Up: The structure of the wing; down: Framing of actuator in the available space in wing.](image)

### III. EXPERIMENTAL SETUP AND RESULTS

The tests for wing characteristics show that the first two measured frequencies are 5.865 Hz (bending frequency) and 14.463 Hz (torsion frequency), while those obtained numerically, by CATIA, are 6.23 and, respectively, 10.21 Hz.

A result of the ANSYS-Matlab calculation applied to a different wing model in the paper [12] is eloquent in itself to see the shape of the modes in their sequence from the base mode (bending, yaw, bending, torsion, bending) (Fig. 3).

The next step was to test the wing in the subsonic wind tunnel at various air speeds. The test to trigger the aeroelastic mechanism of the flutter has led to the following important results: flutter speed: 41 m/s, flutter frequency: 5.8 Hz. In Fig. 4 some sequences of flutter evolution in the subsonic wind tunnel are shown. The wing model was a sacrificial one to certify that the pair aerodynamic tunnel-wing model is compatible for triggering the flutter in the absence of a control loop.

For the synthesis of the antiflutter control law, a first step consisted of the identification operation of the open loop transfer functions $H_{yi}(s), f = 1.2$ from the piezo actuator to two accelerometers who had the role of “seeing” the two basic vibration modes. More specifically, the number of poles and zeros, respectively the coefficients of the numerator and the denominator were determined, so that the frequency response $H_{yi}(io)$ approximates as best as possible the response in the experimental frequency $H_{yi}(io)$ response. For that, the method of receptance is applied.

![Fig. 3. Natural first 5 modes of the wing:a) 1st bending; b) yaw; c) 2nd bending; d) 1st torsion; 3) 3rd bending [12].](image)

![Fig. 4. Sequences of flutter evolution.](image)

The receptance method for poles allocation has been recently developed in the paper [13]. The specificity of the method consists in the elaboration of the control law based on the measurements in the process, and not on the conventional theory based on the space of the states. Practically, the methodology calls for flight measurements of the frequency response, eluding the need for knowing the matrices $M, C, K$ (mass, damping and stiffness). In the following, a brief description of the method for a single input control is given.
The matrix equation of the second order is

\[ \left( Ms^2 + Cs + K \right) x(s) = B(s) \left( r(s) - u(s) \right) \]  
(1)

where

\[ u(s) = - \left( f^T g + g^T \right) x(s) = - (sf + g)^T x \]  
(2)

represents the control law, and \( r(s) \) is a reference signal. The state vector \( x \) has the dimension 4, corresponding to the basic bending and torsion modes. The influence vector \( B(s) \) (by which control is located) is written as a function of \( s \). An usual form is that of PI (proportional-integral)

\[ B(s) = b_1 + \frac{b_2}{s} \]  
(3)

From (1) and (2) it results

\[ \left( Ms^2 + \left( C + B(s)^T s + \left( K + B(s) s \right) g \right) \right) x(s) = B(s) r(s) \]  
(4)

with the consequence of changing the rank of the stiffness matrix, which leads to a significant damping for stability. What is interesting herein is the formula derived from the relation (4) of the characteristic polynomial in closed loop. This will be calculated using the Sherman-Morrison formula in matrix algebra which gives the characteristic polynomial of the closed loop matrix

\[ \tilde{H}(s) := \left( H^{-1} + B(s)(sf + g)^T \right)^{-1} \]  
(5)

\[ = H + HB(s) \left( 1 + (sf + g)^T HB(s) \right) (sf + g)^T H \]

in function of the inverse of the open loop matrix

\[ H(s) = \left( Ms^2 + Cs + K \right)^{-1}. \]  
(6)

The closed loop characteristic polynomial has just the expression \( 1 + (s + f)^T H(s) B(s) \). The problem of poles allocation to the values \( \{\mu_1, \mu_2, \ldots, \mu_n\} \) can be solved as it follows.

