

H-Infinity Design for Automatic Landing System

Serena Cristiana Voicu and Florentin Alin Buțu

Abstract—This study focuses on the H-inf control of aircraft during landing, our aim being also to design a robust control law for a wind shear profile using the H-inf control. This method is based on minimizing a closed loop transfer function norm in order to obtain the stability and robust performance of the system. Some of the design objectives are suggested in other works, but the numerical case study and the numerical implementation of the design procedure are performed and presented in this paper for the first time.

Index Terms—Landing, H-inf, control law, wind shear.

I. INTRODUCTION

Considering the very low visibility conditions in winter in North-West Europe, Great Britain developed seven decades ago, for the first time, a new system, called automatic landing. At these times, visibility could become as low as a few feet and affected the main UK centres, airports such as Heathrow, Gatwick, Birmingham, Glasgow and European cities such as Paris, Brussels, Amsterdam and Milan. After 1945, two state-owned airline corporations were founded-British European Airways (BEA) and British Overseas Airways Corporation (BOAC). During the immediate post-war period, BEA suffered a several number of accidents during landing in conditions of low visibility. That became a very important problem for the pilots: how they could land safely in such conditions. The development of an automatic landing system was international, but the basis of this creation was developed by the Blind Landing Experimental Unit-BLEU. The design of such a system is based on radio navigation, called today Instrument Landing System (ILS) accepted as a standard system in 1947. It provides the vertical and horizontal guidance for a safe landing in conditions of reduced visibility consisting of a number of phases. This kind of system is used to descend along the runway at a glide path of 3° maintaining the aircraft onto the runway centre-line. At 30 ft (9 m) above the ground, the aircraft pitches up to reduce the vertical speed [1]-[4]. There are various categories according to the conditions of visibility which determine the minimum altitude at which the system may be used. The landing categories depend on the decision height (DH), which means the minimum altitude, and the runway

visual range (RVR). The first category, CAT I, specifies a decision height not lower than 200 ft (60 m) and a minimal visibility 1800 ft (550 m). If the runway visibility range is smaller than 1800 ft (550 m) or the runway is not in sight, the pilot is required to execute a missed approach and repeat the landing or divert to an alternative. For CAT II, the minimal decision height is 100 ft (30 m) and the visibility of the runway is minimum 1200 ft (360 m). CAT III is divided into three sub-categories: CAT IIIa, CAT IIIb and CAT IIIc. CAT IIIa allows the pilot to take over control at the point of touchdown making an automatic landing. The minimal decision height is lower than 100 ft (30 m) and the RVR is minimum 700 ft (215 m). For CAT IIIb, the difference is that the pilot takes over control at some distance after touchdown with DH lower than 50 ft (15 m) and RVR 150 ft (45 m). CAT IIIc provides zero visibility, thus the landing is totally automatic but no system has yet been certified to be able to provide such a performance [5]-[10].

This report brings the formulation of the longitudinal dynamics of a transport class aircraft in conditions of wind shear and turbulence. The components of the wind shear are included in the longitudinal dynamic model. Wind shear refers to a difference in wind velocity over short distances which can rapidly cause a lift variation, hence an altitude variation, of the aircraft [11]-[14]. Wind shear is defined as having two components: horizontal and vertical velocity which can affect the aircraft simultaneously. The vertical shear has the most dangerous effect: the change in velocity determines a change in the indicated speed and in thrust requirements. Supposing that the aircraft is using an instrument landing system approach and it is affected by wind shear, it causes a loss of lift. The aircraft is descending and dropping below the glide path. The pilot has to recover this loss of airspeed increasing pitch and power. A very important factor is whether there is enough altitude to correct the path. Therefore, if there is sufficient altitude to recover the desired airspeed and glide path, the pilot is easily able to land safely [2]. The most dangerous forms of wind shear are the microburst and downburst, which have a very short duration of about 3-5 minutes. When wind shear is present at ground level, at the altitude of the runways, it can produce serious disasters changing its airspeed during landing. A serious aircraft landing accident, also presented in [8], occurred on approach to John F. Kennedy International Airport in June 1975 when 113 of the 124 passengers died, accident caused by a microburst. The most recent air catastrophe was also caused by microburst and lack of information on hazardous consequences of the wind shear killing 136 passengers on board on Dallas/Fort Worth International Airport in August 1985. This study focuses on treating this problem using an advanced technique to achieve the proper results. The problem is to design a robust control law for a wind shear profile using H infinity theory. This method is based on minimizing a closed loop transfer function norm to

