Nonlinear Model for Micro-Launcher Attitude Control

Teodor-Viorel Chelaru

***Abstract*—The paper presents aspects regarding six degree of freedom (6DOF) model used for attitude control of the three-stage micro-launcher (ML) with a payload of up to 50 kg. This work uses two separate attitude control models dedicated for different flight phases. In the ascending phases, we will control the attitude angles related to the start frame, and in injection phases, we will control the attitude angles related to the geographical frame. The results analyzed will be the flight parameters in longitudinal, in lateral, and in roll movement. Using this model, the attitude control of the launcher can be evaluated. The novelty of the paper consists in alternative attitude angles used for control and in description of guidance signal.**

***Index Terms*—Mathematical model, micro-launcher, attitude control, guidance signal**

I. INTRODUCTION

The present work is a continuation of the paper [1] where, using three degree of freedom (3DOF) model, based on translational equation, the ascending phase of the micro-launcher (ML) was optimized and performance for Low Earth Orbit was evaluated. The present work proposes to develop the ML model adding the equations of the movement around center of mass (dynamic and kinematic) and the equations of the aerodynamic angles to obtain a six degree of freedom (6DOF) model. From the beginning, we must emphasize that the issue of launcher control is particularly important because, unlike rocket with fins, the launcher is naturally unstable, which leads to the impossibility of motion assessment without the loop control of the vehicle's attitude. The attitude control of ML can be separate in two problems. First consist in choosing the right frames to express the attitude angles and define desired angles in these frames. The second problems consist in obtaining a robust controller, which ensures pursuit of the desired angles by the accomplished angles. In the dedicated works, this problem has been addressed in different ways. In the classical work [2], the translational equations are written in quasi-velocity frame and in start frame, and the kinematic rotational equations are written in relation to the start frame, but attitude control problem does not be approached. In the recent work [3], the translational equations are written in Earth frame known as Earth-centered inertial (ECI) frame. The kinematic rotational equations are written also in relation to ECI frame, which complicates the description of the guidance commands. Work [4] is dedicated to solve the second problem of the launcher control and to obtain a robust controller using μ synthesis technique. In relation to nonlinear motion equations, the problem in this work is formulated in annex A in the body frame. The work is focused on linear form of motion equation, in particular the longitudinal plane. A practical approach is propose in work [5], where the main phases of ascension are defined, which then would help us with the 3DOF model approach of this paper [1]. Work [6], dedicated to reentry vehicle, proposed the use of no inertial geographical frame for express attitude of the vehicle, an idea that we will develop in this paper for the orbital injection phases. The present work intends to seek an answer for first formulated control problem, to choose the right frames with desired angles, and to obtain a preliminary solution for ML control. Summarizing, this present work evaluated the attitude of the launcher using two reference frames and 6DOF calculus model. In ascending fazes, we will use the attitude angles in order (3-2-1) in relation to the start frame, which allow us to consider nullifying the yaw angle, and the link matrix without singularity for vertical position of the ML. In contrast, for injection phases, we will use attitude angle in order (2-3-1) in relation to the geographical frame, which allow to transpose easily the desired attitude angle from the orbital frame to the body frame. As for the translational equations, although we have shown in works [7], [8] that it is possible to work in the linked start frame, the present work used the equations in quasi-velocity frame to obtain a 6DOF model compatible with the developed 3DOF model and to use the previously obtained results, especially regarding the optimization of the ascending phases. Because one of the basic ideas for a micro-launcher is simplicity and low cost and because the avionics and related software are the most expensive, the main purpose of the paper is to get a simple attitude control system based on tracking the desired attitude angles. Although the problem of the evolution of the launchers is not a very new one, with the exception of the Earth frame (ECI), which is the same, the rest of the reference systems used are different for each author or group of authors, which is why we recommend work [9] where the frames used are defined.

II. LAUNCHER MOTION EQUATIONS

Because the translational equations were presented in paper [1], in 3DOF model, we will briefly review the translational equations and focus on rotational equations.

A. Translational Dynamic Equations in Quasi-Velocity Frame

Summarizing the papers [1], [9] to obtain the translational equation in quasi-velocity frame, we start from vector equation:

The manuscript is received on May 31, 2019 and revised on July 1, 2019. This work was funded by STAR program and implemented with the support of ROSA, contract no. 144/2017.

Teodor-Viorel Chelaru is with the University Politehnica of Bucharest, Romania (e-mail: teodor.chelaru@upb.ro).