Let us note

\[ q_k(\mu_k) = H(\mu_k) b(\mu_k) \]  
(7)

Then, for the characteristic equation we have

\[ q_k g^T + \mu_k q_k f = -1, k = 1, \ldots, 2n \]  
(8)

The system of \( 2n \) equations with \( 2n \) unknowns could be written in the matrix form

\[ G \begin{pmatrix} g \\ f \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{pmatrix}, \quad G := \begin{bmatrix} q_1^T & \mu_1 q_1^T \\ q_2^T & \mu_2 q_2^T \\ \vdots & \vdots \\ q_{2n}^T & \mu_{2n} q_{2n}^T \end{bmatrix} \]  
(9)

allowing, thus, the determination of control vectors \( g \) and \( f \) by inversion of the matrix \( G \). The key for control law synthesis by the method of receptance is just the relation (9).

The transfer functions are identified at a specific air velocity. In Fig. 5 it can be observed that the identified transfer function based on a Matlab algorithm follows closely the measured one.

Based on the analytically identified transfer function the control law is determined based on the poles allocation for one or two basic modes; the prescribed poles to be assigned for the closed loop are chosen in a manner to double the damping factor of the bending mode and to increase by 2 Hz its own torque frequency in the idea of increasing the spread of the two modes as they will overlap during flutter. Fig. 6 attests that in the closed loop (with control obtained by the receptance method) an attenuation of the bending mode of approx. 8 dB is obtained.

![Fig. 5. Frequency characteristics of the identified transfer function](image)

![Fig. 6. Amplitude-frequency characteristics of the identified transfer functions, open loop vs closed loop.](image)

IV. A NEW APPROACH: TIME-DELAYED FEEDBACK CONTROL

A simplified two degrees of freedom representation of an airplane wing is used in this work as the aeroelastic system for which flutter suppression is desired. The model is the well-known typical section with trailing edge flap [5], [14], [15]. The two degrees of freedom are the downward vertical displacement \( h \) of the elastic axis and a leading edge up angular rotation \( \alpha \) about this line and are sketched in Fig. 7. These degrees of freedom correspond to the bending and torsion displacements of a high aspect ratio wing under real
The equations of motion describing the plunge and pitch during an aeroelastic response are derived starting from the Lagrange equations [13]

\[
\frac{\partial}{\partial t} \left( \frac{\partial (T - U)}{\partial \dot{q}} \right) - \frac{\partial (T - U)}{\partial q} = \ddot{q}
\]

(10)

where the set of the generalized coordinates is \( q = \{h, \alpha\}^T \).

The corresponding generalized forces are derived here as \( Q_0 = -L^T \) and \( Q_a = M^T \), where \( L^T \) and \( M^T \) are the lift and the moment, respectively. The kinetic energy is given by

\[
T = \frac{1}{2} m \dot{h}^2 + m x_a \dot{h} \dot{\alpha} + \frac{1}{2} I_a \dot{\alpha}^2
\]

(11)

and the potential energy of strain is

\[
U = \frac{1}{2} k_a \alpha^2 + \frac{1}{2} k_h h^2.
\]

(12)

Hence, the governing equations of the aeroelastic system under considerations are

\[
\begin{bmatrix}
m & m x_a b & 0 \\
mx_a b & I_a & 0 \\
0 & 0 & c_h \\
0 & c_h & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\dot{q} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
k_b \\
k_b \\
k_a \\
0
\end{bmatrix}
q = \begin{bmatrix}
-L^T \\
M^T
\end{bmatrix}
+ F(q)
\]

\[
F(q) :=
\begin{bmatrix}
0 \\
0 \\
-\alpha^3
\end{bmatrix}
\]

(13)

where \( m \) is the mass of the typical section, \( I_a \) is the mass moment of inertia about the elastic axis, \( x_a \) is the dimensionless distance between the elastic center and center of mass and \( b \) is the semichord length. \( c_h \) and \( c_a \) are structural dissipation through the structural damping coefficients in pitch and plunge [13], [14]. The aerodynamic lift and moment are given by

\[
L = \rho U^2 b c_{\alpha}^a \left( \alpha + \frac{\dot{h}}{U} \left( 1 - \frac{a}{2} \right) \frac{\dot{\alpha}}{U} \right) + \rho U^2 b c_{\beta}^a \beta
\]

