

An Application of Bilevel Optimization in Pricing and Leasing Strategy

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Abstract—This paper focuses on the finding of an optimal pricing and leasing strategy for both the third-party warehouse and customers, whose conflicts lie on their contradictory objective functions, via bilevel optimization approach. Bilevel optimization is a game-based approach where the third-party warehouse is a leader, who first decides the price of short and long-term rental contracts taking the behavioral reaction of other players, that is, customers and its competitor, into consideration. Once observed, customers then decide which contracts to be chosen so that their objectives are optimized. While complicated, under certain assumptions, we can show that closed-form solutions could be derived. Our experimental results indicate that competitor storage price significantly affects both third-party warehouse's profit and all of its customer's total costs. Besides, large customer tends to suffer more from high opportunity cost; but, the delivery cost charged by the third-party warehouse is crucial for small customer's rental decision.

Index Terms—Bilevel programming, game theory, storage pricing, third-party warehouse.

I. INTRODUCTION

Third-party warehouses have played an important business role in the networks of several supply chains as they provide storage buffer against fluctuating demand and supply. Warehousing and storage leasing industry is also one of the fastest growing businesses nowadays. According to the National Bureau of Statistic (NBS) of China, in 2013, the Chinese national warehousing fixed asset was about 69.2 billion dollars, increasing by 32.7% when compared to 2012's figure [1].

In terms of operations, warehousing companies provide storage services with fixed capacity, measured in pallets, to a wide range of customers; and, storage pricing is crucial for both storage providers and customers as it directly affects their operational costs — and so profitability [2]. In addition, as storage space could be considered a perishable product — it loses value if left unoccupied — and some warehouses are equipped with special equipment, i.e. refrigerated warehouse, utilizing this storage space to its maximum potential is also a typical goal for those warehousing companies [3]. In order to achieve such a goal, effective partnership strategies between warehousing companies and customers are thus required as they could turn opportunity cost into mutual benefit — and be allocated to each player [4].

Long-term storage space leasing schema is one of such strategies, where a warehouse reserves part of its space for long-term leasing, and the rest for contingency use, i.e. short-term leasing. Based on this schema, the warehouse is beneficial from the reduction of potentially idle space, while the customers could enjoy a cheaper unit price from the warehouse [5]. However, as both short and long-term prices, along with the prices set by its competitors, directly affect the decisions made by customers — and so the warehouse's — optimal pricing and leasing strategy is, therefore, needed so that both players could enjoy the surplus gained by a rationale decision making process [6].

In order to better understand the behavior of both warehousing companies and customers in a dynamic setting, in this paper, a bilevel optimization model is developed and applied on a simple system consisting of two warehouses — the third-party warehouse and its competitor — and $|I|$ customers. For simplicity, we assume that the third-party warehouse knows its customers' demand patterns over the planning horizon; and it is responsible for the delivery to the customers. Additionally, the third-party warehouse offers two types of leasing contracts, that is, short and long-term ones, while the competitor provides only one fixed leasing contract. While the price offered by competitor and that of the warehouse's long-term contract are rather stable, the price of the short-term contract gradually rises based on the usage length. For clarity, Fig. 1 shows the comparison of unit storage prices from all leasing contracts.

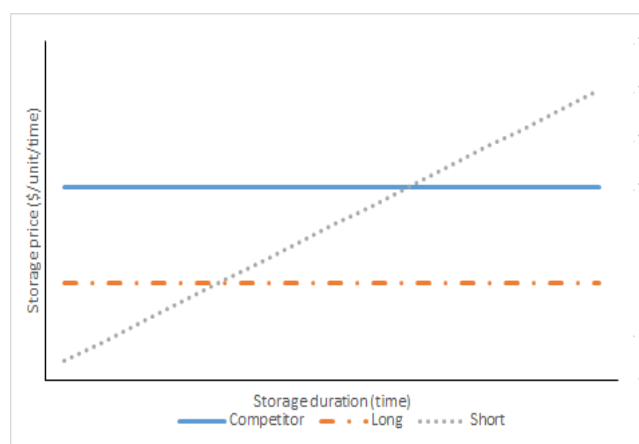


Fig. 1. Warehouse's and competitor's unit storage prices.

