# Analytical Modeling of an <sup>15</sup>N Isotope Separation Column by Second-Order Quasi-Linear Equations with Two Independent Variables

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Abstract—The presented process is an isotope separation column for  $^{15}N$ , which represents a space propagation process modeled by the proposed method. Cohen's equation was approached by a second-order differential equation in relation to time t and also second-order in relation to the spatial variable s. This second order partial derivative equation was developed at an initial stage for which the variables  $F_{0T}(t)$  and  $F_{0S}(s)$ have been limited to two time constants  $(T_1, T_2)$ , or space  $(S_1, S_2)$ , resulting a family of curves of the <sup>15</sup>N  $y_{00}(t, s)$ concentration.

Index Terms—Isotopic separation process, Cohen's equation, transcendent equations, M<sub>pdx</sub> method.

#### I. INTRODUCTION

An isotope separation column for  $^{15}N$  is modeled through a second order partial differential equation which is in relation to two independent variables, time and space. Among the multiple approximate solutions associated with the isotopic separation phenomena expressed in relation to the time (t) and the propagation space (s), it is considered the following overdamped version represented in the following equation [1], [2]:

$$y_{00}(t,s) = \tilde{y}_{00} + F_{0T}(t) \cdot F_{0S}(s) \cdot (\tilde{y}_{ff} - \tilde{y}_{00})$$
 (1)

The two exponentially increasing evolutions represented in the following figure are limited to being approximated by two time constants  $T_1$  respective  $T_2$  and two space constants  $S_1$  respective  $S_2$ .

The ascending exponential functions represented by the following two relations for the final values  $F_{0T}(t) \rightarrow 1$ respective  $F_{0s}(s) \rightarrow 1$  [1]-[4].

$$F_{0T}(t) = 1 - \frac{T_1}{T_1 - T_2} \cdot \varepsilon^{-\frac{t}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \varepsilon^{-\frac{t}{T_2}}$$
(2)  
$$F_{0S}(s) = 1 - \frac{S_1}{S_1 - S_2} \cdot \varepsilon^{-\frac{S}{S_1}} - \frac{S_2}{S_2 - S_1} \cdot \varepsilon^{-\frac{S}{S_2}}$$
(3)

$$F_{0S}(s) = 1 - \frac{s_1}{s_1 - s_2} \cdot \varepsilon^{-\frac{S}{S_1}} - \frac{s_2}{s_2 - s_1} \cdot \varepsilon^{-\frac{S}{S_2}}$$
(3)

Taking into consideration the relations (2) respectively (3),

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the complete expression of the approximating solution represented in relation (1) is presented as follows [2], [4]:

$$y_{00}(t,s) = \tilde{y}_{00} + \left(1 - \frac{T_1}{T_1 - T_2} \cdot \varepsilon^{-\frac{t}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \varepsilon^{-\frac{t}{T_2}}\right) \cdot \left(1 - \frac{s_1}{s_1 - s_2} \cdot \varepsilon^{-\frac{s}{S_1}} - \frac{s_2}{s_2 - s_1} \cdot \varepsilon^{-\frac{s}{S_2}}\right) \cdot (\tilde{y}_{ff} - \tilde{y}_{00})$$
(4)

Considering the elements of the Cartesian system, namely p = 0 and q = 0, it follows that s = r corresponds to the height of the column,  $s_0 = 0$  respective  $s_f = 7$  represents the final height of the column.

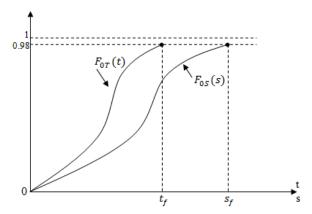


Fig. 1. Evolution of increasing functions.

With  $t = t_0 = 0$  and  $t = t_f = 14$  are noted the initial and final moments of the isotopic separation process.