DOI: 10.7763/IJMO.2019.V9.728

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{\Omega}_V^* \times \mathbf{V} = \frac{\mathbf{N}}{m} + \mathbf{g} + \mathbf{a}_c, \tag{1}$$

where we have grouped aerodynamic force with thrust force, $\mathbf{N} = \mathbf{F} + \mathbf{T}$. $\mathbf{g}$ is the gravity acceleration vector obtained from the so-called "J2" model [1], [2], with radial g_r and polar g_ω components, and $\mathbf{a_c}$ is Coriolis acceleration. The rotational velocity of the quasi-velocity frame in relation the local frame $\mathbf{\Omega}_V^*$ can be express in vector form:

$$\mathbf{\Omega}_V^* = \dot{\boldsymbol{\gamma}} + \dot{\boldsymbol{\chi}} + \dot{\varphi} + \dot{\boldsymbol{\lambda}} \tag{2}$$

with the components along quasi-velocity frame:

$$\begin{aligned} \omega_l^* &= \dot{\lambda}(\cos\varphi \cos\chi \cos\gamma + \sin\varphi \sin\gamma) + \\ &\quad \dot{\varphi}\sin\chi \cos\gamma + \dot{\chi}\sin\gamma \\ \omega_m^* &= \dot{\lambda}(-\cos\varphi \cos\chi \sin\gamma + \sin\varphi \cos\gamma) - \\ &\quad \dot{\varphi}\sin\chi \sin\gamma + \dot{\chi}\cos\gamma \\ \omega_n^* &= \dot{\lambda}\cos\varphi \sin\chi - \dot{\varphi}\cos\chi + \dot{\gamma} \end{aligned} \tag{3}$$

Starting from relation (1), we have obtained in paper [1] the dynamic translational equation that describes the motion of center of mass of the launcher in quasi-velocity frame.

B. Translational Kinematic Equations in Spherical Coordinates

The dynamic equations are complemented with translational kinematic equations:

$$\dot{\varphi} = \frac{V}{R}\cos\chi \cos\gamma \ ; \dot{\lambda} = -\frac{V \sin\chi \cos\gamma}{R\cos\varphi}; \dot{R} = V\sin\gamma \ . \tag{4}$$

which describe the position of center of the mass in spherical coordinates.

C. Dynamic Rotational Equations in Body Frame

Next, we will write the rotation equation in the body frame. Because the body frame is not inertial frame, applying the moment theorem, we obtain:

$$\dot{\mathbf{\Omega}} = \mathbf{J}^{-1}(\mathbf{H} + \mathbf{U}) + \mathbf{J}^{-1}\mathbf{A}_\Omega \mathbf{J}\mathbf{\Omega}, \tag{5}$$

where $\mathbf{H} = [L^A \quad M^A \quad N^A]$ is the aerodynamic moment and $\mathbf{U} = [L^T \quad M^T \quad N^T]$ is the thrust moment.

In relation (5), we used the anti-symmetric rotation matrix:

$$\mathbf{A}_\Omega = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}; \tag{6}$$

and the inertial moment matrix:

$$\mathbf{J} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}; \tag{7}$$

where the inertial moments are given by:

$$\begin{aligned} A &= \int (y^2 + z^2)\,\mathrm{d}m; \\ B &= \int (z^2 + x^2)\,\mathrm{d}m; \\ C &= \int (x^2 + y^2)\,\mathrm{d}m \end{aligned} \tag{8}$$

If we have an axial symmetric configuration, as it is in the ML, the transverse moments are equal ($C = B$), and the dynamic rotational equations become:

$$\begin{aligned} \dot{p} &= \frac{L^T + L^A}{A};\ \dot{q} = \frac{M^T + M^A}{B} + \left(1 - \frac{A}{B}\right) rp \ ; \\ \dot{r} &= \frac{N^T + N^A}{B} - \left(1 - \frac{A}{B}\right) pq \, . \end{aligned} \tag{9}$$

D. Aerodynamic Terms

Taking into account the launcher geometrical symmetry, the polynomial form of the aerodynamic coefficients indicated in the works [10], [11] [12] is:

$$\begin{aligned} C_x^A &= a_1 + a_2(\alpha^2 + \beta^2) + a_3(\alpha^4 + \beta^4) + \\ &\quad + a_4\alpha^2\beta^2 + a_{13}(\hat{y}_0 - \hat{y}_{oc}); \\ C_y^A &= -b_1\alpha - b_2\alpha^3 - b_3\beta^2\alpha - b_4\hat{r}; \\ C_z^A &= b_1\beta + b_2\beta^3 + b_3\alpha^2\beta + b_4\hat{q}; \\ C_l^A &= c_1 + c_2(\alpha^2 + \beta^2)\alpha\beta + c_3\hat{p} + \\ &\quad + c_5(\alpha\hat{q} - \beta\hat{r}); \\ C_m^A &= d_1\beta + d_2\beta^3 + d_3\beta\alpha^2 + d_4\hat{q}; \\ C_n^A &= d_1\alpha + d_2\alpha^3 + d_3\alpha\beta^2 + d_4\hat{r}, \end{aligned} \tag{10}$$