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obtain the stability and robust performance of the system. Some of the design objectives are suggested in [7] but the numerical case study and numerical implementation of the design procedure are performed and firstly presented in this paper.

The paper is structured in several sections. The second part of this work contains preliminaries and problem formulation. After presenting the generalized system design model, the third part will describe design methods with a case study – the Boeing 737 model- in the last section.

II. PRELIMINARIES AND PROBLEM FORMULATION

The general system model used for this problem formulation contains the so called *generalized plant* $T(s)$ and the controller $K(s)$. $T(s)$ includes both the dynamics of the aircraft and the wind shear equations.

A. Plant Model

The glideslope tracking problem was formulated for Boeing 737 aircraft, a similar one to which was involved in the accident on approach to John F. Kennedy International Airport in 1975. The following conditions are used for the mathematical model proposed.

glideslope:	3 degrees
airspeed:	125 knots (211.593 ft/sec)
flaps:	40 degrees
weight:	80,000 lbs
centre of gravity:	25% of the MAC (mean aerodynamic chord)

The state equations for the plant model, $P(s)$ are defined below:

$$\dot{x} = Ax + Bu + B_w d_w, \quad (1)$$

$$y = Cx + Du + D_w d_w + \eta, \quad (2)$$

where:

$$x = [u \quad w \quad q \quad \theta \quad u_w \quad w_w \quad h]^T, \quad (3)$$

$$u = [\delta_e \quad \delta_T]^T, \quad (4)$$

$$d_w = [u_{wg} \quad w_{wg}]^T, \quad (5)$$

$$y = [V_a \quad q \quad \theta \quad h \quad \dot{h}]^T, \quad (6)$$

$$\eta = [\eta_{V_a} \quad \eta_q \quad \eta_\theta \quad \eta_h \quad \eta_{\dot{h}}]^T. \quad (7)$$

The first four states of vector x are the longitudinal states (u and w represent the longitudinal velocity in the x and z directions of the body axes, pitch rate (q), and pitch attitude (θ), u_w and w_w are the longitudinal and vertical components of the wind velocity and h is the altitude of the aircraft along the glideslope. The two components of the wind vary linearly with respect to decreasing altitude by the horizontal and vertical gradients U_z and W_z . The wind turbulence model is represented by d_w with its horizontal and vertical wind gust components: u_{wg} and w_{wg} . The vector u consists of the elevator deflection (δ_e) and the engine thrust (δ_T). The measurement vector, y , has five components: the airspeed (V_a), pitch rate (q), pitch attitude (θ), the aircraft altitude (h), and altitude rate (\dot{h}). The vector η is the measurement noise.

B. Actuator Model

The state model associated with the actuator dynamics is represented as follows:

$$\dot{x}_A = A_A x_A + B_A u_c, \quad (8)$$

$$u_c = C_A x_A, \quad (9)$$

where:

$$x_A = [\delta_e \quad \delta_{th} \quad \delta_T]^T, \quad (10)$$

$$u_c = [\delta_{e_c} \quad \delta_{th_c}]^T. \quad (11)$$

The vector x_A consists of the elevator dynamics, throttle dynamics and engine thrust dynamics. The vector u_c includes the elevator position command (δ_{e_c}) and the throttle position command (δ_{th_c}).