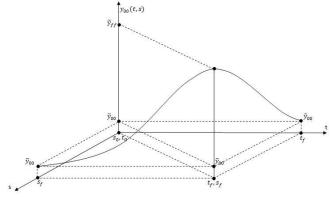


Fig. 2. Representation of the approximating solution

Where with  $\tilde{y}_{00}$  respective  $\tilde{y}_{ff}$  are noted concentrations of the 15N isotope in liquid phase, more precisely:

• 
$$\tilde{y}_{00} = (t_0, s_0) = 0.365\%$$

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•  $\tilde{y}_{ff} = (t_f, s_f) = 8.2\%$ 

The relation represented in (4) is exemplified in Figure 2, this relation becoming:

- $y_{00}(t_0, s_0) = y_{00}(t_0, s_f) = y_{00}(t_f, s_0) = \tilde{y}_{00} = 0.365\%$
- $y_{00}(t_f, s_f) = \tilde{y}_{ff} = 8.2\%$

## II. GENERAL OVERVIEW OF ANALYTICAL MODELING USING THE $\rm M_{PDX}$ METHOD

The simplified representation from Fig. 2 leads to an intuitive interpretation of the evolution  $y_{00}(t,s)$  which represents the concentration of the isotope <sup>15</sup>N in the liquid form, in relation to the time (t) and the propagation height s = r of the separation column. With the approximating solution  $y_{00}(t,s)$  from (1), the partial derivatives expressed in relation (6) lead to much more unitary and systematized expressions than those resulting in  $M_{pdx}$ , presented in [3], directly associated with Cohen's equation.

$$y_{ij} = \frac{\partial^{i+j} y}{\partial t^i \partial s^j} \tag{6}$$

As a result, the following relations will be presented as follows [1], [2], [4]:

$$y_{ij} = F_{iT}(t) \cdot F_{jS}(s) \cdot (\tilde{y}_{ff} - \tilde{y}_{00})$$
(7)
$$F_{iT}(t) = \frac{\partial^{i}F_{0T}(t)}{\partial t^{i}} = (-1)^{i} \left(\frac{1}{T_{1}^{i}} \cdot \frac{T_{1}}{T_{1} - T_{2}} \cdot \varepsilon^{-\frac{t}{T_{1}}} + \frac{1}{T_{2}^{i}} \cdot \frac{T_{2}}{T_{2} - T_{1}} \cdot \varepsilon^{-\frac{t}{T_{2}}}\right)$$
(8)
$$F_{jS}(s) = \frac{\partial^{j}F_{0S}(s)}{\partial s^{i}} = (-1)^{j} \left(\frac{1}{S_{1}^{j}} \cdot \frac{S_{1}}{S_{1} - S_{2}} \cdot \varepsilon^{-\frac{s}{S_{1}}} + \frac{1}{S_{2}^{j}} \cdot \frac{S_{2}}{S_{2} - S_{1}} \cdot \varepsilon^{-\frac{s}{S_{2}}}\right)$$
(9)

The results represented by the relations (7), (8) and (9) allow the partial derivation automatically with propagation indices, which further simplifies the calculations for the new expression of the  $M_{\rm pdx}$ , matrix. It can be shown that the approximate solution  $y_{00}(t,s)$  verifies a second order partial derivatives equation in relation to (t) and (s) such as [5]-[9]:

$$a_{00} \cdot y_{00} + a_{10} \cdot y_{10} + a_{01} \cdot y_{01} + a_{02} \cdot y_{02} + a_{11} \cdot y_{11} + a_{20} \cdot y_{20} = K_{\nu} \cdot \varphi_{0.00} \cdot u_{0}$$

$$(10)$$

where the  $K_y$  represents the proportionality constant,  $\varphi_{0.00} = \varphi_{0.00}(t,s)$ ,  $u_0$  is constant. The coefficients  $(a_{...}...)$  are considered dependent both on time constants on space constants. Since the numerical integration will operate in relation to the time (t), according to which the equation (10) is second order, it results that the state vector consists two variables, respectively [5], [6]:

$$x(2x1) = \begin{array}{c} y_{00} \\ \hline y_{10} \\ \end{array}$$
 (10)

As a result, the new general expression of the matrix of partial derivatives of state vector for the same 1 + M = n + N = 7 become [1]-[4]:

	1	•	<i>M</i> = 6					(11)
	y <sub>60</sub>	У <sub>61</sub>	y <sub>62</sub>	y <sub>63</sub>	y <sub>64</sub>	y <sub>65</sub>	y <sub>66</sub>	Į.
	y <sub>50</sub>	y <sub>51</sub>	y <sub>52</sub>	y <sub>53</sub>	y <sub>54</sub>	y <sub>55</sub>	y <sub>56</sub>	
	y <sub>40</sub>	y <sub>41</sub>	y <sub>42</sub>	y <sub>43</sub>	y <sub>44</sub>	y <sub>45</sub>	y <sub>46</sub>	<i>N</i> = 5
	y <sub>30</sub>	y <sub>31</sub>	y <sub>32</sub>	y <sub>33</sub>	y <sub>34</sub>	y <sub>35</sub>	y <sub>36</sub>	
	y <sub>20</sub>	y <sub>21</sub>	y <sub>22</sub>	y <sub>23</sub>	y <sub>24</sub>	y <sub>25</sub>	y <sub>26</sub>	1
$x_T \mid x_{TS}$	y <sub>10</sub>	y <sub>11</sub>	y <sub>12</sub>	y <sub>13</sub>	y <sub>14</sub>	y <sub>15</sub>	y <sub>16</sub>	1 " "
$I_{dpx} = \begin{array}{c c} x & x_5 \\ \hline \end{array} = \begin{array}{c c} x & x_5 \\ \hline \end{array}$	y <sub>00</sub>	y <sub>01</sub>	y <sub>02</sub>	y <sub>03</sub>	y <sub>04</sub>	y <sub>05</sub>	y <sub>06</sub>	n = 2

The two maximum order of partial derivation related to time (t), more precisely (N) and related to space (s), more precisely (M), are increased in order to provide lower approximation errors for the iterative calculation based on the Taylor Series. The calculation of the vector  $x_T(5x1)$  is obtained from the pivot element  $(y_{20})$  which is derivate by i = 1,2,3, N = 4 in relation to time (t) resulting [7]-[9]:

$$y_{2+i,0} = \frac{1}{a_{20}} [K_y \cdot \varphi_{0.i0} \cdot u_i \cdot (a_{00} \cdot y_{i0} + a_{10} \cdot y_{1+i,0} + a_{01} \cdot y_{i1} + a_{02} \cdot y_{i2} + a_{11} \cdot y_{1+i,1})]$$
(12)

The calculation of the matrix  $x_{ST}(5x1)$  is obtained from the pivot element  $(y_{20})$  which is successively derivate by i = 1,2,3, N = 4 in relation to time (t) and j = 1,2,3,4,5, M = 6 in relation to propagation space (s). As a result, these partial derivatives in relation to (t) and (s) are also obtained from the pivot element in the general form [1], [2]:

$$y_{2+i,j} = \frac{1}{a_{20}} [K_y \cdot \varphi_{0.ij} \cdot u_i \cdot (a_{00} \cdot y_{ij} + a_{10} \cdot y_{1+i,j} + a_{01} \cdot y_{i,i+1} + a_{02} \cdot y_{i,i+2} + a_{11} \cdot y_{1+i,i+1})]$$
(13)

## III. VARIANTS OF APPROXIMATION OF STRUCTURE PARAMETERS BY TRANSCENDENT EQUATIONS

Qualitative evolutions for  $F_{0T}(t)$  and  $F_{0S}(s)$  are presented in the picture below, where  $F_{0T}(t) \rightarrow 1$  and  $F_{0S}(s) \rightarrow 1$  such as:

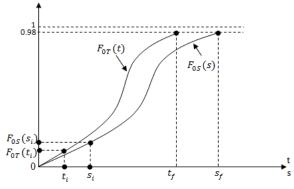


Fig. 3. Qualitative evolution of  $F_{0T}(t)$  and  $F_{0S}(s)$  functions.