Based on this setting, our proposed procedure will first let the warehouse decide on storage pricing, and, once observed, customers will then decide on appropriate inventory planning so that the leasing cost is minimized. As we assume that both players are rationale, the warehouse can then iteratively evaluate the best pricing strategy based on its customer's and

competitor's responses.

II. BILEVEL OPTIMIZATION MODEL

The notations to be used are listed as follows.

Indices

- i is an index for customers ($i = 1, \dots, I$).
- j is an index for planning cycles ($j = 1, \dots, J$).
- t is an index for time periods, where $t \in [0, T]$; and, T denotes the ending period of the whole planning horizon.

Constants and parameters

- CW denotes storage capacity of the warehouse (units).
- C denotes storage price charged by the competitor (\$/unit/time).
- HC denotes inventory holding cost at the warehouse (\$/unit/time).
- OC^c denotes opportunity cost for exorbitant storage space rented by customers (\$/unit/time).
- OC^w denotes opportunity cost for rented-but-unused storage space at the warehouse (\$/unit/time).
- PC denotes penalty cost for excessively rented storage capacity at the warehouse (\$/unit/time).
- d_c denotes delivery cost of the competitor (\$/delivery).
- d_w denotes delivery cost of the warehouse (\$/delivery).
- U_i denotes inventory usage rate of customer i (units/time).
- N_{ij} denotes number of deliveries to customer i in planning cycle j (deliveries).
- T_{ij}^b is the starting time of planning cycle j of customer i .
- T_{ij}^e is the ending time of planning cycle j of customer i .
- T_{ij} is the duration of planning cycle j of customer i .
- Q_{ij} is the quantity of customer i 's product needed to be stored at the warehouse in planning cycle j

(units), where $Q_{ij} = \int_{T_{ij}^b}^{T_{ij}^e} D_i(t) dt$ and $T_{ii}^e = T_{ii+1}^b$.

Functions

- $D_i(t)$ is the demand function of customer i .
- $ZW(p^l, p^s)$ is the profit function of the warehouse, which is a function of short and long-term prices.
- $ZC_i(x_i^l, n_{ij})$ is the total cost function of customer i , which is a function of short (number of deliveries) and long-term contracts.

Decision variables

- p^s is a nonnegative decision variable representing unit price of short-term leasing space at the warehouse (\$/units/time).
- p^l is a nonnegative decision variable representing unit price of long-term leasing space at the warehouse (\$/units/time).

- x_i^l is a nonnegative integer decision variable representing number of long-term leasing space by customer i at the warehouse (units).
- n_{ij} is a nonnegative integer decision variable representing number of deliveries from the warehouse to customer i in planning cycle j (deliveries) whose calculation is based on short-term leasing space ($\frac{n_{ij} Q_{ij}}{N_{ij}}$).
- X_t^p is excess inventory at the warehouse at time t (units).
- D_{ij}^c is number of deliveries from the competitor to customer i in planning cycle j (deliveries).

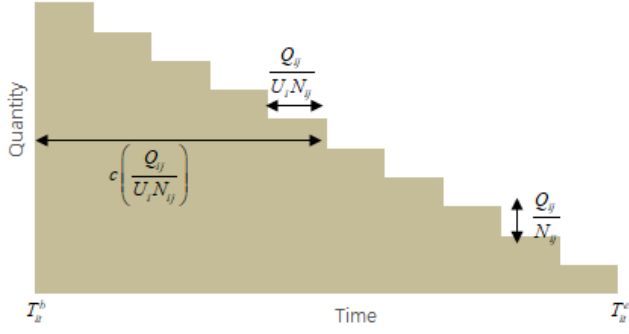
It is worth noting that our proposed bilevel optimization model is based on Stackelberg Game with two main sets of players, that is, warehouses and its customers [7]. For simplicity, there are only two warehouses and $|I|$ customers, each of which has $|J|$ planning cycles whose lengths are not necessarily equal — but with the same planning horizon. All inventory will be stored at the warehouse at the beginning of each planning cycle and gradually withdrawn during such a cycle. And, the process repeats through the end of planning horizon. As customers may lease both short and long-term space in one particular time period, for economical reason, we assume that inventory will be first drawn from the short-term rental space and then the long-term one [8].