C. Performance Weighting Model

The performance weighting is applied to the three individual error signals and its state equations are defined as follows:

$$\dot{x}_{wp} = A_{wp} x_{wp} + B_{wp} z_e, \quad (12)$$

$$z'_e = C_{wp} x_{wp} + D_{wp} z_e; \quad (13)$$

z_e can be written as:

$$z_e = C_{ze} x \mathcal{I}_3 r_c, \quad (14)$$

where:

$$r_c = [h_c \quad V_{ac} \quad \theta_c]^T, \quad (15)$$

r_c is the command vector.

D. Wind Turbulence Model

The state model associated with the wind turbulence through the longitudinal and vertical gust components, u_{wg} and w_{wg} , which represent the vector d_w , can be written as follows:

$$\dot{x}_{wg} = A_{wg} x_{wg} + B_{wg} n_{wg}, \quad (16)$$

$$y_{wg} = C_{wg} x_{wg}, \quad (17)$$

where:

$$x_{wg} = [u_{wg} \quad w_{wg1} \quad w_{wg2}]^T, \quad (18)$$

$$n_{wg} = [n_u \quad n_w]^T, \quad (19)$$

$$y_{wg} = [u_{wg} \quad w_{wg}]^T. \quad (20)$$

Although there is no dynamic weighting for the measurement noise vector, it could be explained by defining an appropriate high-frequency noise spectrum.

III. DESIGN METHOD

The objective of this study is to track the glideslope, despite the conditions of wind shear and turbulence, during landing. This method allows the minimizing of the error in the altitude along the glideslope (h), the airspeed (V_a), and the pitch attitude

(θ), thus a minimizing of the effects of wind shear on the aircraft dynamics.

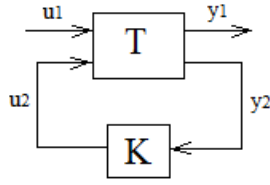


Fig. 1. Block diagram of general system.

The vector of the exogenous inputs, u_1 , consists of the command signal vector (r_c), the measurement noise vector (η), and disturbances vector (n_{wg}). The vector r_c includes the commanded altitude, airspeed and pitch attitude. The components of the vector n_{wg} are white, unit variance noise signals. The vector y_1 contains the controlled output variables (e.g. tracking errors). These two vectors can be written as follows

$$u_1 = \begin{bmatrix} n_u \\ n_w \\ h_c \\ V_{ac} \\ \theta_c \\ \eta_{V_a} \\ \eta_q \\ \eta_\theta \\ \eta_h \\ \eta_i \end{bmatrix} = \begin{bmatrix} \varepsilon_h \\ \varepsilon_{V_a} \\ \varepsilon_\theta \\ \delta_e \\ \delta_{th} \\ \dot{\delta}_e \\ \dot{\delta}_{th} \end{bmatrix}, y_1 = \begin{bmatrix} \delta_e / 7 \\ \delta_{th} / 40 \\ \dot{\delta}_e / 10 \\ \dot{\delta}_{th} / 10 \end{bmatrix}. \quad (21)$$

The vector u_2 is the control input vector and consists of errors associated with altitude, airspeed and pitch attitude, elevator and throttle positions and rates. The vector y_2 represents the measurement vector. The vectors are written as below:

$$u_2 = u_c = \begin{bmatrix} \delta_{ec} \\ \delta_{thc} \end{bmatrix}, y_2 = [V_a \quad q \quad \theta \quad h \quad \dot{h}]^T. \quad (22)$$

The generalized system depicted in Fig. 1 has the state space representation:

$$\begin{aligned} \dot{x}_G &= A_G x_G + B_1 u_1 + B_2 u_2, \\ y_1 &= C_1 x_G + D_{11} u_1 + D_{12} u_2, \\ y_2 &= C_2 x_G + D_{21} u_1 + D_{22} u_2. \end{aligned} \quad (23)$$

where

$$x_G^T = [x \quad x_A \quad x_{wg} \quad x_{wp}]^T = [u \quad w \quad q \quad \theta \quad u_w \quad w_w \quad h]^T, \quad (24)$$

$$\begin{aligned} \dot{x} &= Ax + BC_A x_A + B_w C_{wg} x_{wg}, \\ \dot{x}_A &= A_A x_A + B_A u_c, \\ \dot{x}_{wg} &= A_{wg} x_{wg} + B_{wg} h_{wg}, \\ \dot{x}_{wp} &= A_{wp} x_{wp} + B_{wp} C_{ze} x - B_{wp} r_c. \end{aligned} \quad (25)$$