The inflection abscises correspond to the following relations [1], [2], [5], [6]:

$$t_i = \frac{T_1 \cdot T_2}{T_2 - T_1} \ln \left( \frac{T_2}{T_1} \right) \tag{14}$$

$$s_i = \frac{S_1 \cdot S_2}{S_2 - S_1} \ln \left( \frac{S_2}{S_1} \right)$$
 (15)

For the relations (2), (3), (14) and (15) the following notations are taken into consideration [1], [2], [5], [6]:

$$\lambda_T = \frac{T_2}{T} > 1 \tag{16}$$

$$\mu_{T} = \frac{t_{f}}{t_{1} + t_{2}} > 1$$

$$\lambda_{S} = \frac{S_{2}}{S_{1}} > 1$$

$$\mu_{S} = \frac{s_{f}}{S_{1} + S_{2}} > 1$$
(17)
(18)

$$\lambda_S = \frac{S_2}{S} > 1 \tag{18}$$

$$\mu_S = \frac{s_f}{s_1 + s_2} > 1 \tag{19}$$

The approximation of these parameters is based on knowing the abscises  $(t_i)$ ,  $(t_f)$  and ordinate  $F_{0T}(t) \rightarrow 1$ resulting the aproximation of both  $T_1$  and  $T_2$ , but also knowing the abscises  $(s_i)$ ,  $(s_f)$  and ordinate  $F_{0S}(t) \rightarrow 1$ resulting the approximation of both  $S_1$  and  $S_2$ . This is based on the numerical solution of the following transcendent equation [1], [2], [5], [6]:

$$F_{0T}(t_f) - \left(1 + \frac{1}{\lambda_{T-1}} \cdot \varepsilon^{coef1} - \frac{\lambda_T}{\lambda_{T-1}} \varepsilon^{coef2}\right) = DIF \quad (20)$$

where:

$$coef1 = -\frac{\lambda_T \cdot ln\lambda_T}{\lambda_T - 1} \cdot \frac{t_f}{t_i}$$

$$coef2 = -\frac{ln\lambda_T}{\lambda_T - 1} \cdot \frac{t_f}{t_i}$$
(21)

$$coef2 = -\frac{ln\lambda_T}{\lambda_{T-1}} \cdot \frac{t_f}{t_i}$$
 (22)

The start of calculations started with  $\lambda_T = 1 + \Delta \lambda_T$  for  $\Delta \lambda_T = 10^{-2} \sim 10^{-4}$  providing iterative increasing for  $(\lambda_T)$ until the difference (DIF) changes its mark, at which time the (DIF) is canceled, and  $(\lambda_T)$  corresponds to the solution of the transcendent equation. From (16), (17), (18) and (19) results the following relations which corresponds to the approximation of time constants  $(T_1)$  and  $(T_2)$  with an error, as low as the step  $(\Delta \lambda_T)$ . [1], [2]:

$$\mu_T = \frac{\lambda_T ln \lambda_T}{\lambda_T^2 - 1} \cdot \frac{t_f}{t_i} \tag{23}$$

$$T_1 = \frac{t_f}{\mu_T(1+\lambda_T)} \tag{24}$$

$$T_2 = \lambda_T \cdot T_1 \tag{25}$$

The program developed for the numerical integration of transcendental equations is presented in the following [1], [2]:

<u>L1</u>:  $t_f = 25$ ,  $F_{0Tf} = 0.979$ ,  $t_i = 3$ ,  $F_{AS} = 230$ ,  $F_{0Ti} =$ informative

L2: 
$$\Delta \lambda = 0.001$$
,  $\lambda = 1 + \Delta \lambda$ ,  $\lambda_{max} = 20$ 

$$\begin{split} \text{L3: coef1} &= -\frac{\lambda \text{ln}\lambda}{\lambda - 1} \cdot \frac{t_f}{t_i} \ \text{coef2} = -\frac{\text{ln}\lambda}{\lambda - 1} \cdot \frac{t_f}{t_i} \\ \text{L4: DIF1} &= \left[ F_{0T}(t_f) - \left( 1 + \frac{1}{\lambda_{T-1}} \cdot \epsilon^{\text{coef1}} - \frac{\lambda_T}{\lambda_{T-1}} \epsilon^{\text{coef2}} \right) \right] \cdot \\ F_{AS} \\ \text{L5: } \lambda &= \lambda + \Delta \lambda \\ \text{L6: coef1} &= -\frac{\lambda \text{ln}\lambda}{\lambda - 1} \cdot \frac{t_f}{t_i} \ \text{coef2} = -\frac{\text{ln}\lambda}{\lambda - 1} \cdot \frac{t_f}{t_i} \end{split}$$

