Upper Central Incisor Orthodontic Movement Approximation through Mathematical Simulation Programs

Bunta Olimpia, Vlad Muresan, and Tiberiu Colosi

Abstract—**Orthodontic tooth movement is strictly dependent on force application point. In relation to this point, the Resistance center and Rotation center notions, express the obtained tooth movement and its' biological implications. There for, a certain resistance of the supporting tooth tissues, can ease or make mode difficult a bodily tooth movement. The proposed model, presents, based on two programs in a series sequence, an accessible variant of orthodontic tooth movement, with the possibility of initial data adaptation in order to obtain a numerical simulation and analogical modeling of the orthodontic movement of a wide dental dimensional category.**

Index Terms—**Orthodontics, tooth movement, upper central incisor, rotation center, apex movement.**

I. Introduction

The orthodontic tooth movement is closely correlated with the Center of resistance of the tooth. As the center of mass, it is positioned at the level of the tooth root, so in the interior of the supporting alveolar bone, and it conditions the obtained tooth movement [1]-[3]. If a force would be applied at the level of the Resistance center, the movement obtained would be a pure translation. Orthodontics, applying forces on the exposed dental surfaces- teeth crowns, either vestibular or lingual/palatal surfaces, determines the appearance of a new point, the Rotation center [4].

This point transforms the ideal orthodontic movement – pure translation – into a rotation or roto-translation. It is unanimously accepted that the roto-translation type of movement is the one closer to the ideal variant, pure translation.

There for, the orthodontist has to generate adapted orthodontic forces in order to determine a tooth movement closer to a translation.

The Resistance center and the Rotation center notions, as terms borrowed from the mechanical field and applied to the dentistry field, have a considerable age in the specialty literature – 1917, determining an increasingly early interest for a more suitable and correct orthodontic treatment-1962 [5].

There are multiple studies in the orthodontic literature that use mathematical programs in order to predetermine the orthodontic tooth movement. Dathe et all have tried to explain the concept of Center of Resistance as it is found in the history of the technical field. Having the theoretical data, the authors have tried to idealize the periodontal ligaments to a linear elastic suspension. Using mathematical formalisms they tried to demonstrate that the Center of Resistance point exists only in a two dimensional environment [5].

Other studies have tried to determine the Resistance Center (CR) and the Rotation Center on the basis of a rigid body embodied in an elastic environment, with the help of the Finite Element analysis. [6] Others have approached the same Finite Element Method (FEM) in order to determine the CR of an upper right central incisor for the 3 dimensional variant, and mathematical methods for the 2 dimensional variant.[7] Geramy A. *et all*, Have published a study in which they wanted to determine the influence of the moment-force ratio on the position of the Rotation Center. The study was conducted on a central mandibular incisor.They concluded that a single Rotation can be determined for any number of tooth position. Nevertheless, this point does not always act as a Rotation Center, during the tooth movement process.[8]

The literature abounds in mathematical and engineering means of elucidation of the biological processes that take place in the orthodontic treatments. The most frequently used mean is of course the Finite Element Method.

The purpose of this paper is to present an orthodontic tooth movement model, for the case of the upper central incisor, mentioning the exact implicated parameters, the result obtained and their interpretation, using partial differential equation.

II. The DINORT05(06) Program

The DINORT05(06) program, is based on two distinct programs, ORTODC05(06) and SCTUY03(04), run in a sequence series.

At the base of these two programs is an elliptical parabolic tooth, with the ray (ρ), the height (r_f) and the volume Vol= $\frac{1}{2}\pi\delta^2 r_f$. The rotation parabolic axis, having the structure parameters P=Q= ρ and the length r_f at the level of the incisal edge of the central maxillary incisor, and the length equaling zero, at the level of the tooth apex, represents the geometrical element used in this study.

For the length axis

$$r_f = s_f^1 = s_f + \tilde{s}^1 + \tilde{s} \tag{1}$$

At the $t = t_0 = 0$ moment- the beginning of the orthodontic treatment, the notations $y_{\alpha 0}^1$ (incisal edge of the

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tooth), $y_{\alpha 0}$ and y_{β} (tooth apex) are considered. It is also considered that the whole length

$$s_f = s_f^1 - (s_f + \tilde{s}) \qquad (2)$$

is located inside the plastic environment, which is subject to deformation. The point of force application, with the activation force $u_0 = u_0(t_0)$ of the spring, is considered to be situated at the $(s_f + \tilde{s})$. Due to the $u_0 = u_0(t_f)$ force, at the end of the orthodontic activation cycle, respectively at $t = t_f$, the y_β point may move in the same direction with the $u_0(t_f)$ force, resulting a roto-translation type movement, or it may move in the opposite direction, resulting a rotation type movement.

Therefor, the analogical model of the $y^1{}_{00}(t,s)$ deformation – tooth axis orthodontic movement – is approximated through:

$$y_{00}^1(t,s) = K_y \cdot (1 - \frac{T_1}{T_1 - T_2}\varepsilon^{\frac{-t}{T_1}} - \frac{T_2}{T_2 - T_1}\varepsilon^{\frac{-t}{T_2}}) \cdot (\gamma_0 + \gamma_1 s) \cdot u_0(t) \qquad (3)$$

where the elastic force $u_0(t)$ can be expressed in relation to the two time constants T_1 and T_2 [9-27].