The vectors x, x_A and x_{wg} are expressed in equations (3), (10) and (18), respectively. The vector x_{wp} contains the states associated with the performance weighting functions for the errors which are contained in the vector y_1 . The elements of the vectors n_{wg} and r_c are defined in equations (19) and (15).

The above system can be written in matrix form as follows:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_A \\ \dot{x}_{wg} \\ \dot{x}_{wp} \end{bmatrix} &= \begin{bmatrix} A & BC_A & B_w C_{wg} & 0 \\ 0 & A_A & 0 & 0 \\ 0 & 0 & A_{wg} & 0 \\ B_{wp} C_{ze} & 0 & 0 & A_{wp} \end{bmatrix} \begin{bmatrix} x \\ x_A \\ x_{wg} \\ x_{wp} \end{bmatrix} + \\ &+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{wg} & 0 \\ 0 & -B_{wp} \end{bmatrix} \begin{bmatrix} n_{wg} \\ r_c \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \delta_{ec} \\ \delta_{thc} \end{bmatrix}, \\ \begin{bmatrix} z'_e \\ \delta_e / 7 \\ \delta_{th} / 40 \\ \dot{\delta}_e / 10 \\ \dot{\delta}_{th} / 10 \end{bmatrix} &= \begin{bmatrix} D_{wp} C_{ze} & 0 & 0 & C_{wp} \\ 0 & [1/7 \quad 0] C_A & 0 & 0 \\ 0 & [0 \quad 1/40] C_A & 0 & 0 \\ 0 & 1/10 [1 \quad 0 \quad 0] A_A & 0 & 0 \\ 0 & 1/10 [0 \quad 0 \quad 1] A_A & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_A \\ x_{wg} \\ x_{wp} \end{bmatrix} + \\ &+ \begin{bmatrix} 0 & -D_{wp} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} n_{wg} \\ r_c \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/10 [1 \quad 0 \quad 0] B_A \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{ec} \\ \delta_{thc} \end{bmatrix}, \\ \begin{bmatrix} V_a \\ q \\ \theta \\ h \\ \dot{h} \end{bmatrix} &= [C \quad 0 \quad 0 \quad 0] \begin{bmatrix} x \\ x_A \\ x_{wg} \\ x_{wp} \end{bmatrix} + [0 \quad I_5] \begin{bmatrix} n_{wg} \\ r_c \\ \eta \end{bmatrix} + [0 \quad 0] \begin{bmatrix} \delta_{ec} \\ \delta_{thc} \end{bmatrix}. \end{aligned} \quad (26)$$

The matrix of the generalized system has the following form:

$$T = \begin{bmatrix} A_G & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}. \quad (27)$$

The matrices A_w and B_w for the plant model are very important because they contain the wind shear gradients and they are given as follows:

$$A = \begin{bmatrix} A_{aa} & A_{aw} & 0 \\ A_{wa} & A_{ww} & 0 \\ A_h & 0 & 0 \end{bmatrix}, \quad (28)$$

$$B_w = \begin{bmatrix} -a_{11} & -a_{12} \\ \vdots & \vdots \\ -a_{71} & -a_{72} \end{bmatrix}, \quad (29)$$

where:

$$A_{aa} = \begin{bmatrix} 0.4179E-01 & 0.98126E-01 & 0.49565E+01 & 0.32155E+02 \\ 0.29344E+00 & 0.76071E+00 & 0.21178E+03 & 0.96749E+00 \\ 0.30862E-03 & 0.49839E-02 & -0.52858E+00 & -0.37077E-03 \\ 0.00000E+00 & 0.00000E+00 & 0.10000E+01 & 0.00000E+00 \end{bmatrix}$$