where the aerodynamic angles α, β will be obtained later using differential equations. Using reference force and moment defined by standard [13], we obtain aerodynamic forces and moments:

$$\begin{aligned} X^A &= F_0 C_x^A;\ Y^A = F_0 C_y^A;\ Z^A = F_0 C_z^A; \\ L^A &= H_0 C_l^A;\ M^A = H_0 C_m^A;\ N^A = H_0 C_n^A \, . \end{aligned} \tag{10}$$

E. Thrust Terms

Considering that the roll command δ_l is given by separate reaction control system (RCS) and pitch δ_m, and yaw δ_n commands are given through the angular deflection of themain rocket motor by TVC. The thrust components according [9], [14] are given by:

$$\begin{aligned} X^T &= T\cos\delta_m \cos\delta_n; \\ Y^T &= -T\cos\delta_m \sin\delta_n;\ Z^T = T\sin\delta_m \end{aligned} \tag{11}$$

with moment command given by:

$$L^T = T_r d\delta_l;\ M^T = -x_T Z^T \cong -x_T T\sin\delta_m; \tag{12}$$

$$N^T = x_T Y^T \cong -x_T T \cos\delta_m \sin\delta_n$$

F. Rotational Kinematic Equations

The kinematic equations complete the dynamic equations, allowing obtaining a first-order ordinary differential equation system.

1) Euler kinematic equation in start frame

If we want to obtain Euler angles for rotation from start frame to body frame, we will use the kinematic Euler equations:

$$\begin{bmatrix}\dot\phi & \dot\psi & \dot\theta\end{bmatrix}^T = \mathbf{W}_A \begin{bmatrix}p & q & r\end{bmatrix}^T. \tag{13}$$

where the rotation velocity components of the body frame $\begin{bmatrix}p & q & r\end{bmatrix}^T$ are the solution of dynamic rotation equation (5), and the link matrix is given by:

$$\mathbf{W}_A = \begin{bmatrix}1 & \sin\phi\tan\psi & \cos\phi\tan\psi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\psi & \cos\phi\sec\psi\end{bmatrix}. \tag{14}$$

One can observe that, due a rotation order (3-2-1) and orientation of the start frame, the link matrix $\mathbf{W}_A$ has no singularity for vertical position of the ML.

2) Euler kinematic equation in geographical frame

In order to obtain Euler angles between geographical frame and the body frame, we must take in consideration that the geographical frame is a non-inertial one.
Considering the vector expression:

$$\mathbf{\Omega} = \mathbf{\Omega}_{BG} + \mathbf{\Omega}_{GI}. \tag{15}$$

where $\mathbf{\Omega} = \begin{bmatrix}p & q & r\end{bmatrix}^T$ is rotation of the body frame in relation to inertial frame, define by components in body frame, and $\mathbf{\Omega}_{BG} = \begin{bmatrix}p_b & q_b & r_b\end{bmatrix}^T$ is rotation on the body frame in relation to geographical frame defined by components in body frame and $\mathbf{\Omega}_{GI} = \begin{bmatrix}p_g & q_g & r_g\end{bmatrix}^T$ is rotation of the geographical frame in relation to inertial frame, defined by components in geographical frame.

In this case, matrix expression of the angular velocity of the body frame related to geographical frame is given by:

$$\begin{bmatrix}p_b \\ q_b \\ r_b\end{bmatrix} = \begin{bmatrix}p \\ q \\ r\end{bmatrix} - \mathbf{A}_G \begin{bmatrix}p_g \\ q_g \\ r_g\end{bmatrix}. \tag{16}$$

where $\begin{bmatrix}p & q & r\end{bmatrix}^T$ are the solution of dynamic rotation equation (5), and the rotation matrix $\mathbf{A}_G$ in order (2-3-1) is given by relation :

$$\mathbf{A}_G = \begin{bmatrix} c\psi_g c\theta_g & s\theta_g & -s\psi_g c\theta_g \\ s\psi_g s\phi_g - c\psi_g s\theta_g c\phi_g & c\theta_g c\phi_g & c\psi_g s\phi_g + s\psi_g s\theta_g c\phi_g \\ s\psi_g c\phi_g + c\psi_g s\theta_g s\phi_g & -c\theta_g s\phi_g & c\psi_g c\phi_g - s\psi_g s\theta_g s\phi_g \end{bmatrix}.$$