Based on the literature [9], solving a bilevel optimization problem is quite difficult as there is no efficient algorithm to solve large-scale problems to optimality. Therefore, the following assumptions are imposed for tractability.

- (1) The unit of storage space for both warehouses are the same.
- (2) The capacity of the third-party warehouse is limited.
- (3) The competitor warehouse has unlimited capacity.
- (4) There must be no beginning and ending inventory at the third-party warehouse.
- (5) Shortages at customers are not allowed.
- (6) The number of vehicles is sufficiently large.
- (7) There is no delivery lead time.
- (8) Demand of customer i is generated from the following function, $D_i(t) = \mu + \alpha \sin(\pi t / T)$, where μ is an average demand and α is a standard deviation.
- (9) Long-term price is a function of short-term price, $p^l = kp^s$ where k is a constant $k \geq 1$.

Fig. 2 shows the movement of customer i 's inventory at the third-party warehouse, where the inventory of customer i is fully replenished at the beginning of planning cycle j (T_{ij}^b) and delivered by Q_{ij} / N_{ij} units every $Q_{ij} / U_i N_{ij}$ time periods. As the inventory of Q_{ij} / N_{ij} units will be stored at the warehouse for $c \cdot Q_{ij} / U_i N_{ij}$ time periods, where $c = (1, 2, 3, \dots, n_{ij})$, total inventory stored at the warehouse by customer i in planning cycle j could be calculated by (1).

$$\text{Total Inventory} = \left(\frac{Q_{ij}}{N_{ij}} \right) \left(\frac{Q_{ij}}{U_i N_{ij}} \right) \left(\sum_{c=1}^{n_{ij}} c \right) \quad (1)$$


 Fig. 2. Inventory movement of customer i at the warehouse.

A. An Optimization Model for the Third-Party Warehouse

The objective of the third-party warehouse is to determine the optimal storage pricing strategy that maximizes its total profit, subject to two practical constraints that limit the prices of both short and long-term rental contracts, as follows.

$$\text{Maximize } ZW(p^l, p^s) = \sum_i \sum_j S_{ij} + \sum_i \sum_j L_{ij} + \sum_i \sum_j D_{ij} + \sum_i \sum_j OC_{ij}^w - \sum_i \sum_j H_{ij} - \sum_j P_{ij} \quad (2)$$

$$S_{ij} = \frac{p^s Q_{ij}^2}{2U_i N_{ij}^2} (n_{ij}^3 + 2n_{ij}^2 + n_{ij}) - \frac{p^s Q_{ij}^3}{6U_i^2 N_{ij}^3} (2n_{ij}^3 + 3n_{ij}^2 + n_{ij}) \quad (3)$$

$$L_{ij} = p^l x_i^l T_{ij} \quad (4)$$

$$D_{ij} = (n_{ij} + x_i^l \frac{N_{ij}}{Q_{ij}}) d_w \quad (5)$$

$$OC_{ij}^w = OC^w \left[\frac{x_i^l Q_{ij}}{2U_i N_{ij}} \left(\frac{x_i^l N_{ij}}{Q_{ij}} - 1 \right) + x_i^l \left(T_{ij} - \frac{Q_{ij}}{U_i} \right) \right] \quad (6)$$