$$A_{aw} = \begin{bmatrix} 0.41790E-01 & -0.98126E-01 \\ 0.29344E+00 & 0.76071E+00 \\ 0.30862E-03 & 0.49839E-02 \\ 0.00000E+00 & 0.00000E+00 \end{bmatrix}$$

$$A_{wa} = \begin{bmatrix} a_{w11} & a_{w12} & a_{w13} & a_{w14} \\ a_{w21} & a_{w22} & a_{w23} & a_{w24} \end{bmatrix} \quad (30)$$

$$\begin{aligned}
 a_{w11} &= 0.028944U_z + 0.0008384W_z, a_{w12} = 0.999U_z + 0.02894W_z, \\
 a_{w13} &= 0, \\
 a_{w14} &= -210.89U_z - 17.188W_z, a_{w21} = -0.0008384U_z + 0.0289W_z, \\
 a_{w22} &= -0.028944U_z + 0.9991W_z, a_{w23} = 0, a_{w24} = 17.188 - 210.89W_z, \\
 A_{ww} &= \begin{bmatrix} a_{w15} & 0 \\ 0 & a_{w26} \end{bmatrix}, a_{w15} = a_{w16} = -0.01, \\
 A_h &= [-0.02895 \quad -0.999 \quad 0 \quad 211.3].
 \end{aligned}$$

The vertical and horizontal wind shear gradients U_z and W_z vary as follows: $-0.25 \leq U_z \leq 0.39$ (Increasing Tailwind); $-0.47 \leq W_z \leq 0.33$ (Increasing Downdraft). The actuator state model, which is expressed by the equations (8) and (9), has the following matrix:

$$A_A = \begin{bmatrix} -\omega_{\delta_e} & 0 & 0 \\ 0 & -\omega_{\delta_{th}} & 0 \\ 0 & K_{eng} \omega_{\delta_{eng}} & -\omega_{\delta_{eng}} \end{bmatrix}, B_A = \begin{bmatrix} \omega_{\delta_e} & 0 \\ 0 & \omega_{\delta_{th}} \\ 0 & 0 \end{bmatrix}, C_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (31)$$

The performance aim was expressed relative to the controlled vector (y_1), which contains controlled variables and tracking errors. The requirements were formulated relative to the following time response characteristics: Time Constant: 1.0 sec; Steady-State Error: 0.1%, Step Response Overshoot: 10%. The requirements defined above are used to express a performance weighting function (W_p), as below:

$$W_p = \frac{K_p (s + \omega_1)}{s + \omega_2}. \quad (32)$$

The weighted control position and rate signals are based on the actuator position and rate limits which are written below:

$$\begin{aligned}
 \delta_e &\leq \pm 10 \text{ deg} \\
 \text{Elevator Position: } &Trim = 3.2 \text{ deg} \\
 |\delta_{e \max}| &= 7 \text{ deg} \\
 10 \leq \delta_{th} &\leq 60 \text{ deg} \\
 \text{Throttle Position: } &Trim = 17.6 \text{ deg} \\
 |\delta_{th \max}| &= 40 \text{ deg}
 \end{aligned}$$

Elevator Rate Limit: 10deg/sec

Throttle Rate Limit: 10deg/sec

The weightings of the control positions and rates can be selected to be:

$W_{\delta_e} = 1/7, W_{\delta_{th}} = 1/40, W_{\delta_{e \text{ (rate)}}} = 1/10, W_{\delta_{th \text{ (rate)}}} = 1/10$. The matrices which are included in the performance weighting model are defined as follows:

$$\begin{aligned}
 A_{wp} &= \begin{bmatrix} -0.001 & 0 & 0 \\ 0 & -0.001 & 0 \\ 0 & 0 & -0.001 \end{bmatrix}, B_{wp} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \\
 C_{wp} &= \begin{bmatrix} 9.99 & 0 & 0 \\ 0 & 9.99 & 0 \\ 0 & 0 & 9.99 \end{bmatrix}, D_{wp} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}. \quad (33)
 \end{aligned}$$