where $c \equiv \cos; s \equiv \sin$ and the components of the rotation of the geographical frame are given by:

$$\begin{bmatrix}p_g \\ q_g \\ r_g\end{bmatrix} = \begin{bmatrix}\dot\lambda_s \cos\varphi \\ \dot\lambda_s \sin\varphi \\ -\dot\varphi\end{bmatrix}. \tag{17}$$

with:

$$\dot\lambda_s = \dot\lambda + \Omega_p. \tag{18}$$

The derivatives $\dot\lambda, \dot\varphi$ as derivatives of latitude and longitude angles along geographical frame were previously defined by relations (4) for translational equations.
In this case, we can write kinematic Euler equations:

$$\begin{bmatrix}\dot\phi_g & \dot\psi_g & \dot\theta_g\end{bmatrix}^T = \mathbf{W}_G \begin{bmatrix}p_b & q_b & r_b\end{bmatrix}^T, \tag{19}$$

where the rotation velocity components of the body frame $\begin{bmatrix}p_b & q_b & r_b\end{bmatrix}^T$ related to geographical frame are previously defined by (16), and the link matrix is given by:

$$\mathbf{W}_G = \begin{bmatrix}1 & -\cos\phi_g \operatorname{tg}\theta_g & \sin\phi_g \operatorname{tg}\theta_g \\ 0 & \cos\phi_g \sec\theta_g & -\sin\phi_g \sec\theta_g \\ 0 & \sin\phi_g & \cos\phi_g\end{bmatrix}, \tag{20}$$

One can observe that, due the rotation order (2-3-1) and the orientation of the geographical frame, the link matrix $\mathbf{W}_G$ has singularity for vertical position of the ML. For this reason, we use Euler angles relatively to start frame in ascending phases and Euler angles relatively to geographical frame in injection phases. The equations (19) that express attitude in geographical frame are equivalent to the equations (13) that express attitude in the start frame and theoretically can be analytically written in relations that link that two groups of angles. But, the cod robustness, in 6DOF model, will be used to solve simultaneously both groups of equations, and in different flight phases, the cod robustness will be used alternatively for the control, which is one of the groups of angles.

G. The Aerodynamic Angles

From the previous relation, we can observe that, in order to obtain the components of the aerodynamic and thrust force in quasi-velocity frame, we need the aerodynamic angles α, β^* and μ . To get them in the form of differential relationships, we can start from vector relation:

$$\mathbf{\Omega} = \mathbf{\Omega}_V^* + \dot{\boldsymbol{\mu}} + \dot{\boldsymbol{\beta}}^* + \dot{\boldsymbol{\alpha}} + \mathbf{\Omega}_p \tag{21}$$

Projecting along axis of body frame result:

$$\begin{bmatrix}p \\ q \\ r\end{bmatrix} = \mathbf{A}_{\alpha\beta^*\mu} \begin{bmatrix}0 \\ \omega_m^* \\ \omega_n^*\end{bmatrix} + \mathbf{A}_{\alpha\beta^*} \begin{bmatrix}\omega_l^* + \dot\mu \\ \dot\beta^* \\ 0\end{bmatrix} + \begin{bmatrix}0 \\ 0 \\ \dot\alpha\end{bmatrix} + \begin{bmatrix}\Omega_{pp} \\ \Omega_{pq} \\ \Omega_{pr}\end{bmatrix} \tag{22}$$

where:

$$\begin{bmatrix}\Omega_{pp}\\ \Omega_{pq}\\ \Omega_{pr}\end{bmatrix}=\Omega_p\mathbf{A}_I\begin{bmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{bmatrix}. \quad (23)$$

where $\mathbf{A}_I$ is the rotation matrix in order (3-2-1) from start frame to body frame [9], $\mathbf{A}_{\alpha\beta^*\mu}$ is rotation matrix from quasi-velocity frame to the body frame [9], and $\mathbf{A}_{\alpha\beta^*}$ is rotation matrix from the aerodynamic frame to the body frame [9].

Replacing the rotation matrices where we get the following system of differential equations:

$$\begin{aligned}&\dot{\mu}=p^*\frac{\cos\alpha}{\cos\beta^*}-q^*\frac{\sin\alpha}{\cos\beta^*}-\omega_l^*-\\&\quad\left(\omega_m^*\sin\mu-\omega_n^*\cos\mu\right)\tan\beta^*\\&\dot{\beta}^*=p^*\sin\alpha+q^*\cos\alpha-\omega_m^*\cos\mu-\\&\quad\omega_n^*\sin\mu\\&\dot{\alpha}=-p^*\cos\alpha\tan\beta^*+q^*\sin\alpha\tan\beta^*+r^*+\\&\quad\omega_m^*\frac{\sin\mu}{\cos\beta^*}-\omega_n^*\frac{\cos\mu}{\cos\beta^*}\end{aligned} \quad (24)$$

with rotation velocity along body axis:

$$p^*=p-\Omega_{pp};\ q^*=q-\Omega_{pq};\ r^*=r-\Omega_{pr}. \quad (25)$$

where the components of the rotation velocity in the body frame p,q,r result from dynamic equation around center of mass (5), and components of the angular velocity in quasi-velocity frame $\omega_l^*,\omega_m^*,\omega_n^*$ are given by relation (3).