$$H_{ij} = HC \left[x_i^l \left(\frac{Q_{ij}}{U_i} - \frac{x_i^l}{U_i} \right) + \frac{Q_{ij}}{2U_i N_{ij}} \left(\frac{x_i^l N_{ij}}{Q_{ij}} + 1 + \frac{Q_{ij}}{N_{ij}} (n_{ij}^2 + n_{ij}) \right) \right] \quad (7)$$

$$P_{ij} = PC \sum_t X_t^p \quad (8)$$

Subject to:

$$0 \leq p^s \leq C \quad (9)$$

$$0 \leq p^l \leq C \quad (10)$$

The warehouse's objective (2), is simply a finding of prices for both short and long-term contracts (p^s and p^l) that maximize its total profit, where (3) – (6) denote the revenues from short-term storage, long-term storage, delivery, and customer's opportunity cost; lastly, (7) and (8) denotes inventory holding and penalty costs at the warehouse, respectively.

- Short-term leasing cost (3) is calculated by the summation of total inventory multiplied by the short-term rental cost, which is an increasing function of storage time ($p^s t$ where $0 < t < T$).

- Customer's opportunity cost (6) is calculated from vacant space rented by the customers from all possible scenarios, i.e. when there is no demand.
- Inventory holding cost (7) is calculated from both short and long-term rental contracts based on total inventory (1).

Lastly, inequalities (9) and (10) help limit the ranges of p^s and p^l that should not exceed the price set by its competitor.

B. An Optimization Model for the Customers

As the objective of customers is to minimize their total costs, once the prices of all rental contracts are observed, each customer then decides on the amount of short and long-term leasing space.

$$\text{Minimize } ZC_i(x_i^l, n_{ij}) = \sum_j S_{ij} + \sum_j L_{ij} + \sum_j D_{ij} + \sum_j OC_{ij}^c + \sum_j C_{ij} + \sum_j D_{ij}^c \quad (11)$$

$$OC_{ij}^c = OC^c \left[\frac{x_i^l Q_{ij}}{2U_i N_{ij}} \left(\frac{x_i^l N_{ij}}{Q_{ij}} - 1 \right) + x_i^l \left(T_{ij} - \frac{Q_{ij}}{U_i} \right) \right] \quad (12)$$

$$C_{ij} = \frac{CQ_{ij}^2 D_{ij}^c (D_{ij}^c + 1)}{2U_i N_{ij}^2} \quad (13)$$

$$D_{ij}^c = (N_{ij} - n_{ij} - x_i^l \frac{N_{ij}}{Q_{ij}}) d_c \quad (14)$$

Subject to:

$$1 \leq n_{ij} \leq N_{ij} \quad (15)$$

$$0 \leq x_i^l \leq Q_{ij} \quad (16)$$

$$\frac{x_i^l N_{ij}}{Q_{ij}} + n \leq N_{ij} \quad (17)$$

The objective of customer i (11) is simply total cost minimization, where the cost function consists of short-term storage cost, long-term storage cost, and delivery cost at the warehouse — Equations (3) – (5), respectively. Equation (12) shows the opportunity cost from leasing idle space, which is similar to (6) but with different parameters. Similar to (3), Equation (13) denotes the cost of leasing space from competitor but with constant price. Equation (14) denotes delivery cost from competitor and Inequalities (15) - (17) limit the ranges of x_i^l and n_{ij} .

III. SOLUTION PROCEDURE

Below is the pseudo code of our proposed methodology for solving storage pricing and leasing problems based on bilevel optimization.