L6: coef1 = 
$$-\frac{1}{\lambda - 1} \cdot \frac{1}{t_i}$$
 coef2 =  $-\frac{1}{\lambda - 1} \cdot \frac{1}{t_i}$   
L7: DIF2 =  $[F_{0T}(t_f) - \left(1 + \frac{1}{\lambda_{T-1}} \cdot \epsilon^{\text{coef1}} - \frac{\lambda_{T}}{\lambda_{T-1}} \epsilon^{\text{coef2}}\right)]$ 

L8:  $DIF = DIF1 \cdot DIF2$ L9: IF DIF < 0 goto L12

L10:If  $\lambda \ge \lambda_{max}$  goto L16

L11:goto L3

L12: 
$$\mu = \frac{\lambda \ln \lambda}{\lambda^2 - 1} \cdot \frac{t_f}{t_i}$$
,  $T_1 = \frac{t_f}{\mu(1 + \lambda)}$ ,  $T_2 = \lambda \cdot T_1$ 

$$\begin{split} &L13 \colon F_{0Tf}(t) = 1 - \frac{T_1}{T_1 - T_2} \cdot \epsilon^{-\frac{t_f}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \epsilon^{-\frac{t_f}{T_2}} \\ &L14 \colon F_{0Ti}(t) = 1 - \frac{T_1}{T_1 - T_2} \cdot \epsilon^{-\frac{t_i}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \epsilon^{-\frac{t_i}{T_2}} \end{split}$$

L14: 
$$F_{0Ti}(t) = 1 - \frac{T_1}{T_1 - T_2} \cdot \epsilon^{-\frac{t_i}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \epsilon^{-\frac{t_i}{T_2}}$$

L15: Print  $\mu$ ,  $T_1$ ,  $T_2$ ,  $F_{0Tf}$ ,  $F_{0Ti}$ 

L16: STOP

L17: 
$$t = 0$$
,  $\Delta t = t_f/10$ 

L18: 
$$F_{0T}(t) = 1 - \frac{T_1}{T_1 - T_2} \cdot \varepsilon^{-\frac{t}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \varepsilon^{-\frac{t}{T_2}}$$

$$L19: y_{00} = F_{0T} \cdot F_{AS}$$

L20: 
$$F_{1T}(t) = \frac{1}{T_1 - T_2} \cdot \epsilon^{-\frac{t}{T_1}} - \frac{1}{T_2 - T_1} \cdot \epsilon^{-\frac{t}{T_2}}$$

$$L19. y_{00} - F_{0T} \cdot F_{AS}$$

$$L20: F_{1T}(t) = \frac{1}{T_1 - T_2} \cdot \varepsilon^{-\frac{t}{T_1}} - \frac{1}{T_2 - T_1} \cdot \varepsilon^{-\frac{t}{T_2}}$$

$$L21: F_{2T}(t) = \frac{1}{T_1(T_1 - T_2)} \cdot \varepsilon^{-\frac{t}{T_1}} - \frac{1}{T_1(T_2 - T_1)} \cdot \varepsilon^{-\frac{t}{T_2}}$$

L22: IF  $t \ge t_f + \Delta t$  goto L25

L23: Print t,  $F_{0T}$ ,  $F_{1T}$ ,  $F_{2T}$ ,  $y_{00}$ 

L24:  $t = t + \Delta t$  goto L18

L 25: STOP

L26: END

The transcendent equation calculates  $T_1$  and  $T_2$  from  $t_i$ ,  $t_f$ ,  $F_{0T}(t_f)$  respectively  $F_{AS}$ , the value of the asymptote. The corresponding curves are shown in the figure below:

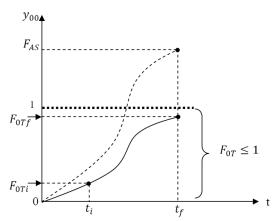


Fig. 4. Evolution of  $F_{0Ti}(t)$  and  $F_{0Tf}(t)$  function of the value of the asymptote.