The translational equations, from [1] as 3DOF model, together with rotational equations (9) and (13) or (9) and (19) with auxiliary relations (24) describe uncontrolled movement of the launcher, grouped in the so-called 6DOF model. As we said in the introduction, because the launcher is unstable, the system cannot be integrated in this form, and it is necessary to add the control loop that will be the subject of the next item.

III. Relations for Guidance and Control

In order to obtain the guidance command for ML, we start with the simple forms of the command signals:

$$\begin{aligned}&u_\phi=k_u^\phi\tilde{\phi};\ u_\theta=k_u^\theta\tilde{\theta};u_\psi=k_u^\psi\tilde{\psi};\\&u_p=k_u^p p;\ u_q=k_u^q q;u_r=k_u^r r\\&u_\alpha=k_u^\alpha\alpha;\ u_\beta=k_u^\beta\beta\\&u_{\delta n}=k_u^{\delta m}\delta_n\ \ u_{\delta m}=k_u^{\delta m}\delta_m\end{aligned} \quad (26)$$

with the significance u_ϕ (roll signal), u_θ (pitch signal), u_ψ (yaw signal), u_α (incidence signal), u_β (sideslip signal), $u_{\delta n}$ (pitch angular deflection signal), and $u_{\delta m}$ (yaw angular deflection signal). Using signal commands, resuming paper [10], [15], [9] we obtain the guidance commands:

A. Ascending Phases

In these phases, the guidance commands are based on attitude angles related to the start frame having the form:

$$\begin{aligned}&u_l=-u_\phi-u_p+u_\theta\sin\psi;\\&u_m=-u_\psi\cos\phi-u_\theta\sin\phi\cos\psi-u_q-u_\beta-u_{\delta m};\\&u_n=u_\psi\sin\phi-u_\theta\cos\phi\cos\psi-u_r-u_\alpha-u_{\delta n},\end{aligned} \quad (27)$$

with the relative parameters:

$$\tilde{\theta}=\theta-\theta_d;\ \tilde{\psi}=\psi-\psi_d;\ \tilde{\phi}=\phi-\phi_d, \quad (28)$$

During this phase, a *coasting* sequence may occur, when the thrust is stopped and launcher has ballistic evolution. In this case, only roll control is ensured by RCS. Switching from one phase to the other, the desired angles $\theta_d\ \ \psi_d$ values can be considered as flight parameters in the ascending evolution and can be the subject of a parametric optimization. The end of the ascending evolution is considered when we reached the flight parameters (velocity, position, attitude) that allow the start of orbital injection maneuvers.

B. Injection Phases

In these phases, guidance commands are based on attitude angles related to the geographical frame having the form:

$$\begin{aligned}&u_l=-u_\phi-u_p+u_\psi\sin\theta_g;\\&u_m=-u_\psi\cos\phi_g\cos\theta_g-u_\theta\sin\phi_g-u_q-u_\beta-u_{\delta m};\\&u_n=u_\psi\sin\phi_g\cos\theta_g-u_\theta\cos\phi_g-u_r-u_\alpha-u_{\delta n},\end{aligned} \quad (29)$$

with the relative parameters:

$$\tilde{\theta}=\theta_g-\theta_d;\ \tilde{\psi}=\psi_g-\psi_d;\ \tilde{\phi}=\phi_g-\phi_d, \quad (30)$$

where the reference values are $\theta_d;\ \psi_d\ \ \phi_d$.

As shown in paper [1] for the orbital injection, the optimal maneuvers can be obtained from Gauss perturbing equations [16]. Defining δ_1, which is the angular deflection of the thrust vector, relative to the perpendicular direction on $\mathbf{r}$ in the orbit plane and δ_2, which is the angular deflection of the thrust vector outside the orbit plane, we have obtained:

1) *Optimal maneuver for increase major semiaxis:*

$$\delta_1=\arctan\left(\frac{e}{\zeta}\sin\psi\right);\delta_2=0. \quad (31)$$

2) *Optimal maneuver for decrease eccentricity:*

$$\delta_1=\arctan\left(\frac{e\zeta\sin\psi}{\zeta^2-f^2}\right);\delta_2=0. \quad (32)$$

3) Optimal maneuver for increase/decrease orbit inclination:

$$\delta_1 = 0\,; \delta_2 = \pm\frac{\pi}{2} \tag{33}$$

where e is the eccentricity, ψ is the eccentric anomaly, $\zeta^2 = 1 - e^2$, and $f = 1 - e\cos\psi$.