The data run in the program by the orthodontist are:

- t=0 (the initial moment);
- $t_f = 4 weeks$ (final time – the duration of a orthodontic activation cycle);
- $t_{fu} = t_f$;
- $s_f^1 = 24mm$ (medium length of an upper central incisor);
- $\Delta t = t_f / 10$ (extraction of the data pace) ;
- $\tilde{s} = 4$ (distance from the force application point – the bracket- to the inferior part of the free fixed gingiva);
- $\tilde{s}^1 = 4$ (distance from the incisal edge of the tooth , to the force application - the bracket);
- $y_\alpha^1 = 1$ mm (maximum crown movement accepted/ activation cycle);
- $K_y = 0.495$ (proportionality coefficient obtained in relation (1) and scaled independently in the program ORTOD05(06));
- $K_u = 100$ (maximum value of the applied force u_0 - in grams force);
- $\tilde{u}_f = 10$ (force at the t_f moment).

By completing the DINORT05(06) program, the main evolutions for $(t, u_0, y_\alpha^1, s_c, y_\beta)$ can be obtained.

The DINORT05(06) program, from line 10 to line 370, contains the scaling program ORTOD05(06) from line 10 to line 170 , which can be used independently as well. All the initial calculus data:

$t_0, t_f, t_{fu}, s_f^1, \Delta t, y_\alpha^1 K_y, K_u, u_0, s_f, y_\beta$ are given by the orthodontist.

From these, only (y_β) - the movement of the tooth apex- has a fictional value.

There for, the scaling program ORTOD05(06), can generate an exact result only for $y_{00}^1(t, s_0)$. (K_y) is scaled.

The same DINORT05(06) program , from line 180 to line 360, contains the SCTUY03(04) program, which approximates the rotation center (s_c) and the (y_β) deformation, based on the solution of the equilibrium equation , in mechanical moments : $M_E = \sigma \cdot M_P$ - expressed in grams force · mm, uses the σ coefficient. This coefficient reflects the resistance (higher or lower) of the deformed environment.

III. Results

TABLE I: Values Obtained with the Dinort5(6) for The Movement of the Upper Central Incisor (u_0 [Grams Force]; t[Weeks]; s_c [mm]; y_α^1 [mm]; y_β [mm])

	t	0	0.8	1.6	2.4	3.2	4
	u_0	100	56.8	25.5	14.5	11.2	10.33
σ	y_α^1	0	0.33	0.54	0.62	0.77	1.0002
0.4	s_c	177.872	148.64	95.888	62.576	49.152	42.8
0.4	y_β	0.303	0.515	0.605	0.584	0.603	0.664
0.3	s_c	207.2	176.192	117.712	79.584	65.504	61.344
0.3	y_β	0.307	0.525	0.627	0.627	0.676	0.784
0.2	s_c	256.432	222.544	154.912	109.264	95.536	101.664
0.2	y_β	0.312	0.536	0.651	0.67	0.744	0.893

TABLE II: L_E Values Obtained at the Level of the y_α^1 Movement of the Upper Central Incisor (u_0 [Grams Force]; t[Weeks]; y_α^1 [mm]; L_E [Grams Force · mm])

t	0	0.8	1.6	2.4	3.2	4
u_0	100	56.8	25.5	14.5	11.2	10.33
y_α^1	0	0.33	0.54	0.62	0.77	1.0002
L_E	0	18.744	13.77	8.99	8.624	10.332

TABLE III: M_E Values Obtained for the Situation $\sigma = 0.3$ During the Upper Central Incisor Movement (u_0 [Grams Force]; T[Weeks]; s_c [mm]; M_E [Grams Force · mm])

t	0	0.8	1.6	2.4	3.2	4
u_0	100	56.8	25.5	14.5	11.2	10.33

S_c	207.2	176.192	117.712	79.584	65.504	61.344
M_E	2072	10007.70	3001.65	1153.96	733.64	633.68

	0	5	6	8	4	3

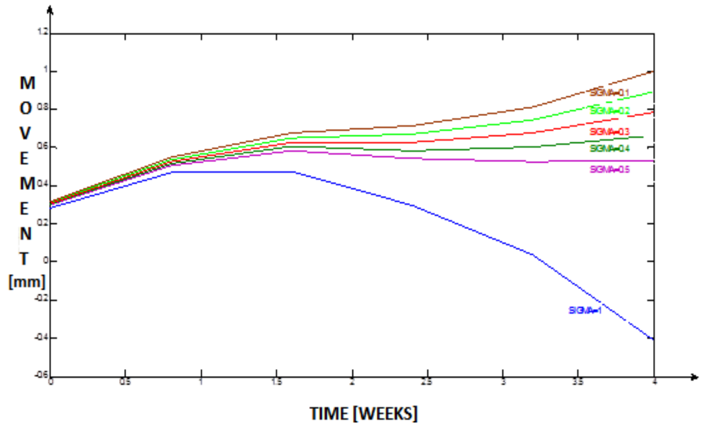

Fig. 1. Evolution of the y_β movement in relation to time at $\sigma = 1.0.5,0.4,0.3,0.2,0.1$