Pseudo code for the Proposed Bilevel Optimization model

- 1: **Input** parameters of warehouses and customers
- 2: **Find** values of x_i^l, n_{ij} by minimizing the following function

$$F(x_i^l, n_{ij}) = S_{ij} + L_{ij} + D_{ij} + OC_{ij}^c + C_{ij} + D_{ij}^c$$
- 3: Differentiate $F(x_i^l, n_{ij})$ with respect to both x_i^l and n_{ij}

4: **Find** a closed form solution of x_i^l, n_{ij} and p^s
 5: Let $z = 0$
 6: while $z \leq N_{ij}$ for each i and j let $n_{ij} = z$
 7: **Find** p^s for each value of n_{ij}
 8: $z + 1$
 9: end
 10: **Find** all possible x_i^l for each value of Q_{ij}, N_{ij} and for all values of p^s
 11: if x_i^l is not an integer, $x_i^l = \lceil x_i^l \rceil$
 12: **Find** value n_{ij} for each x_i^l
 13: if n_{ij} is not an integer

$$n_{ij} = \arg \min_{\{ \lceil n_{ij} \rceil, \lfloor n_{ij} \rfloor \}} \{ F(\lceil n_{ij} \rceil), F(\lfloor n_{ij} \rfloor) \}$$

 14: else $n_{ij} = n_{ij}$
 15: if number of deliveries $\frac{x_i^l N_{ij}}{Q_{ij}}$ is not an integer,

$$\frac{x_i^l N_{ij}}{Q_{ij}} = \left\lceil \frac{x_i^l N_{ij}}{Q_{ij}} \right\rceil$$

 16: **Eliminate** infeasible solutions
 17: **Find** x_i^l, n_{ij} that minimize total cost of customer for each value of p^s
 18: **Find** value of $ZW(p^s, p^l)$
 19: **Find** value of $ZW(p^s, p^l)$
 20: **Return** optimal values of p^s, x_i^l, n_{ij}

From Line 2, given a leasing strategy of one particular customer in one planning cycle, its total cost could be defined by Equation (18).

$$F(x_i^l, n_{ij}) = S_{ij} + L_{ij} + D_{ij} + OC_{ij}^c + C_{ij} + D_{ij}^c \quad (18)$$

By differentiating $F(x_i^l, n_{ij})$ with respect to x_i^l and n_{ij} , we can then obtain $F'(x_i^l)$ and $F'(n_{ij})$ as shown in Equations (19) and (20).

$$F'(x_i^l) = \frac{C + OC^c}{U_i} x_i^l + \left(\frac{p^l Q_{ij}}{2U_i} \right) (N_{ij} + 1) - BC(N_{ij} - n_{ij} - \frac{1}{2} - \frac{OC^c}{2C}) + (d_w - d_c) \frac{N_{ij}}{Q_{ij}} + kp^s T_{ij} \quad (19)$$

$$F'(n_{ij}) = \frac{Q_{ij}^3}{U_i^2 N_{ij}^3} \left(\frac{p^s}{2} \right) n_{ij}^2 + \left(\frac{CQ_{ij}^2}{U_i N_{ij}^2} - \frac{p^s Q_{ij}^3}{2U_i^2 N_{ij}^3} \right) n_{ij} + \frac{Q_{ij}^3}{U_i^2 N_{ij}^3} \left(\frac{p^s}{12} \right) + \frac{CQ_{ij}^2}{2U_i N_{ij}^2} \left(\frac{2N_{ij} x_i^l}{Q_{ij}} - 2N_{ij} + 1 \right) + d_w - d_c \quad (20)$$

$$n_{ij} = \left(\frac{1}{2} - \frac{C}{p^s B} \right) + \sqrt{\left(\frac{C}{p^s B} \right)^2 + \frac{1}{12} - \left(\frac{C}{p^s B} \right) \left(\frac{2N_{ij} x_i^l}{Q_{ij}} - 2N_{ij} + 2 \right) + (d_c - d_w) \frac{2}{p^s AB}}, \quad (28)$$

$$p^s = \frac{(4B - 6)Cn_{ij} + 2C(6 - 4B) \left[2N_{ij} + 1 - \left(\frac{2}{B(C + OC)} \right) \left[OC \left(\frac{B}{2} + \frac{Q_{ij}}{U_i} - T_{ij} \right) - \frac{BC}{2} (2n_{ij} - 2N_{ij} - 1) \right] \right] + \frac{6 - 4B}{A(C + OC)} (d_c - d_w)(5C + OC)}{(9 + 4B^2 - 12B)n_{ij}^2 + (12 + 4B^2 - 14B)n_{ij} - 3B + \frac{4}{6}B^2 + 3 - 4C(6 - 4B) \left(\frac{kT_{ij}}{B(C + OC)} \right)} \quad (29)$$