Taking into account that through a simple rotation along y_g axis with air-path track angle χ we can overlap orbital frame with the geographical frame, we can impose optimal pitch and yaw command for injection in circular orbit:

$$\theta_d = \delta_1\,;\; \psi_d = \delta_2 \cos\chi \tag{34}$$

where:

$$\delta_2 = -k_3\left(i - i_d\right) \tag{35}$$

with i (orbital inclination angle) and i_d (desired inclination angle).

For guidance, command application is necessary to know the gain constants introduced by relations (26) when we expressed the command signals in simple form. These values will be specified in the next item.

In order to obtain angular deflection for TVC and an equivalent roll command, considering the system delay, we define the actuator equation system in scalar form:

$$\begin{aligned} \dot{\delta}_l &= -\frac{\delta_l}{\tau_{\delta l}} + \frac{u_l}{\tau_{\delta l}}; \\ \dot{\delta}_m &= -\frac{\delta_m}{\tau_{\delta m}} + \frac{u_m}{\tau_{\delta m}}; \\ \dot{\delta}_n &= -\frac{\delta_n}{\tau_{\delta n}} + \frac{u_n}{\tau_{\delta n}}; \end{aligned} \tag{36}$$

where the gains are included in the command signals (26). The time constants for the relation (36) will be specified in the next item.

V. Input Data for ML Model

The input data used are taken from paper [1]. In Fig. 1, we have P/L (payload) and ST (stage). All sizes are in meters.

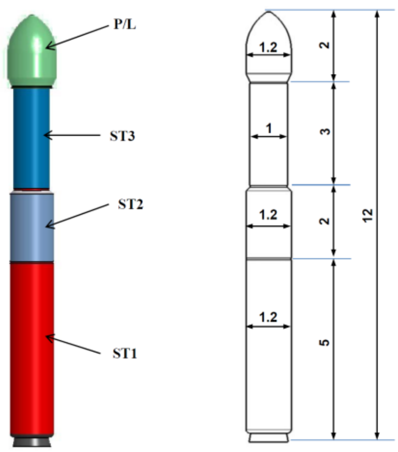

Fig. 1. ML configuration [1].

In Fig. 2, mass (m), center of mass (xcm), inertial moments (A,B), and stage operating parameter (itr) in time were presented.

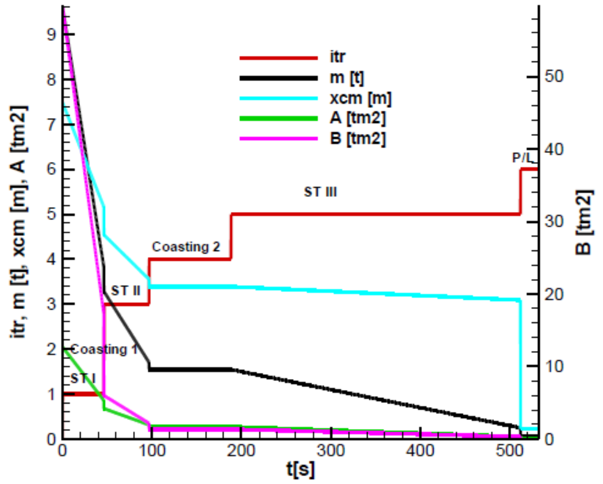

Fig. 2 Mass characteristics.

From Fig. 2, due to the model hypothesis, we can observe a linear variation of the mass characteristics between characteristic points.

In Fig. 3, thrust (T) and stage operating parameter (itr) in time were presented.

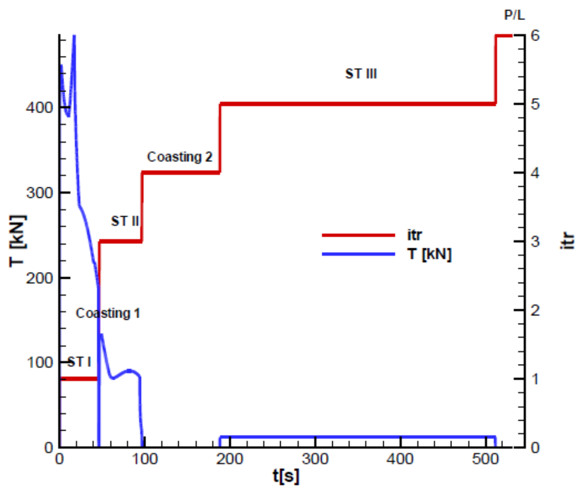

Fig. 3. Thrust characteristics.

