Nonlinear Fault-Tolerant Trajectory Tracking Control of a Quadrotor UAV

Seyyed Ali Emami and Afshin Banazadeh

Abstract—An adaptive fault-tolerant sliding mode control is presented in this paper. A regular sliding mode controller is designed as the inner loop of the control structure, while in the outer loop the desired trajectory components are converted to the desired attitude. The problem of identification of the faulty subsystems’ dynamics is converted to the mathematical problem of determining the unknown coefficients of a Linear-In-Parameter (LIP) model. Subsequently, an effective observer is developed in the paper based on the well-known Online Sequential Extreme Learning Machine (OS-ELM) approach in order to identify the dynamics of faulty subsystems and afterward, a disturbance observer is proposed to estimate the effects of external disturbances on the dynamic model. Furthermore, the stability of the closed-loop system is analyzed based on the Lyapunov theorem. The introduced control structure is applied to a quadrotor Unmanned Aerial Vehicle (UAV) for tracking a predefined trajectory in the 3D environment. The simulation results demonstrate that using the proposed control scheme, the air vehicle can follow the desired trajectory in the presence of simultaneous actuator faults and external disturbances.

Index Terms—Adaptive control, OS-ELM, quadrotor UAV, sliding mode control, trajectory tracking.

I. INTRODUCTION

Sliding Mode Control (SMC) has been used widely in the control structure of different nonlinear dynamic systems [1]. It is easy to implement and can deal with nonlinear part of dynamic systems, satisfactorily. In addition, due to the robustness of the SMC, it can be used satisfactorily to maintain the closed-loop performance in the presence of model uncertainties and external disturbances [2].

SMC has been also employed in the structure of Fault-Tolerant Controls (FTCs), extensively [3], [4]. However, using a regular SMC, the closed-loop performance drops significantly in the presence of multiple actuator faults and it may lead to the closed-loop instability. There are different approaches in the literature, which have addressed the design of a reliable FTC scheme using the combination of the robustness of SMCs with the capabilities of adaptive control approaches. A robust backstepping SMC approach has been introduced in [5] for a quadrotor UAV in the presence of model uncertainties and external disturbances. The control structure consists of a regular SMC as the inner loop and the backstepping control approach as the outer loop of the closed-loop system. Also, an adaptive fault estimation block has been employed only in the take-off and landing mode in order to avoid the crash. An adaptive fault-tolerant SMC has been proposed in [2] for a multi-rotor Unmanned Aerial Vehicle (UAV) in the presence of simultaneous actuator faults. Two separate control modules have been introduced in the paper, which have been developed by combining both the adaptive SMC and the control allocation scheme. However, the proposed approach suffers from different complexities. Authors in [6], have proposed an adaptive fuzzy SMC for a quadrotor UAV. The control structure includes a PID control for the position tracking and an adaptive SMC with a parallel fuzzy system in the attitude control block.

Almost all of the aforementioned control structures suffer from several complexities. This is despite the fact that if we are capable of identifying the dynamics of the faulty system, then it is possible to redesign the control structure based on the newly identified dynamic system, satisfactorily. More specifically, in the case of quadrotor UAVs, the system faults can be usually modeled using a set of four multiplicative gains, which correspond to each rotor. Thus, the problem of identifying the faulty system can be converted to the mathematical problem of determining the unknown coefficients of a Linear-In-Parameter model.

Online Sequential Extreme Learning Machine (OS-ELM) is an iterative identification approach, which has been proposed for the first time in [7] for updating the unknown parameters of an ELM. An ELM is a neural network with a single hidden layer, where only the weights of the output layer of the network should be updated in the training process and the weights and biases of the hidden layer are chosen as constant random numbers. Different variety of OS-ELM approach has been introduced in the literature [8], [9]. An efficient version of OS-ELM approach called to OS-ELM with the forgetting factor has been introduced in [10], which can be used satisfactorily for identification of time-variant dynamic systems. Also, the sufficient conditions for the convergence of the OS-ELM approach to desired identification accuracy can found in [7].