To check whether the critical points from $F'(x_i^l) = 0$ and $F'(n_{ij}) = 0$ are indeed global minimizers, we need to check the definiteness of Hessian matrix of $F(x_i^l, n_{ij})$ whose second derivatives with respect to x_i^l and n_{ij} are shown as follows.

$$F''(n_{ij}) = \frac{p^s Q_{ij}^3}{U_i^2 N_{ij}^3} \left(n_{ij} - \frac{1}{2} \right) + \frac{CQ_{ij}^2}{U_i N_{ij}^2} \quad (21)$$

$$F''(x_i^l) = \frac{C + OC^c}{U_i} \quad (22)$$

$$F''(x_i^l, n_{ij}) = \frac{CQ_{ij}}{U_i N_{ij}} \quad (23)$$

$$F''(n_{ij}, x_i^l) = \frac{CQ_{ij}}{U_i N_{ij}} \quad (24)$$

Based on the determinants of all leading principal minors, (25) and (26), it could be seen that $F(x_i^l, n_{ij})$ is strictly convex. Therefore, the critical points from $F'(x_i^l) = 0$ and $F'(n_{ij}) = 0$, i.e. (27) and (28), are indeed the optimal solution to $F(x_i^l, n_{ij})$.

where $A = \frac{Q_{ij}^2}{U_i N_{ij}^2}$ and $B = \frac{Q_{ij}}{U_i N_{ij}}$. By substituting (27) into (28), we can derive the short-term rental price (p^s) as shown in (29).

$$|F''(n_{ij})| = \frac{p^s Q_{ij}^3}{U_i^2 N_{ij}^3} \left(n_{ij} - \frac{1}{2} \right) + \frac{CQ_{ij}^2}{U_i N_{ij}^2} > 0 \quad (25)$$

$$\begin{vmatrix} F''(n_{ij}) & F''(x_i^l, n_{ij}) \\ F''(n_{ij}, x_i^l) & F''(x_i^l) \end{vmatrix} = \frac{p^s Q_{ij}^3}{U_i^2 N_{ij}^3} \left(n_{ij} - \frac{1}{2} \right) > 0 \quad (26)$$

$$x_i^l = \left(\frac{U_i}{C + OC^c} \right) \left[\frac{OC \left(\frac{B}{2} + \frac{Q_{ij}}{U_i} - T_{ij} \right)}{2Q_{ij}} - \frac{CAN_{ij}}{2Q_{ij}} (2n_{ij} - 2N_{ij} - 1) + (d_c - d_w) \frac{N_{ij}}{Q_{ij}} - kp^s T_{ij} \right] \quad (27)$$

Since $p^l = kp^s$, warehouse pricing function (29) defines both short and long-term rental contracts for the warehouse; while (27) and (28) are closed-form solutions for customer reaction.

IV. NUMERICAL EXAMPLE

For simplicity, we test our proposed model on a simple system consisting of two warehouses — the third-party and its competitor — and two different customers, one being a big customer (C1) having more average demand and variation, and another is a comparably small one (C2) with less demand and variation. The model is executed over 360 days, with 3 seasons and 12 planning cycles. The input parameters are drawn from previous research [8].

