

Preliminaries of Structural Parameters Approximation Through Transcendence Equations for an ^{15}N Isotope Separation Column

Muresan V., Sas D., Clitan I., and Unguresan M.-L.

Abstract—The ^{15}N isotope separation column process presented in this article represents a space propagation process modeled by second-order quasi-linear equations with two independent variables. The new approach of Cohen's equation using a second-order differential equation in relation to time variable t and spatial variable s is presented. For this stage of development, for variables $F_{0T}(t)$ and $F_{0S}(s)$ there is a limitation of two time constants (T_1, T_2), or space constants (S_1, S_2). Also, $F_{0T}(t)$ and $F_{0S}(s)$ evolutions are exponentially increasing, resulting that the phenomenon is cumulative, both in relation to time and in relation to the propagation space. In this case, results a family of curves of the ^{15}N $y_{00}(t, s)$ concentration.

Index Terms—Isotopic separation process, cohen's equation, transcendent equations, structural parameters approximation.

I. INTRODUCTION

The space propagation process presented in this article represents an ^{15}N isotope separation column process which is modeled by second-order quasi-linear equations with two independent variables.

Using a second-order differential equation in relation with both time and space variables, a new approach of Cohen's equation is presented.

In the following equation, it is considered the overdamped version of approximating solutions associated with the isotopic separation phenomena [1]-[3]:

$$y_{00}(t, s) = \tilde{y}_{00} + F_{0T}(t) \cdot F_{0S}(s) \cdot (\tilde{y}_{ff} - \tilde{y}_{00}) \quad (1)$$

For this stage of development, for variables $F_{0T}(t)$ and $F_{0S}(s)$ there is a limitation of two time constants (T_1, T_2), or space constants (S_1, S_2). The evolution of $F_{0T}(t)$ and $F_{0S}(s)$ is represented in the following figure:

Also, $F_{0T}(t)$ and $F_{0S}(s)$ evolutions are exponentially increasing, resulting that the phenomenon is cumulative, both in relation to time and in relation to the propagation space. In this case, results a family of curves of the ^{15}N $y_{00}(t, s)$ concentration.

Considering the final values $F_{0T}(t) \rightarrow 1$ and $F_{0S}(s) \rightarrow 1$

[1]-[5] the ascending exponential functions are represented as follows:

$$F_{0T}(t) = 1 - \frac{T_1}{T_1 - T_2} \cdot e^{-\frac{t}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot e^{-\frac{t}{T_2}} \quad (2)$$

$$F_{0S}(s) = 1 - \frac{S_1}{S_1 - S_2} \cdot e^{-\frac{s}{S_1}} - \frac{S_2}{S_2 - S_1} \cdot e^{-\frac{s}{S_2}} \quad (3)$$

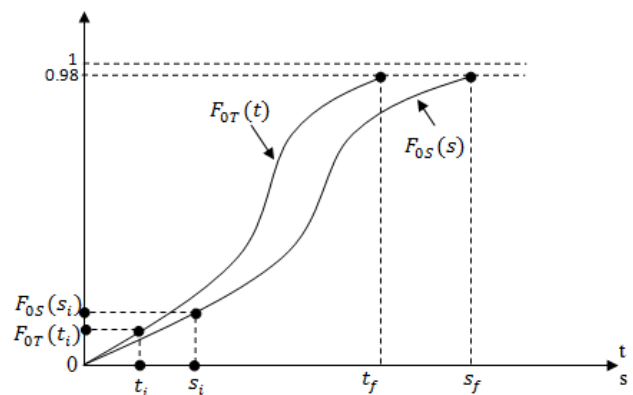


Fig. 1. Increasing evolution for $F_{0T}(t)$ and $F_{0S}(s)$ [3].