(14)

\[
M = \rho U^2 b e_{m}^a \left( \alpha + \frac{\dot{h}}{U} \left( 1 - \frac{a}{2} \right) \frac{\dot{\alpha}}{U} \right) + \rho U^2 b e_{m}^\beta \beta
\]

(15)

where \( \rho \) is the density of the air, \( U \) is the velocity of the free air stream, \( c_{\alpha}^a \), \( c_{\beta}^a \) are the lift and moment coefficients per angle of attack and \( c_{m}^\beta , c_{m}^\beta \) are the lift and moment coefficients per flap deflection and \( a \) is the dimensionless distance between the midchord and the elastic axis. After substituting the lift and moment into the equations of motion one obtains the standard, structural type, second order multidimensional linear time invariant system

\[
M \ddot{q} + C \dot{q} + K q = B_0 \beta + \tilde{F}(q), \quad q = \begin{bmatrix}
h \\
a
\end{bmatrix}
\]

(16)

\[
M := \begin{bmatrix}
m & m b x \alpha \\
mb x \alpha & J_a
\end{bmatrix}, \quad C := C_0 + C_1(U)
\]

(17)

\[
C := \begin{bmatrix}
C h + \rho b c_{\alpha}^a & \rho b^2 \left( 1/2 - a \right) c_{\alpha}^a U \\
-\rho b^2 c_{m}^a U & c_a - \rho b^3 \left( 1/2 - a \right) c_{m}^a U
\end{bmatrix}
\]

where \( k_q = B_0 \beta \) is introduced. The following relations are successively obtained

\[
X + V^T C V X + V^T K V X = V^T B_0 \beta + V^T F(X)
\]

\[
\dot{X} + \text{diag}(2 \zeta \omega_n) \dot{X} + \text{diag}(\omega_n^2) X = B_0 \beta(t - \tau) + \tilde{F}(X),
\]

\[
i = 1, \ldots, 4, x = [X^T \ X^T]
\]

Finally, we obtain the state space form of a nonlinear system with control delay

\[
\dot{x}(t) = Ax(t) + B_1 u(t - \tau) + F(x(t))
\]

where \( k_q \) is the gain of aileron deflection \( \beta \). Next, a modal transformation is made \( q = V \tilde{X} \), where \( V \) is the modal vector matrix, and then a static cause-effect interaction by using the analogy between thermal and piezoelectric equations

\[
\tilde{X} + \text{diag}(2 \zeta \omega_n) \tilde{X} + \text{diag}(\omega_n^2) \tilde{X} = \tilde{B}_0 \beta(t - \tau) + F(x),
\]

\[
i = 1, \ldots, 4, x = [X^T \ X^T]
\]

The actuation \( \beta \) was introduced as a pure time delay \( \tau \). The mathematical model (19) will be studied further in the framework of equilibrium stability for nonlinear systems with delay on control.

V. CONCLUSION

A first contribution of the paper is the development and validation by tests in aerodynamic tunnel of a control law based on the receptance method. The purpose was to find an active control solution to increase the values at which flutter occurs. The tests were performed in the INCAS subsonic wind tunnel at various air velocities showing good attenuation results about 8 dB. Consequently, critical flutter speed can be increased by using a succession of such gains of control law.

A second contribution consists in preparing a mathematical model of a nonlinear system with delayed control. The synthesis of the control law will be done in a later work by the predictive control method [16], [17].
REFERENCES


Daniela Enciu is PhD student in applied mathematics at the University Politehnica of Bucharest. She is an employee at the Mechatronics Unit of National Institute for Aerospace Research “Elie Carafoli” – INCAS, Bucharest, Romania. Her areas of interest are advanced materials, nanotechnology, structural health monitoring (SHM) based on smart materials, applied mathematics, digital and acoustic microscopy, scanning laser Doppler vibrometry. Some of her achievements are: demonstration of the functionality of carbon nanotubes thin films as strain sensors, implementation of optical fibers as sensing systems for the detection of microstrains, development of a PWAS based SHM method for mechanical damages detection. Drd. Daniela Enciu was awarded in 2012 with “Caius Iacob” Prize for the BSc. Thesis entitled Nanomechanics. Moreover, she was awarded again in 2018 with the Gold Medal and the Special Prize from the Turkish Patent and Trademark Office for the patent no. ROI131152B1/2017 at the 46th Edition of the International Invention Salon held at Geneva.