Motivated by the above discussion, a nonlinear fault-tolerant control approach is introduced in this paper for a quadrotor UAV. A new formulation of the system dynamic model is developed in order to convert the identification problem of the faulty system to the well-known problem of online identification of the ELM parameters. Subsequently, an effective disturbance observer is employed to estimate the existent external disturbances and finally, an SMC approach is designed based on the identified dynamic model of the quadrotor UAV.
II. NONLINEAR DYNAMIC MODEL OF A QUADROTOR UAV

The position and attitude of the air vehicle with respect to the inertial reference frame are denoted respectively, by \( \mathbf{r} = [x \ y \ z]^T \) and \( \Phi = [\phi \ \theta \ \psi]^T \), where \( \phi \), \( \theta \), and \( \psi \) represent the roll, pitch, and yaw angle. Accordingly, the nonlinear dynamic model of a conventional quadrotor UAV (using the small angle approximation) can be formulated as follows [11], [12]:

\[
\begin{align*}
\dot{x} &= -\frac{F}{m} (\cos \phi \sin \theta \cos \phi + \sin \phi \sin \theta) + \ddot{d}_1, \\
\dot{y} &= -\frac{F}{m} (\sin \phi \sin \theta \cos \phi - \cos \phi \sin \theta) + \ddot{d}_2, \\
\dot{z} &= -\frac{F}{m} (\cos \phi \cos \theta) + g + \ddot{d}_3, \\
\dot{\phi} &= \frac{1}{I_x} (u_\phi + (l_y - l_z) \dot{\theta} \psi) + \ddot{d}_4, \\
\dot{\theta} &= \frac{1}{I_y} (u_\theta + (l_z - l_x) \dot{\phi} \psi) + \ddot{d}_5, \\
\dot{\psi} &= \frac{1}{I_z} (u_\psi + (l_x - l_y) \dot{\theta} \phi) + \ddot{d}_6,
\end{align*}
\]

where \( I_x \), \( I_y \), \( I_z \), \( g \), and \( \ddot{d}_i \ (i \in \{1, \ldots, 6\}) \) represent respectively, the diagonal terms of the moments of inertia, the gravity acceleration, and the disturbance components which include the effects of external disturbances and model uncertainties. Further, we have:

\[
[\begin{array}{c}
\begin{bmatrix}
F \\
u_\phi \\
u_\theta \\
u_\psi
\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
0 & l & 0 & -l \\
\frac{c}{l} & -c & c & -c
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3
\end{bmatrix}
\]

(7)

where \( l \) and \( T_i \ (i \in \{1, \ldots, 4\}) \) represent the arm length of the quadrotor and the thrust force generated by each rotor, respectively. Also, \( c = \frac{k_D}{k_T} \), where \( k_D \) and \( k_T \) denote the drag factor and the thrust coefficient [13].

From (1)-(7), it can be seen that only four independent system outputs can be controlled by the system inputs \( (T_i) \). In other words, the quadrotor is an under-actuated system. An effective approach to develop a precise trajectory tracking control in such cases is to tune the air vehicle’s roll and pitch angles based on the horizontal components of the desired acceleration of the air vehicle (\( \ddot{x}_{des} \) and \( \ddot{y}_{des} \)). More precisely, from (1) and (2), we have:

\[
(\ddot{x}) = -\frac{F}{m} (\cos \phi \sin \theta \cos \phi + \sin \phi \sin \theta) + \ddot{d}_1.
\]

(8)

Thus, the desired values of the roll and pitch angles can be determined as follows:

\[
(\cos \phi_{des} \sin \theta_{des}) = -\frac{m}{F} (\sin \phi \sin \theta - \cos \phi \cos \theta) \dot{\phi}_{des} - \ddot{d}_1.
\]

Consequently, in the rest of the paper, the system states are considered as \( \mathbf{x} = [z \ \phi \ \theta \ \psi]^T \). Accordingly, the system dynamic model can be represented in the affine form as follows:

\[
\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) + \mathbf{B}(\mathbf{x}, \mathbf{u}) \mathbf{u} + \mathbf{d},
\]

where \( \mathbf{x}_{n \times 1} \), \( \mathbf{u}_{m \times 1} \), and \( \mathbf{d} \) represent the system state variables, the control inputs, and the external disturbances, respectively. Here, \( \mathbf{f} \) demonstrates the nonlinear internal dynamics of the system, while \( \mathbf{B} \) represents the control gains of the system. Also, we have:

\[
\mathbf{u} = [T_1 \ T_2 \ T_3 \ T_4]^T,
\]

(10)

(11)

(12)

In the following section, the Sliding Mode Control (SMC) formulation is developed for the introduced dynamic model (10).