The approximating solution represented in relation (1) may be rewritten considering relations (2) and (3) such as [1]-[5]:

$$y_{00}(t, s) = \tilde{y}_{00} + \left(1 - \frac{T_1}{T_1 - T_2} \cdot e^{-\frac{t}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot e^{-\frac{t}{T_2}}\right) \cdot \left(1 - \frac{S_1}{S_1 - S_2} \cdot e^{-\frac{s}{S_1}} - \frac{S_2}{S_2 - S_1} \cdot e^{-\frac{s}{S_2}}\right) \cdot (\tilde{y}_{ff} - \tilde{y}_{00}) \quad (4)$$

The inflection abscises correspond to the following relations [1]-[3], [5]-[7]:

$$t_i = \frac{T_1 \cdot T_2}{T_2 - T_1} \ln\left(\frac{T_2}{T_1}\right) \quad (5)$$

$$s_i = \frac{S_1 \cdot S_2}{S_2 - S_1} \ln\left(\frac{S_2}{S_1}\right) \quad (6)$$

In addition, it can also be shown that for the decreasing evolutions of $F_{0T}(t)$ and $F_{0S}(t)$ defined by the relations (7) and (8) and exemplified in figure 2, where $F_{0T}(t) \rightarrow 0$ and $F_{0S}(t) \rightarrow 0$, the inflection abscises t_i respectively s_i correspond to those defined in the relations (5) and (6) [1-2].

$$F_{0T}(t) = \frac{T_1}{T_1 - T_2} \cdot e^{-\frac{t}{T_1}} + \frac{T_2}{T_2 - T_1} \cdot e^{-\frac{t}{T_2}} \quad (7)$$

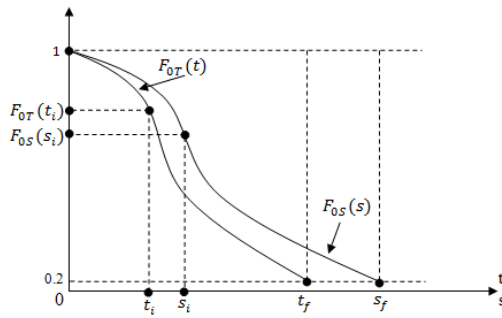
$$F_{0S}(s) = \frac{S_1}{S_1 - S_2} \cdot e^{-\frac{s}{S_1}} + \frac{S_2}{S_2 - S_1} \cdot e^{-\frac{s}{S_2}} \quad (8)$$

Further, in this paper it will be presented only the case for increasing evolution of the $F_{0T}(t)$ and $F_{0S}(s)$.

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 Fig. 2. Decreasing evolution for $F_{0T}(t)$ and $F_{0S}(s)$

II. APPROXIMATION OF STRUCTURAL PARAMETERS BY TRANSCENDENT EQUATIONS

Considering the relations (2), (3), (5) and (6) there are taken into consideration the following notations [1]-[3], [6]-[8]:

$$\lambda_T = \frac{T_2}{T_1} > 1 \quad (9)$$

$$\mu_T = \frac{t_f}{T_1 + T_2} > 1 \quad (10)$$

$$\lambda_S = \frac{S_2}{S_1} > 1 \quad (11)$$

$$\mu_S = \frac{s_f}{S_1 + S_2} > 1 \quad (12)$$

Based on knowing the abscises (t_i), (t_f) and ordinate $F_{0T}(t) \rightarrow 1$, but also the abscises (s_i), (s_f) and ordinate $F_{0S}(s) \rightarrow 1$ the approximating parameters noted above determine the approximation of both T_1 and T_2 , respectively S_1 and S_2 . It will be considered the following transcendent equation [1]-[3], [6], [7]:

$$F_{0S}(s_f) - \left(1 + \frac{1}{\lambda_S - 1} \cdot \varepsilon^{coef1} - \frac{\lambda_S}{\lambda_S - 1} \varepsilon^{coef2}\right) = DIF \quad (13)$$

where:

$$coef1 = -\frac{\lambda_S \ln \lambda_S}{\lambda_S - 1} \cdot \frac{s_f}{s_i} \quad (14)$$

$$coef2 = -\frac{\ln \lambda_S}{\lambda_S - 1} \cdot \frac{s_f}{s_i} \quad (15)$$

Calculations start with $\lambda_S = 1 + \Delta\lambda_S$ for $\Delta\lambda_S = 10^{-2} \sim 10^{-4}$ providing iterative increasing for (λ_S) until the difference (DIF) changes its mark and (λ_S) fulfills to the solution of the transcendent equation.