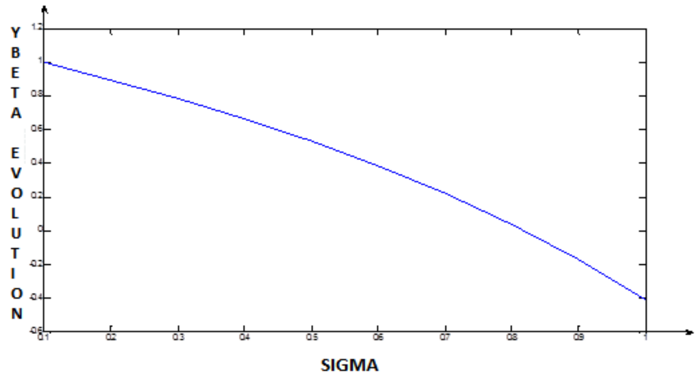

Fig. 2. The y_β evolutions for all the σ variants ($\sigma = 0.1-1$), at the $t_f = 4$ moment

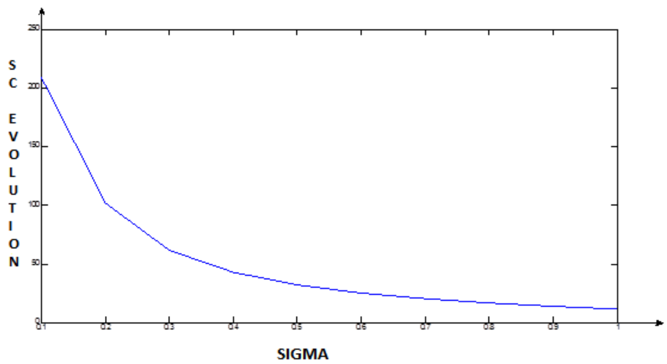

Fig. 3. Rotation center evolution at the $t_f = 4$ moment for all σ variants ($\sigma = 0.1-1$).

IV. Interpretation of the Results

In can be observed from the results of the y_β evolution for all σ variants, $\sigma = 0.1-1$, at $t_f = 4$, that y_β is higher (the dental apex presents a more ample movement) for lower σ variants, and automatically y_β is lower (the dental apex present a less ample movement) for higher σ variants.

The variant that resembles the ideal movement of the upper central incisor, at the level of y_α (the incisal edge of the tooth) and also at the level of y_β (tooth apex) , for an initial application force of 100 grams , is the one obtained at $\sigma = 0.3$.

The (σ) coefficient reflects the higher or lower resistance of the deformed medium. If $\sigma = 1$, the two moments are equal. If $\sigma > 1$, the plastic medium develops a higher resistant moment .

If $\sigma < 1$, the plastic medium develops a lower resistant moment. There for, as shown in Fig. 2, the tooth movement obtained at $\sigma < 1$ shoes a more ample y_β and so an obvious apex movement, in the range of +1mm, the maximum allowed.

In Fig. 1, it is obvious the difference between the movement obtained at $\sigma = 1$ and $\sigma = 0.3$. The y_β evolution at $\sigma = 1$ shows a rotation type movement, meanwhile the y_β evolution at $\sigma = 0.3$ shown a roto-translation type movement.

The rotation center descends from its' high positioning at the t=0 moment, 207.2mm, to a more proximal to the tooth apex position, at the $t_f = 4$ moment , 61.344 mm (Table I, Fig. 3).

These result contradict the believes that the orthodontic tooth movement occurs around a point situated inside the tooth . Positioning the rotation center of the tooth movement outside the tooth morphology, is a more relevant and mathematical approach, and there for a more logical situation.

The lines 40 and 80 of the program correspond to the summed moments $(M_E ; M_P)$ but mutually offset with an advance step (Δs_c) for the rotation center (s_c).

If in the program line 90 the sing of the (DIF1XDIF2) product is inverted, the (s_c) value of the rotation center results.

Table II shows the elastic L_E, expressed in $[grf \cdot mm]$ that appears at the level of the y_α^1 movement. Table III shows the results obtained for the elastic moment $M_E[grf \cdot mm]$, that appears at the level of the (s_c) movement, for $\sigma = 0.3$.

V. CONCLUSION

The synthesis program DINORT05(06) , containing the two serial programs , corresponds to a unitary and systematized program , including the (K_y) scaling – from the ORTOD05(06)program, and the rotation center and (y_β) movement- from the SCTUY03(04) program. This mathematical approach using partial differential equation offers an improved and more precise solution of determination of the Rotation center, and of predetermining the accurate position of the tooth apex.

The medical interpretation of this phenomena can elucidate the orthodontic tooth movement in its' support system. Modifying the initial data in the program, it can be used for all types of tooth morphologies, there for improving orthodontic treatment outcome.

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