- $T = 360$ days
- $T_{ij} = 30$ days
- $HC = \$0.1/\text{unit/day}$
- $C = \$1/\text{unit/day}$
- $OC^w = \$0.5/\text{unit/day}$
- $OC^c = \$0.5/\text{unit/day}$
- $CW = 20,000$ units
- $PC = \$1.2/\text{unit/day}$
- $d_w = \$50/\text{delivery}$
- $d_c = \$155/\text{delivery}$
- $U_1 = 250$ units/day
- $U_2 = 100$ units/day
- $D_{1j} = 218 + 55\sin(\pi t / 120)$
- $D_{2j} = 98 + 7\sin(\pi t / 120)$
- $N_{1j} = (165, 215, 240, 200)$
- $N_{2j} = (80, 85, 90, 95)$
- N_{ij} is for a season and is repeated over the planning horizon.
- The initial inventory is set to zero

Based on this setting, the optimal pricing and leasing decisions of the warehouse and all customers, along with the sensitivity analysis on some key parameters, are shown in Table I.

TABLE I. DECISIONS MADE BY THE WAREHOUSE AND ALL CUSTOMERS

Parameters	p^s	ZW	ZC1	ZC2	x_1^l	x_2^l
Basic	0.00856	1,715,003	1,578,806	302,397	518	206
$d_w = 15$	0.00837	1,432,014	1,356,222	240,749	707	282
$d_w = 85$	0.00876	2,010,271	1,810,627	367,070	329	131
$d_w = 120$	0.00895	2,257,044	1,998,432	430,016	139	60
$d_w = 155$	0.00931	2,432,850	2,130,514	481,731	11	0
$C = 0.3$	0.00785	441,623	497,138	242,708	2207	387
$C = 0.65$	0.00550	1,019,612	984,589	203,897	690	272
$C = 1.35$	0.01168	2,461,491	2,219,043	407,235	410	165
$C = 1.7$	0.01482	3,227,636	2,878,044	514,116	337	137
$OC^c = 0.1$	0.00823	1,540,539	1,415,462	270,382	724	296
$OC^c = 0.3$	0.00842	1,662,885	1,529,819	289,020	605	244
$OC^c = 0.7$	0.00867	1,751,707	1,614,474	312,905	451	177
$OC^c = 0.9$	0.00876	1,740,224	1,604,351	322,340	399	156

A. Warehouse's Delivery Charge (d_w)

Expectedly, from Table I, the warehouse's profit (ZW) significantly rises as warehouse's deliver charge (d_w) increases. Since the increase of d_w also affects the decisions made by the customers, accordingly, the customers will lease less amount of long-term space and rely more on its competitor. In order to compensate revenue loss, the third-party warehouse needs to react by increasing the price for short-term contract (p^s) resulting in higher values of customers' cost.

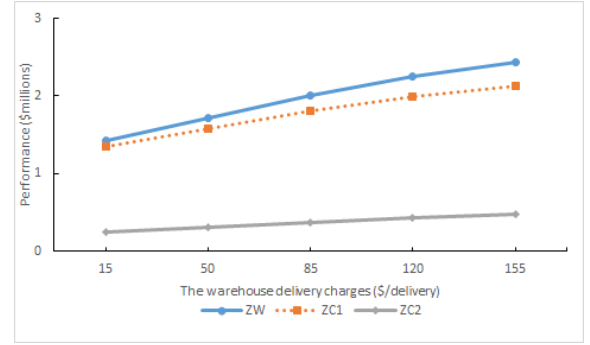


Fig. 3. The impact of change on the warehouse's delivery fare.

If we vary the value of d_w from 50 to 85, the warehouse needs to increase the value of p^s from 0.00856 to 0.00876; but this would increase the total costs of customer 1 (ZC1) and 2 (ZC2) by 14.69% and 21.39%.

Competitor Storage Price (C)

From Table I and Fig. 4, the third-party warehouse significantly gains more when the competitor's storage price (C) increases — as it has a good chance to charge customers more by increasing the value of p^s . For example, if we increase the value of C from 1 to 1.7, ZW increases by 88.2% due to the abrupt rise on short-term leasing (p^s) from 0.00856 to 0.01482 — resulting in higher values of ZC1 and ZC2. However, when C is decreasing to a certain point, i.e. $C = 0.3$, the warehouse will not lower the value of p^s to compete with the competitor; but, it rather sets p^s at a certain high price to attract more long-term leasing customers.