The wind disturbance weighting matrix, $W_g(s)$, is associated with the wind gust disturbance vector, d_w , defined in equation (5). The Dryden turbulence model provides low pass turbulence weighting functions for the longitudinal and vertical wind gust components, u_{wg} and w_{wg} . The Dryden wind turbulence model uses the linear velocity and angular components of gust as variable. The Dryden wind turbulence

model is defined by power spectral densities and is used to add turbulence to the model by passing band-limited white noise through appropriate forming filters. According to the Military Specification MIL-F-8785C and Military Handbook MIL-HDBK-1797, turbulence is a stochastic process. For an aircraft flying at airspeed V_a in a turbulent atmosphere, the transfer functions are expressed as below; the model is represented in Fig. 2.

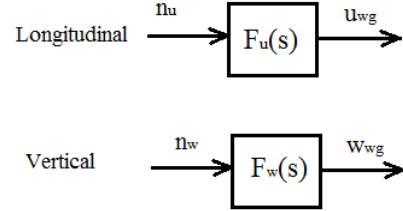


Fig. 2. Dryden turbulence model.

$$F_u(s) = \sigma_u \sqrt{\frac{L_u}{\pi V_a}} \left[\frac{1}{1 + \frac{L_u}{V_a} s} \right], \quad F_w(s) = \sigma_w \sqrt{\frac{L_w}{2\pi V_a}} \left[\frac{1 + \sqrt{3} \left(\frac{L_w}{V_a} \right) s}{\left(1 + \frac{L_w}{V_a} s \right)^2} \right]$$

The wind disturbance weighting matrix, $W_g(s)$, is determined by the expression: $W_g(s) = \text{diag}(F_u(s), F_w(s))$. The state equations of the disturbance weighting are defined in equations (16) and (17). The matrices A_{wg} , B_{wg} and C_{wg} are defined below:

$$\begin{aligned}
 A_{wg} &= \begin{bmatrix} -\frac{V_a}{L_u} & 0 & 0 \\ 0 & -\frac{V_a}{L_w} & 0 \\ 0 & \sigma_w \frac{1}{\sqrt{L_w}} \sqrt{\frac{3V_a}{2\pi}} & -\frac{V_a}{L_w} \end{bmatrix}, \quad B_{wg} = \begin{bmatrix} \sigma_u \frac{1}{\sqrt{L_u}} \sqrt{\frac{V_a}{\pi}} & 0 \\ 0 & -V_a \frac{1}{L_w} \left(1 - \frac{1}{\sqrt{3}} \right) \\ 0 & \sigma_w \frac{1}{\sqrt{L_w}} \sqrt{\frac{3V_a}{2\pi}} \end{bmatrix}, \\
 C_{wg} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
 \end{aligned}$$

IV. A CASE STUDY – BOEING 737 MODEL

Introducing the parameters for the selected aircraft model, the matrices included in the state equations for the plant model are written below. The numerical values used in this case study are given in [7]. For the weighting transfer matrix W_p chosen in this paper is $W_p s = 0.1 s + 2000 / s + 0.001$ which frequency response is presented in Fig. 3. The values of vertical and horizontal wind shear gradients U_z and W_z are chosen 0.1ft/sec for both the gradients. These values correspond to and increasing tailwind simultaneous with an increasing downdraft. The used numerical values in simulations are:

$$A = \begin{bmatrix} -0.0042 & 0.0981 & -4.9565 & -32.155 & 0.0418 & -0.0981 & 0 \\ -0.2934 & -0.7607 & 211.78 & 0.9675 & 0.2934 & 0.7607 & 0 \\ -0.0003 & -0.005 & -0.5286 & -370.77 & 0.0003 & 0.005 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.003 & 0.1028 & 0 & -22.8078 & -0.001 & 0 & 0 \\ 0.0028 & 0.097 & 0 & -19.3702 & 0 & -0.001 & 0 \\ -0.029 & -0.9996 & 0 & 211.3 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.004 & 0.0004 \\ -0.1725 & 0 \\ -0.021 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.9997 & 0.0234 & 0 & 0 & -0.9997 & -0.0234 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -0.028956 & -0.99958 & 0 & 211.3 & 0 & 0 & 0 \end{bmatrix}$$