Ioan Ursa earned his PhD from the Institute of Mathematics of the Romanian Academy. He is the author of over 30 ISI Thomson Reuters papers on electrohydraulic actuators design, active and semiactive control, smart aerospace structures, structural health monitoring (SHM), linear and nonlinear control synthesis, intelligent control systems, and real-time simulation with R-L. He was awarded in 2004, with the Romanian Academy “Aurel Vlaicu” Prize for the book Active and Semiactive Control (Romanian Academy Publishing House, 2002). Also, the paper From robust control to antiwindup compensation of electrohydraulic servo actuators (1998), authors I. Ursa et al. has obtained Award for Excellence (1999) from the journal Aircraft Engineering and Aerospace Technology, Great Britain. Dr. Ursa was awarded the Gold Medal and the Special Prize from the Turkish Patent and Trademark Office for the patent no. ROI131152B1/2017 at the 46th Edition of the International Invention Salon held at Geneva, 2018.

George Tecuceanu is the head of INCAS Mechatronics Department. Over 40 years of scientific career. He was graduated University Politehnica Bucharest from Faculty of Automatic Control and Computer Science. His research interest are theoretical and applied control synthesis. Mathematical modeling, numerical methods, optimization, modern methods of synthesis of automatic flight control systems and electro-hydraulic servomechanisms. He obtained the excellence award from Aircraft Engineering and Aerospace Technology Journal, Great Britain, in 1999.

Dragos Daniel Ion Guta is the head of INCAS Systems Department. Over 15 years of scientific career, PhD, 2008, University Politehnica Bucharest - Faculty of Power Engineering. His research interest are hardware in the loop (HIL) real time simulation, fluid power systems, mechatronics, industrial software. Publications: 6 articles in ISI Journals and Proceedings, over 20 articles in other BDI Peer-Review Journals.

Recent achievements:
2) Concurrent engineering by hardware-in-the-loop simulation with R-T, Workshop, Radu Puhalschi, Daniela Vasiliu, Daniel Dragos Ion Guta and Bogdan Mihaescu, 19th European Concurrent Engineering Conference;

Andrei Halanay is a full professor in the Department of mathematics and informatics at the University Politehnica of Bucharest. His research is dedicated to operator theory, stability and bifurcation analysis of equilibria in ordinary and delay differential equations, geometric control theory, controllability for heat equations with memory. He has worked on models of cell dynamics in leukemias. He is also involved in researches on shape optimization and fluid-structure interaction. Prof. Halanay collaborates with the researcher from INCAS in the framework of CONTUR project regarding the flutter mitigation and time-delay mathematical modeling.

Mihai Tudose is a senior researcher at the Mechatronics Unit of National Institute for Aerospace Research “Elie Carafoli” – INCAS, Bucharest, Romania. He graduated Polytechnic Institute of Bucharest, domain Electronics at Faculty of Electronic and Telecommunications. His areas of interest are automation, programming, electronics, embedded systems, control theory and signal processing, artificial intelligence, sensors and measurement. He has professional experience in the following areas: avionics research and design for Romanian aircrafts IAR 93, IAR 99, IAR 316, IAR 330; research, design, type approval and manufacturing of various onboard equipment as static inverters, power supply, accelerometers, autopilot for IAR 99 and IAR 330 aircrafts, flight data recorder systems used on MIG 21, IAR99 aircrafts and IAR330 Puma Socat helicopters. Eng. Tudose was awarded the Gold Medal and the Special Prize from the Turkish Patent and Trademark Office for the patent no. ROI131152B1/2017 at the 46th Edition of the International Invention Salon held at Geneva, 2018.