This solution does make a good sense when the opportunity cost of customers is low, as both C1 and C2 will react to the warehouse decision by leasing a lot of long-term space due to such a high price of p^s .

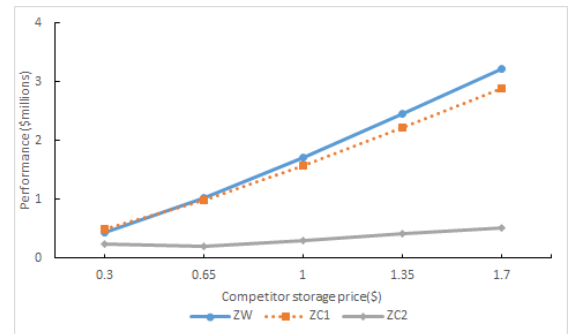


Fig. 4. The impact of change on competitor's storage price.

B. Customer's Opportunity Cost (OC^c)

As we increase customer's opportunity cost (OC^c), the warehouse's profit is observed to rise and then decline as shown in Fig. 5. Intuitively, by increasing customer's opportunity cost, customers should have leased less long-term space and relied more on short-term contracts resulting in higher operational cost.

However, we observe that ZC1 decreases when we increase OC^c , which seems particularly strange in practice. But, this could be explained by the reaction of the third-party warehouse as it would increase the value of p^s to compensate revenue loss. However, if such a cost is too high, the customers will lean toward a cheaper alternative offered by the competitor resulting in a lower cost at customer and a lower profit at the third-party warehouse.

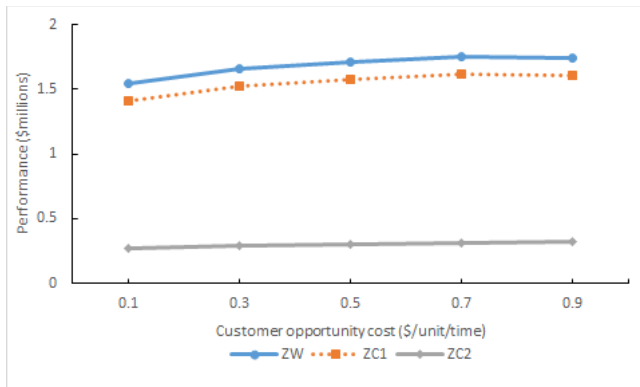


Fig. 5. The impact of change on customer's opportunity cost.

V. CONCLUSION

In this paper, we develop a game-based bilevel optimization model for the determination of optimal pricing and leasing strategy for both the third-part warehouse and its customers. A numerical study is also conducted on a small-scale problem with two customers, along with the sensitivity analysis on some important parameters. Our numerical results indicate that the competitor's storage price has remarkable impact on both the warehouse's profit and customers' total cost. Moreover, the warehouse's delivery charge has more impact on small customer — as it could attract more long-term space demands by reducing delivery cost; but, this is not the case for a large customer, with more demand and high variation, as it tends to focus more on opportunity cost.

While we can improve pricing and leasing strategy by the proposed schema, some limitations are worth to be mentioned. Firstly, we assume that long-term price is a function of short-term price, where $p^l = kp^s$. This however does not hold true in general as the long-term prices depend

on several factors, such as seasonality and the amount of space to be leased. Accordingly, it would be interesting to see the changes as this assumption has been relaxed and replaced by some more realistic functions. Secondly, we assume that all customers are the same in terms of bargaining power as we apply the same leasing price to all customers. But, in practice, large customers should be differently treated; and, this would affect both decisions made by the warehouse and other players. Finally, dynamic pricing strategy on different customers with different characteristics should be further studied.

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