For the actuator model the following values are used (see [7])

$$\omega_{\delta_e} = 16 \text{ rad/s}, \omega_{\delta_{th}} = 10 \text{ rad/s}, \omega_{\delta_{eng}} = 0.5 \text{ rad/s},$$

$$K_{eng} = 596 \text{ lbs/deg}, V_a = 211.53 \text{ ft/s}.$$

The matrices A_A , B_A are defined as follows:

$$A_A = \begin{bmatrix} -16 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 298 & -0.5 \end{bmatrix} \quad B_A = \begin{bmatrix} 16 & 0 \\ 0 & 10 \\ 0 & 0 \end{bmatrix}.$$

The matrices for the performance weighting model are defined in equations (33). For the Dryden disturbance models (34)-(36), the following turbulence specifications for the wind profile and for the altitude $h = 200.0656 \text{ ft}$ have been used

$$L_u = 306.595 \text{ ft}, L_w = 106.037 \text{ ft}, \sigma_u = 7.477 \text{ ft/s},$$

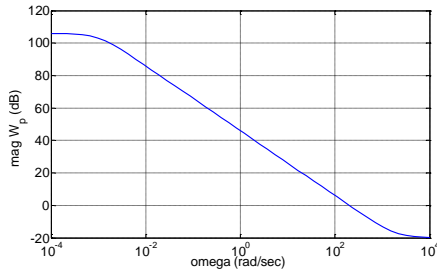
$$\sigma_w = 7.342 \text{ ft/s}.$$


Fig. 3. Magnitude of the weighting function W_p .

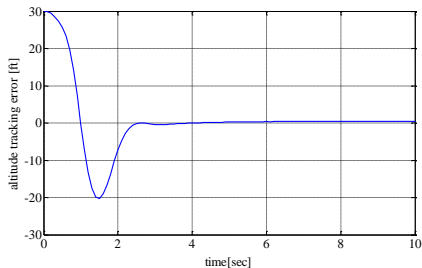


Fig. 4. Time response of the altitude tracking error.

In Figs. 3-5 there are presented the magnitude of the weighting function W_p , the time response of the altitude tracking error, and the pitch angle tracking error time response. These time responses have been obtained for an initial altitude deviation with respect to the ideal glide slope of 30ft and an initial deviation of the pitch angle of 10 degrees, respectively. In these numerical simulations a value of 0.1ft/sec for the wind

shear gradients U_z and W_z have been considered, corresponding to increasing tailwind and to increasing downdraft.

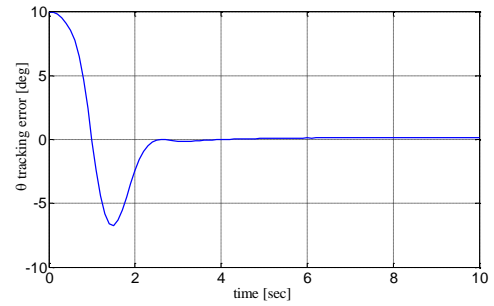


Fig. 5. The pitch angle tracking error time response.

V. CONCLUSION

This study has presented the problem formulation of a glideslope tracking through severe wind shear and gusts conditions. The parameters were based on the reconstruction of the wind profile that caused the landing accident at Kennedy International Airport in 1975. The Dryden turbulence model has provided weighting information and the wind gusts were included as disturbance inputs to the plant. Using the H infinity theory, stability and robust performance has been obtained. This system has been able to allow the aircraft to track the glideslope, despite severe conditions, during the landing. This advanced theory allows minimizing of the effects of wind shear and turbulence on the aircraft dynamics.

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