The following relations corresponds to the approximation of space constants (S_1) and (S_2) with an error, as low as the step ($\Delta\lambda_S$) is [1]-[3]:

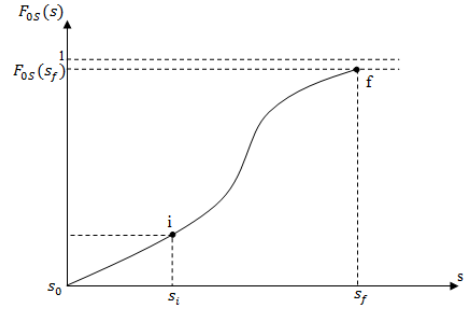
$$\mu_S = \frac{\lambda_S \ln \lambda_S}{\lambda_S^2 - 1} \cdot \frac{s_f}{s_i} \quad (16)$$

$$S_1 = \frac{s_f}{\mu_S(1 + \lambda_S)} \quad (17)$$

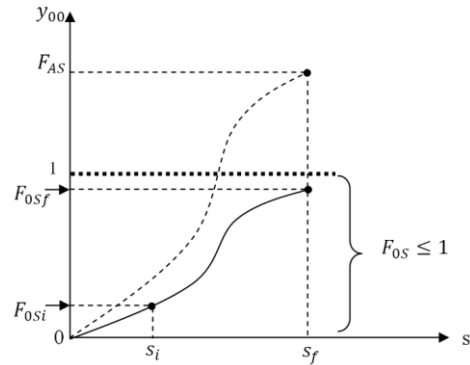
$$S_2 = \lambda_S \cdot S_1 \quad (18)$$

At the same time, it can be shown that there is only one value (λ_S) for which the curve represented in the following

figure passes simultaneously through the two points (i) and (f), where for (f) corresponds $F_{0S}(s) \rightarrow 1$, such as $F_{0S}(s) = 0.95 \div 0.99$.


 Fig. 3. Evolution of $F_{0S}(s)$

The transcendent equation calculates S_1 and S_2 from s_i , s_f , $F_{0S}(s_f)$ respectively F_{AS} , the value of the asymptote. The corresponding curves are shown in the figure below:


 Fig. 4. Evolution of $F_{0Si}(s)$ and $F_{0Sf}(s)$ function of the value of the asymptote.

The program developed for the numerical integration of transcendental equations is presented in the following [1]-[3]:

L1: s_f = given, F_{0Sf} = given, s_i = given, F_{AS} = given, F_{0Si} = informative

L2: $\Delta\lambda = 0.001$, $\lambda = 1 + \Delta\lambda$, $\lambda_{\max} = 20$

L3: $coef1 = -\frac{\lambda \ln \lambda}{\lambda - 1} \cdot \frac{s_f}{s_i}$ $coef2 = -\frac{\ln \lambda}{\lambda - 1} \cdot \frac{s_f}{s_i}$

L4: $DIF1 = [F_{0S}(s_f) - \left(1 + \frac{1}{\lambda_S - 1} \cdot \varepsilon^{coef1} - \frac{\lambda_S}{\lambda_S - 1} \varepsilon^{coef2}\right)] \cdot F_{AS}$

F_{AS}

L5: $\lambda = \lambda + \Delta\lambda$

L6: $coef1 = -\frac{\lambda \ln \lambda}{\lambda - 1} \cdot \frac{s_f}{s_i}$ $coef2 = -\frac{\ln \lambda}{\lambda - 1} \cdot \frac{s_f}{s_i}$