III. FORMULATION OF THE SLIDING MODE CONTROL

If the desired states of the system is denoted by \( \mathbf{x}_{des} \), then the tracking error can be calculated as follows:

\[
\mathbf{e} = \mathbf{x} - \mathbf{x}_{des}.
\]

(13)

Accordingly, the sliding surface can be formulated as follows [14]:

\[
\mathbf{s} = \dot{\mathbf{e}} + \Lambda \mathbf{e},
\]

(14)

where \( \Lambda \) is a diagonal positive definite matrix. The Lyapunov function can then be defined as follows:

\[
V = \frac{1}{2} \mathbf{s}^T \mathbf{s}.
\]

(15)

Consequently, the time derivative of the introduced Lyapunov function is obtained as follows:

\[
\dot{V} = \mathbf{s}^T \dot{\mathbf{s}} = \mathbf{s}^T (\dot{\mathbf{e}} + \Lambda \dot{\mathbf{e}}) = \mathbf{s}^T (\dot{\mathbf{x}} - \dot{\mathbf{x}}_{des} + \Lambda \dot{\mathbf{e}}) = \mathbf{s}^T \{ \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{B}(\mathbf{x}, \dot{\mathbf{x}}) \mathbf{u} + \mathbf{d} - \mathbf{x}_{des} + \Lambda \dot{\mathbf{e}} \}.
\]

(16)

Thus, if we have \( n = m \), then using the following input command:

\[
\mathbf{u}_c = \mathbf{B}^{-1} (\ddot{\mathbf{x}}_{des} - \dot{\mathbf{f}}(\mathbf{x}, \dot{\mathbf{x}}) - \mathbf{d} - \Lambda \dot{\mathbf{e}} - \mathbf{K} \mathbf{s}),
\]

(17)

we have:

\[
\dot{V} = -\mathbf{K} \mathbf{s}^T \mathbf{s}.
\]

(18)
where $K$ is a positive definite matrix. Eq. (18) guarantees the asymptotical stability of the closed-loop system.

However, in the presence of actuator faults, determining the accurate system dynamics and external disturbances is not easy at all. An effective observer is introduced in the following section in order to estimate the precise values of the system parameters in the presence of internal faults and external disturbances.

### IV. THE PROPOSED ADAPTIVE SMC APPROACH

Using (10), the dynamic equation of the $i$th system state $(x_i)$ can be obtained as follows:

$$
\dot{x}_i = f_i + B_i u + d_i,
$$

where $f_i$ and $B_i$ represent the $i$th row of $f$ and the $i$th row of $B$, respectively. Also, $d_i = \tilde{d}_{i+2}$. Considering a multiplicative actuator fault, the $j$th rotor thrust can be formulated as follows:

$$
T_j = \alpha_j T_{jc},
$$

where $T_{jc}$ represents the $j$th input command and $0 \leq \alpha_j \leq 1$ is the corresponding fault coefficient. Thus, (19) can be reformulated as follows:

$$
y_i := \tilde{x}_i - f_i = \left[1 \ B_i \bigcirc u_i^T \right] \left[\begin{array}{c} d_i \\ \alpha \end{array}\right] =: \Theta_i \alpha,
$$

where,

$$
\Theta_i = \left[1 \ B_i \bigcirc u_i^T \right],
\quad \alpha_i = \left[\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{array}\right]^T,
\quad u_i = \left[\begin{array}{cccc} T_{1c} & T_{2c} & T_{3c} & T_{4c} \end{array}\right]^T,
$$

and $\bigcirc$ is used for the element-wise multiplication. Generally, $\alpha$ is an unknown vector which should be identified. An online learning approach has been introduced in [7] for updating the unknown coefficient of a neural network called Online Sequential Extreme Learning Machine (OS-ELM). The OS-ELM has been developed based on the Recursive Least Square (RLS) optimization. The OS-ELM approach can now be applied to the obtained dynamic equation (21). More precisely, the unknown vector $\alpha$ should be updated in each iteration after obtaining a new set of system data as follows [15]:

$$
\alpha_k = \alpha_{k-1} + \kappa_k \varepsilon_k,
\quad P_k = \left(I - \kappa_k \Theta_k \right) P_{k-1} \frac{1}{\lambda},
$$

where,

$$
\varepsilon_k = y_{ik} - \Theta_{ik} \alpha_{k-1},
\quad \kappa_k = \frac{\varepsilon_k^T \varepsilon_k}{\lambda + \Theta_{ik}^T P_{k-1} \Theta_{ik}},
$$

where subscript $k$ denotes the $k$th time step and $I$ represents the identity matrix. Also, $0 < \lambda \leq 1$ is a constant forgetting factor which is used to regulate the adaptation rate of the identification algorithm to the newly obtained data.

The first component of $\alpha$ corresponds to the external disturbance, while the other components of $\alpha$ represent the multiplicative fault coefficients of each rotor. Accordingly, the unknown fault gains, as well as the disturbance term, can be determined iteratively using the aforementioned optimization algorithm.