From Fig. 3, we can observe the irregularly of the thrust force for the first two stages due to geometry of the solid propellant. In contrast, the thrust force for the third stage is constant, due to liquid rocket motor.

The gain values used in command signals from relation (26) are:

$$k_\delta^\phi = 10.;\; k_\delta^\theta = 80.;\, k_\delta^\psi = 800.;\; k_\delta^p = 20.;$$
$$k_\delta^q = 60.;\, k_\delta^r = 60.;$$
$$k_\delta^\alpha = 20.;\; k_\delta^\beta = 20.;\, k_\delta^{\delta n} = 5.\;\; k_\delta^{\delta m} = 5.$$

The time constants for the relation (36) are:

$$\tau_{\delta l} = \tau_{\delta m} = \tau_{\delta n} = 0.1$$

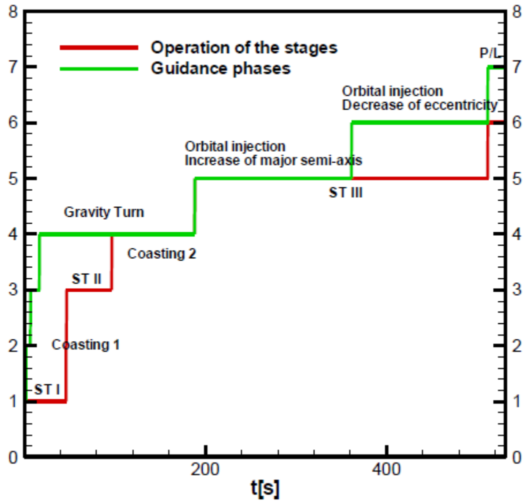

Fig. 4. Operation of the stages and guidance phases.

VI. Test Case Description

As test case, we choose a polar orbit, with the following initial conditions: geographical orientation, azimuth angle $\psi_0 = 90°$ (to the North); geocentric latitude, $\varphi = 0°$ (equatorial latitude); altitude, $h_0 = 1m$; initial velocity, $V_0 = 1[m/s]$; and initial pitch angle, $\theta_0 = 90°$. Payload mass is $MPL = 50[kg]$. Regarding flight parameters as can be seen in Fig. 4, we have two sequences of flight events: the first is link on stage operation and the second on guidance phases. Typical for three-stage launcher, from the point of view of stage operations, we have the functionality of three rocket engines separated by two coasting duration. The first coasting, between the first stage and second stage, has a small duration necessary for stage separation. The second coasting between the second and third stages has a significant duration in order to increase altitude and ensure better functionality of the third stage with liquid engine. Regarding guidance phases, we have two groups: first for ascending flight and second for orbital injection. Ascending flight starts with a vertical evolution, is followed by inclination manoeuver to an imposed pitch angle, continues with an evolution with constant pitch angle, and is finalized by gravity turn evolution. Orbital injection contains two phases: first for increase of major semiaxis and second for decrease of eccentricity. The flight parameters were the subject of optimization for 3DOF model, being described in paper [1]. For the end time of vertical flight $t_1 = 2s$ and the end time of the inclination decrease $t_2 = 7s$ corresponding to the minimal value of performance index defined in paper [1], we obtained the following: duration of inclination flight with constant pitch angle $\Delta_1 = 10[s]$; coasting duration between the second and third stage, $\Delta_2 = 91.7[s]$; increasing major semiaxis duration on the third stage $\Delta_3 = 172.9[s]$; and pitch imposed for the first inclination $\theta_1 = 58°$ .These parameters lead to a circular polar orbit with the altitude $h_p = 496[km]$ and inclination $i = 80°$. The test case defined is summarized in Fig. 4.

Using these parameters, we obtain a circular orbit. The flight parameters are described in the next item.

VII. Results

Fig. 5 shows the pitch thrust deflection angle (*dn*), the climb angle (*ga*) and pitch angle (Ted (desired) and *Te* (accomplished)), and incidence angle (*alfa*). Guidance phase (*ic*) was also represented. One can observe that thrust deflection angle (*dn*), except in the initial phases, is generally null, with some fluctuations when the ML changes the guidance phase. The pitch accomplished angle follows the pitch desired angle, and the climb angle is close to the pitch angle. The incidence angle is small except for the second phase when ML tilts to the desired pitch angle and during the sixth phase (circularization).