L7: $DIF2 = [F_{0S}(s_f) - \left(1 + \frac{1}{\lambda_S - 1} \cdot \varepsilon^{coef1} - \frac{\lambda_S}{\lambda_S - 1} \varepsilon^{coef2}\right)] \cdot F_{AS}$

F_{AS}

L8: $DIF = DIF1 \cdot DIF2$

L9: IF $DIF < 0$ goto L12

L10: If $\lambda \geq \lambda_{\max}$ goto L16

L11: goto L3

The correction of the time constants T_1 and T_2 in relation to the propagation depth s becomes variable also in this case due to the fact that from Fig. 5 it is observed that the evolution of the approximating solution $y_{00}(t, s_f)$ presents a delay from the final moment $t_{f\alpha}$ to the final moment $t_{f\beta}$.

Since $t_{f\beta} > t_{f\alpha}$ is due to the progressive increase of time constants T_1 and T_2 in relation to propagation depth s , it is justified to use corrections.

IV. APPROXIMATION OF STRUCTURE PARAMETERS BY TABULAR METHOD

The tabular method may be considered an ideal method whenever experimentally can be completed tables such as the one below, both from t_0 to t_f and from s_0 to s_f , either for a real process or for an approximating solution, which corresponds, for example, with the relation (23) associated with figure 5, respectively [1]-[5]:

$$y_{00}(t, s) = \tilde{y}_{00} + F_{0T}(t) \cdot F_{0S}(s) \cdot (\tilde{y}_{ff} - \tilde{y}_{00}) \quad (23)$$

TABLE I: STRUCTURAL PARAMETERS IN TABULAR FORM

$s \backslash t$	t_0	...	t_k	...	t_f
s_0	y_{00}	...	y_{k0}	...	y_{f0}
...
s_j	y_{0j}	...	y_{kj}	...	y_{fj}
...
s_f	y_{0f}	...	y_{kf}	...	y_{ff}

From Table I, for the line $j = \text{constant}$ the following curve could be represented [4], [5]:

$$y_{00}(t, s_j) = \tilde{y}_{00} + F_{0T}(t) \cdot F_{0S}(s_j) \cdot (\tilde{y}_{ff} - \tilde{y}_{00}) \quad (24)$$

Also, for column $k = \text{constant}$ the following curve could be represented [4], [5]:

$$y_{00}(t_k, s) = \tilde{y}_{00} + F_{0T}(t_k) \cdot F_{0S}(s) \cdot (\tilde{y}_{ff} - \tilde{y}_{00}) \quad (25)$$

where $\tilde{y}_{00} = y_{00}(t_0, s_0)$ and $\tilde{y}_{ff} = y_{00}(t_f, s_f)$

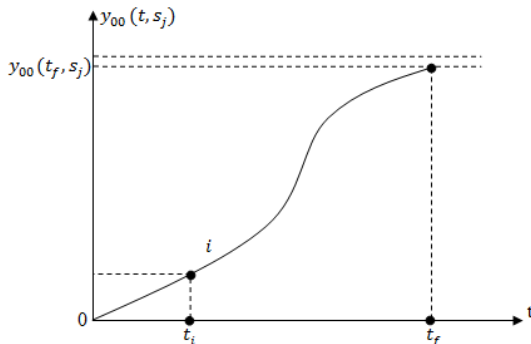


Fig. 6. Evolution of the approximating solution.

Thus, it can be observed that from the relations (24) and

(25) the following figures may be represented. From Fig. 6 result time constants $T_1(s_j)$ and $T_2(s_j)$ meanwhile from Fig. 7 result space constants $S_1(t_k)$ and $S_2(t_k)$ but also $y_{00}(t_k, s_j)$.