Notice that the introduced observer (26)-(29) can be applied to the dynamic equations of all the four system states $(x, \phi, \theta, \psi)$. Thus, the vector $\alpha$ can be estimated using different ways. This, in turn, leads to more reliable identification. Indeed, the fault coefficients of the system inputs can be updated in each step using the average of the obtained values of $\alpha$ for all the four dynamic equations of the system states. As can be observed in (3)-(7), the dynamic equation of $\phi$ includes only the second and fourth control inputs, while the dynamic equation of $\theta$ consists of the first and third control inputs. On the other hand, the dynamic equations of $z$ and $\psi$ include all the four control inputs. Consequently, there are three different dynamic equation for each system input in order to identify the corresponding fault coefficient of the system inputs.

As mentioned earlier, the disturbance term of each dynamic equation can be estimated using the introduced approach. However, in practice, it is more effective to update the disturbance term $d_i$ at each time step as follows:

$$
\hat{d}_i(k) = \tilde{x}_i(k) - f_i(k) - B_i(k) (\alpha(k) \bigcirc u_i(k)),
$$

where $d_i$ represents the estimated value of $d_i$. Accordingly, the proposed adaptive SMC approach can be summarized as follows:

**Algorithm 1:**

1. Set $t = k$.
2. Calculate the desired roll and pitch angles using (9).
3. Update the fault coefficients using (26)-(29).
4. Estimate the disturbance term $d_i$ using (30).
5. Calculate the input command using (17).
6. Divide the obtained input command of each rotor to the corresponding component of $\alpha$ and apply the obtained vector to the system.
7. Set $t = k + 1$ and go to step 1.

### V. SIMULATION RESULTS

The numerical values of the system parameters are taken from [13], which are listed in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>Mass</td>
<td>0.58 kg</td>
</tr>
<tr>
<td>$l$</td>
<td>Arm length</td>
<td>0.25 m</td>
</tr>
<tr>
<td>$(I_0, I_1, I_2)$</td>
<td>Moments of inertia</td>
<td>(0.01, 0.01, 0.02) kg·m²</td>
</tr>
<tr>
<td>$k_D$</td>
<td>Drag factor</td>
<td>2.82e−7</td>
</tr>
<tr>
<td>$k_T$</td>
<td>Thrust coefficient</td>
<td>1.5e−5</td>
</tr>
</tbody>
</table>

It should be noted that the desired value of the yaw angle and its time derivatives are assumed zero in the current study. Also, to simplify the control structure, it is assumed that $\dot{\phi}_{des} = \dot{\theta}_{des} = 0$ and $\ddot{\phi}_{des} = \ddot{\theta}_{des} = 0$ in the entire simulation time. On the other hand, to provide a more effective trajectory tracking, the desired values of $\tilde{x}$ and $\tilde{y}$ in (9) are substituted by

$$
\dot{x}_{des} \leftarrow \dot{x}_{des} - k_{1x} (x - x_{des}) - k_{2x} (\dot{x} - \dot{x}_{des}),
\quad \dot{y}_{des} \leftarrow \dot{y}_{des} - k_{1y} (y - y_{des}) - k_{2y} (\dot{y} - \dot{y}_{des}),
$$

where $x$ and $y$ represent the yaw and pitch angles, respectively.
where $k_{1x}, k_{2x}, k_{1y}, k_{2y}$ are some positive constants. Here, the numerical values of the control parameters are chosen as: $k_{1x} = k_{1y} = 2$ and $k_{2x} = k_{2y} = 3$. Further, we have $K = \text{diag}([1, 20, 20, 20])$, and $\Lambda = \text{diag}([1, 50, 50, 50])$. Also, the sampling time of the simulations is chosen as $T_s = 10\text{ms}$ and the desired trajectory of the system is defined as follows:

$$x_{\text{des}}(t) = \sin \left( \frac{2\pi}{5} t + \frac{\pi}{4} \right),$$

$$y_{\text{des}}(t) = \sin \left( \frac{2\pi}{5} t + \pi \right),$$

$$z_{\text{des}}(t) = 2\sin \left( \frac{2\pi}{5} t \right).$$

Further, the upper bound of the thrust force and the upper bound of the change rate of the thrust force for each rotor are set to be $4N$ and $40 \frac{N}{s}$, respectively. To evaluate the performance of the proposed control structure, three different scenarios are studied in the following subsections.