Formally identical, space constants are also approximated resulting the following equations:

$$F_{0S}(s_f) - \left(1 + \frac{1}{\lambda_{S^{-1}}} \cdot \varepsilon^{coef1} - \frac{\lambda_S}{\lambda_{S^{-1}}} \varepsilon^{coef2}\right) = DIF \quad (26)$$

$$coef1 = -\frac{\lambda_{S} \cdot ln\lambda_{S}}{\lambda_{S} - 1} \cdot \frac{s_{f}}{s_{i}}$$
 (27)

$$coef2 = -\frac{ln\lambda_S}{\lambda_{S}-1} \cdot \frac{s_f}{s_i}$$
 (28)

$$\mu_S = \frac{\lambda_S \ln \lambda_S}{\lambda_S^2 - 1} \cdot \frac{s_f}{s_i} \tag{29}$$

$$S_1 = \frac{s_f}{\mu_S(1+\lambda_S)} \tag{30}$$

$$S_2 = \lambda_S \cdot S_1 \tag{31}$$

### IV. CORRECTION OF STRUCTURAL PARAMETERS IN RELATION TO PROPAGATION DEPTH

For the usual propagation case with stationary values  $y_{\alpha} = y_{00}(t_{f\alpha}, s_0)$  and  $y_{\beta} = y_{00}(t_{f\beta}, s_f)$ , it is observed that  $t_{f\beta} > t_{f\alpha}$ . As a result, for this propagation inertia, which is increasing at the output  $s_f$  in comparison to the input  $s_0$ , it follows that for the time constants the following relation is noted [1], [2], [5], [6]:

$$T_{1\beta} + T_{2\beta} > T_{1\alpha} + T_{2\alpha}$$
 (32)

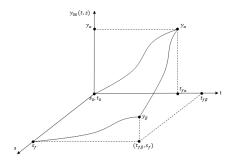


Fig. 5. Evolution of structural parameters.

The increasing trend for  $T_1$ , which is also formally identical for  $T_2$ , is approximated either as a dotted line, like the relation (33) or as a full exponential, like (34), as follows:

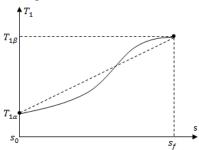


Fig. 6. Increasing evolution of  $T_1$ 

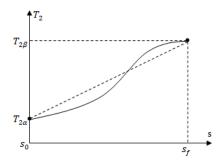


Fig. 7. Increasing evolution of  $T_2$  $T_1 = J_0 + J_1 \cdot s$ 

$$T_1 = J_0 + J_1 \cdot \varepsilon^{-\frac{s}{S_1}} + J_2 \cdot \varepsilon^{-\frac{s}{S_2}}$$
 (34)

From the conditions that the relations (33) and (34) verify the values  $T_1(s_0) = T_{1\alpha}$  and  $T_1(s_f) = T_{1\beta}$ , but also the values  $T_2(s_0) = T_{2\alpha}$  and  $T_2(s_f) = T_{2\beta}$  results the following approximations represented in the relations, where (35) and (36) are linear approximations and (37) and (38) are exponentially approximations[1], [2], [5], [6]:

$$T_1 = T_{1\alpha} + \frac{T_{1\beta} - T_{1\alpha}}{s_f} \cdot s$$
 (35)

$$T_2 = T_{2\alpha} + \frac{T_{2\beta} - T_{2\alpha}}{s_f} \cdot s \tag{36}$$

$$T_1 = T_{1\alpha} + \frac{T_{1\beta} - T_{1\alpha}}{\frac{s_f}{s_{1+s}} - \frac{s_f}{s_2}} \cdot \varepsilon^{-\frac{s}{s_1}}$$

$$(37)$$

$$T_2 = T_{2\alpha} + \frac{T_{2\beta} - T_{2\alpha}}{\sum_{\epsilon=1}^{s} - \frac{s}{S_2}} \cdot \varepsilon^{-\frac{s}{S_2}}$$
 (38)