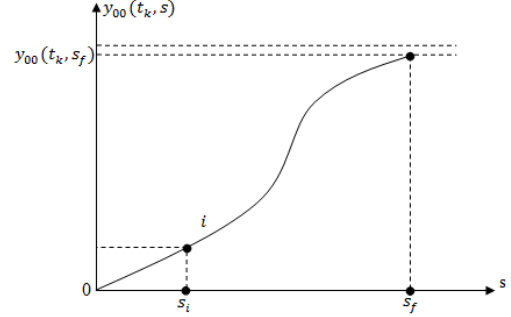


Fig. 7. Evolution of the approximating solution

Due to the fact that from both figures was obtained the triplet of values $[t_i, t_f, (\tilde{y}_{ff} - \tilde{y}_{00})]$ and $[s_i, s_f, (\tilde{y}_{ff} - \tilde{y}_{00})]$ in the following it is still possible to use the transcendent equation method for ascending evolutions.

As a result, from $[t_i, t_f, (\tilde{y}_{ff} - \tilde{y}_{00})]$ are obtain $T_1(s_j)$ and $T_2(s_j)$ and from $[s_i, s_f, (\tilde{y}_{ff} - \tilde{y}_{00})]$ are obtain $S_1(t_k)$ and $S_2(t_k)$. Finally, an analytical solution or approximating solution can be obtained in the vicinity of the point (t_k, s_j) such as [1]-[5]:

$$y_{00}(t, s) = \tilde{y}_{00} + \left(1 - \frac{T_1}{T_1 - T_2} \cdot \varepsilon^{\frac{t}{T_1}} - \frac{T_2}{T_2 - T_1} \cdot \varepsilon^{\frac{t}{T_2}}\right) \cdot \left(1 - \frac{S_1}{S_1 - S_2} \cdot \varepsilon^{\frac{s}{S_1}} - \frac{S_2}{S_2 - S_1} \cdot \varepsilon^{\frac{s}{S_2}}\right) \cdot (\tilde{y}_{ff} - \tilde{y}_{00}) \quad (26)$$

where $T_1 = T_1(s_j)$, $T_2 = T_2(s_j)$, $S_1 = S_1(t_k)$, $S_2 = S_2(t_k)$.

Considering the figures 6 and 7, from $t_i = 0$ days to $t_f = 14$ days, and from $s_i = 0$ to $s_f = 7$ meters, both $F_{0T}(t)$ and $F_{0S}(s)$ evolutions are exponentially increasing, resulting that the phenomenon is cumulative, both in relation to time and in relation to the propagation space.

It should be mentioned the fact that for possible corrections, the expressions $F_{0T}(t)$ and $F_{0S}(s)$ can be completed with formally identical components $\frac{t_i}{t_i - t_{i-1}} \varepsilon$ respectively $\frac{s_i}{s_i - s_{i-1}} \varepsilon$, of a number of times, so the final evolution $y_0(t, s)$ to become almost overlay on the rigorous evolution investigated in the literature.

V. CONCLUSION

A space propagation process is modeled by second-order quasi-linear equations with two independent variables, one in relation to time t and the other one in relation to propagation space s .

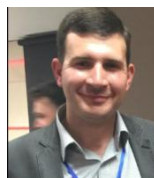
By using Cohen's equation with a new approach, more precisely using a second-order differential equation in relation to time variable t and spatial variable s , it is modeled an ^{15}N isotope separation column process.

For both $F_{0T}(t)$ and $F_{0S}(s)$ is a limitation of two time constants (T_1, T_2) , or space constants (S_1, S_2) , at this stage of development. For possible corrections, both expressions $F_{0T}(t)$ and $F_{0S}(s)$ should be fulfilled with formally identical components $\frac{t_i}{t_i - t_{i-1}} \varepsilon$ respectively $\frac{s_i}{s_i - s_{i-1}} \varepsilon$, so the final evolution $y_0(t, s)$ to achieve a better accuracy.

Also, $F_{0T}(t)$ and $F_{0S}(s)$ evolutions are exponentially increasing, resulting a family of curves of the ^{15}N $y_{00}(t, s)$ concentration, taking into consideration the fact that the phenomenon is cumulative, both in relation to time and in relation to the propagation space.

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