A. No Fault / No Disturbance

In the first scenario, the performance of the closed-loop system is evaluated in the nominal flight condition. The desired trajectory, as well as the real trajectory of the system, is shown in Fig. 1. As seen in the figure, the quadrotor can satisfactorily follow the desired trajectory. Indeed, there is no significant difference between the performance of a regular SMC and the proposed adaptive control approach, in this scenario.

B. Single Fault + External Disturbance

In the second scenario, a single fault and some external disturbances are considered in the system dynamic model. More specifically, it is assumed that $\alpha_3 = 0.5$ from beginning the flight. Also, some external disturbances are applied to the air vehicle as follows:

$$d_1(t) = 0.2\sin \left( \frac{2\pi}{10} t + \frac{\pi}{4} \right),$$

$$d_3(t) = 0.2\sin \left( \frac{2\pi}{10} t \right) + w_\phi(t),$$

where $w_\phi(t)$ is a white noise signal with the power of $-20$ dBW. In the current study, we use only the dynamic equation
of \( \theta \) to identify the fault coefficients of the first and third rotors. Again, the desired trajectory of the system and the real trajectory are illustrated in Fig. 2.

![Fig. 4. The estimated values of the first and third fault coefficients in the presence of a single fault and external disturbances.](image1)

![Fig. 5. The estimated values of the first and third fault coefficients in the presence of multiple faults and external disturbances.](image2)

![Fig. 6. The desired trajectory and the real trajectory in the presence of multiple faults and external disturbances.](image3)

![Fig. 7. The system position and attitude in the presence of multiple faults and external disturbances.](image4)

![Fig. 8. The applied inputs to the system in the presence of multiple faults and external disturbances.](image5)

As seen, the introduced observer can satisfactorily identify the fault coefficient of the third rotor and consequently, the air vehicle can suitably follow the desired trajectory. It is notable that the forgetting factor \( \lambda \) is chosen as 1 in the current study. Several simulations have been performed in order to evaluate the closed-loop performance in the presence of sudden rotor faults during the flight simulation (not from beginning the flight). It has been observed that the assumption of \( \lambda = 1 \) in the control structure leads to acceptable trajectory tracking even if the accuracy of the estimated fault coefficients (\( \alpha_i \)) is not as well as that of the aforementioned scenario. Also, the use of the smaller forgetting factor leads to better adaptation, but it may result in less robustness to the actuator faults.

A. Multiple faults + External Disturbance

Finally, in the third scenario, two different faults are
considered in the dynamic model of the quadrotor. More specifically, it is assumed that $\alpha_1 = 0.8$ and $\alpha_2 = 0.5$ from beginning the flight. Also, the introduced external disturbances in the previous scenario are also applied to the system. The estimated values of the fault coefficients are demonstrated in Fig. 5. As seen in the figure, the introduced observer can identify both of the fault coefficients, satisfactorily even in the presence of external disturbances.

In addition, the desired trajectory of the system, as well as the real trajectory, is illustrated in Fig. 6. To demonstrate the high capability of the proposed control approach in comparison with the regular SMC, the quadrotor trajectory using the regular SMC is also shown in Fig. 6. As can be observed in the figure, the proposed adaptive SMC approach leads to quite better performance compared to the conventional SMC and the quadrotor UAV using the proposed control approach can satisfactorily follow the desired trajectory even in the presence of multiple rotor faults and external disturbances.

Also, the system position, the system attitude, and their desired values are shown in Fig. 7, which shows the acceptable performance of the proposed control approach even in the presence of multiple rotor faults and external disturbances. Further, the applied inputs to the system, in this scenario, are shown in Fig. 8. As seen, all the input commands are within the permissible range.

Finally, the Mean Square tracking Error (MSE) of all the aforementioned scenarios are listed in Table II.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
<td>0.015</td>
</tr>
<tr>
<td>3</td>
<td>0.022</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

A novel adaptive sliding mode control was proposed in this research. An OSELM-based observer was introduced in order to identify the faulty subsystems of a quadrotor UAV. Subsequently, a disturbance observer was proposed to compensate for the effects of external disturbances on the closed-loop performance. It was shown that the proposed observer can satisfactorily estimate the fault coefficients even in the simultaneous presence of multiple rotor faults and external disturbances. As seen in the results, the proposed adaptive SMC approach improved the fault-tolerant capability of the regular sliding mode controller, significantly. Accordingly, the closed-loop system using the proposed adaptive SMC can acceptably follow a desired trajectory (in the 3D environment) in the presence of multiple rotor faults and external disturbances.

REFERENCES


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