However, for both approximations, linear or exponential, the same approximate solution remains valid [1], [2], [5], [6]:

$$y_{00}(t,s) = K_y \cdot \left(1 - \frac{T_1}{T_1 - T_2} \cdot \varepsilon^{-\frac{t}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \varepsilon^{-\frac{t}{T_2}}\right) [y_\beta + t]$$

$$(y_{\alpha} - y_{\beta}) \cdot (\frac{S_1}{S_1 - S_2} \cdot \varepsilon^{-\frac{S}{S_1}} - \frac{S_2}{S_2 - S_1} \cdot \varepsilon^{-\frac{S}{S_2}})]$$
 (39)

If  $t_{f\beta}=t_{f\alpha}$  then  $T_{1\beta}=T_{1\alpha}$  and  $T_{2\beta}=T_{2\alpha}$  which emphasise the same propagation inertia in relation with time t, both on input  $s=s_0$  and on output  $s=s_f$ .

Through a specialized detail for  $F_{0T}(t)$  and  $F_{0S}(s)$  with more than two time and space constants, the achieved performances for the concentration  $y_{00}(t,s)$  are very close to the results experimentally found in the literature.

#### V. CONCLUSION

In this article, the presented process is an isotope separation column for  $^{15}N$ , which represents a space propagation process modeled by the proposed method, more precisely a new approach of Cohen's equation, by a second-order differential equation in relation to time t and also second-order in relation to the spatial variable s.

At the initial stage of development, for the partial derivative equation of second order in relation to time t and space constant s, the variables  $F_{0T}(t)$  and  $F_{0S}(s)$  are limited at two, where time constants are  $(T_1, T_2)$  and space constants are  $(S_1, S_2)$ , in this way resulting a family of curves of the  $^{15}N$   $y_{00}(t, s)$  concentration.

#### REFERENCES

- [1] T. Coloşi, M. L. Ungureşan, E. H. Dulf, and R. C. Cordoş, "Introduction to analogical modeling and numerical simulation with (Mpdx) and taylor series distributed parameters processes," *Editura Galaxia Gutenberg*, Cluj-Napoca, Romania, 2009
- [2] T. Coloşi, M. Abrudean, M. L. Ungureşan, and V. Mureşan, "Numerical simulation of distributed parameters processes," Springer, 2013

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- [3] D. Sas and Z. Kovendi, "Preliminaries in (Mpdx) method associated with Cohen equations," in Proc. 2016 IEEE International Conference on Automation, Quality and Testing, Robotics (AQTR) Cluj-Napoca, Romania
- [4] T. Coloşi, M. Abrudean, M. L. Ungureşan, and V. Mureşan, "PID control of distributed parameter processes through the (M<sub>pdx</sub>) method with approximating solutions," in *Proc. Conference of Information Technologies and Control Methods with Power Systems, Cluj-Napoca*, Romania. 2013.
- [5] D. Sas and M. Secara, "Preliminaries in numerical simulation of isotope separation closed-loop processes," in Proc. 2015 International Conference on Engineering of Modern Electric Systems (EMES) Oradea, Romania.
- [6] T. Colosi, E. Dulf, and M. Abrudean, "Modeling and numerical simulation of PDE using the (Mdpx) matrix, associated to taylor series," in *Proc. 2006 IEEE International Conference on Automation, Quality* and Testing, Robotics, vol. 1, May 2006.
- [7] D. C. Dumitrache, I. Inoan, and B. D. Schutter, "An analytic model for a 13C isotope separation process by cryogenic distillation," *Journal of Process Control*, vol. 24, no. 5, 2014, pp. 463-474.
- [8] Z. Kovendi, V. Mureşan, M. Abrudean, I. Clitan, M. L. Ungureşan, and T. Coloşi, "Modeling a chemical exchange process for the <sup>13</sup>C isotope enrichment," in *Proc.* 10<sup>th</sup> edition of the International Concerence on Aerospace, Robotics, Mechanical Engineering, Manufacturing Systems, Biomechanics and Robots for Neurorehabilitation (OPTIROB), 27-29 Iunie, 2015, Jupiter, România, Applied Mechanics and Materials, vol. 772, 2015, pp. 27-32.
- [9] I. Clitan, V. Muresan, T. Colosi, and M. Unguresan, "Control structure design for an 18 O isotope separation column," in *Proc. 20th International Conference on Automation, Quality and Testing, Robotics (AQTR 2016).*



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