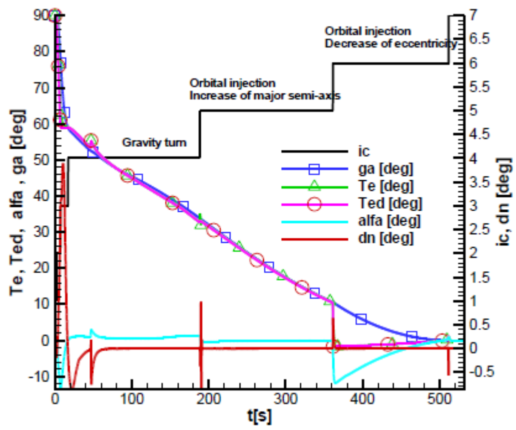

Fig. 5. Longitudinal motion parameters.

Fig. 6 shows the yaw thrust deflection angle (*dm*), the yaw angle (*psd* (desired) and *ps* (accomplished)), the glissade angle (*beta*), the air-path track angle (*hi*), the orbit inclination (*inc*), and guidance phase (*ic*). We can observe that similar longitudinal motion in the yaw thrust deflection is generally null, except for the beginning of the fifth phase when the injection phases begin. The yaw accomplished angle (*ps*) follows the yaw desired angle (*psd*), which finally assures the desired orbit inclination (*ic*). The glissade angle has significant values only in injection phases.

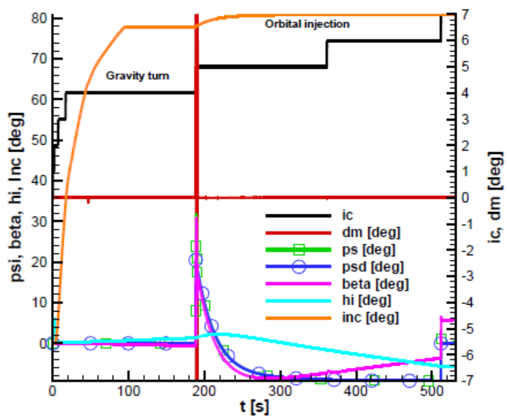

Fig. 6. Lateral motion parameters.

Fig. 7 shows the roll velocity (*p*), roll angle (*fi*), equivalent roll deflection (*dl*) controlled by RCS, and the guidance phase parameter (*ic*). One can observe that, despite the oscillation, the roll parameters remain in a restricted area around the zero value.

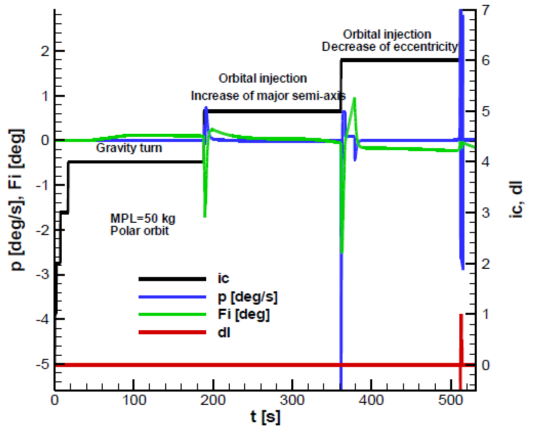

Fig. 7. Roll motion parameters.

VIII. Conclusions

As we said at the beginning, the paper's objective is to build a 6DOF model able to solve attitude control problem of the ML. In order to solve this problem, similar with paper [1], we separated the launcher's evolution in two groups of phases, the first group being the ascending phases until the launcher or the upper stage of it is in the optimal position to make orbital injection and the second phases group when the upper stage performs orbital maneuvers and payload injection. For each group of phases, we developed a separate calculus model. For the ascending phases, we controlled the attitude angles in relation to the start frame, and in injection phases, we controlled the attitude angles in relation to the geographical frame. Despite different models used for controlling each flight phases, for unitary approach, we actually use a unitary 6DOF model with translation equation in quasi-velocity frame and dynamic rotation equation in body frame. The difference between the flight fazes is done by rotation kinematic equation. The test case build and the results obtained prove the correctness of the developed model, including the alternative used of the different attitude angles for different flight phases.

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Teodor-Viorel Chelaru worked 19 years in research activity in field of guided missile in Electromecanica - Ploieşti. During this period, he led a research laboratory of aerodynamics, dynamic flight and control and participated to more than 10 projects, 5 of them being successful finalized. Between 2000 - 2016 he worked in CN Romarm SA in research department. After 2005 he come in University Politehnica of Bucharest where become an associate professor. Now he is a professor in UPB, and CCAS Manager.