Three Dimensional Modified Star Grain Design and Burnback Analysis

Almostafa Abdelaziz and Liang Guozhu

***Abstract*—The determination of grain geometry is an important and critical step in the design of solid propellant rocket motor. In this study the design process involved parametric geometry modeling in CAD, MATLAB coding of performance prediction and 2D star grain ignition experiment. The 2D star grain burnback achieved by creating new surface via each web increment and calculating geometrical properties at each step. The 2D star grain is further modified to burn as a tapered 3D star grain. Zero dimensional method used to calculate the internal ballistic performance. Experimental and theoretical results were compared in order to validate the performance prediction of the solid rocket motor. The results show that the usage of 3D grain geometry will decrease the pressure inside the combustion chamber and enhance the volumetric loading ratio.**

***Index Terms*—Burn back, three dimensional grain, star grain.**

I. Introduction

Grain design is the most imperative in completing the design of solid rocket motor (SRM). Geometry modeling, burnback analysis and performance prediction are the main tasks performed in the ballistic grain design process. Burnback analysis can be accomplished by using analytical method, or CAD software. The star grain configuration is applicable to a relatively wide range of web fraction and volumetric loading fraction and offers the possibility to design motor that offers good performance with realistic and reasonable range of web fraction and volumetric loading ratio [1].

The star grain configuration is applicable to a relatively wide range of web fractions and volumetric loading fractions. It provides reasonably neutral burning characteristics in two dimensions without the need of end effects and slots; thus, minimum insulation for protection of the chamber wall is required. In view of these characteristics, the star grain design has been widely used in solid rocket applications.

The seven independent geometric variables (six dimensionless variables) that define the star grain geometry make it a very analytic configuration with seemingly endless mathematical relationships of practical interest. This analytic potential has resulted in many published works on methods of star analysis over the past years [2, 3]. These methods are characterized by graphs from which relationships among volumetric loading and sliver. A more recent star analysis that relates structural and ballistic characteristics was reported [4].

Design of star grain is presented by Barrere [5] He defined non-dimensional equation for the design of two-dimensional Star grains. Ricciardi [6] described the complete set of analytical expressions for star grain geometries including convex and concave star geometries. Ricciardi also [7] described Geometric parameters of star grain and their effects on internal ballistics and their future trends were studied Guanglin [8] provides a generalized approach to determine the geometry parameters and the performance prediction required for Star grain analysis. Antoine Lefebvre [9] present an analysis of the burning comportment of star shape as well as parameter recommendation to achieve better performance. Francois Bouquet [10] work on calculation of the performance and burning behavior of star grain in detail. Willcox [11] developed a computational computer code capable in designing 3D solid rocket motor. This code works by simulation the evolution of the burning surface by using commercial computer-aided design (CAD) programs.

The burnback analysis is used in the study of modeling of the solid rocket grain, the grain geometry is based on CAD software that has the advantage to handle with the parameters of the grain configuration specially the complex shapes such as 3D star grain, finocyl grain, slotted grain [12]. Gokay Puskula [13] investigated in his study the grain burnback analysis of different 3D grains (star grain, finocyl and wagon wheel) the method used is solid modeling of the propellant grain.

The burnback methodology used in this study is solid modeling of propellant grain, the solid propellant grain at the beginning of the operation is modeled parametrically, the parameters that change during the burnback process are decided and for every burn step they are modified. The software used is INVENTOR which is used to draw the grain geometry and investigate the burnback analysis at each burning step which calculates the area burning at each increment.

The experiment of 2D star grain geometry will be compered by the data obtained from the theoretical results from MATLAB program, by modification the shape of the star grain to be burned as 3D star grain geometry, we will be able to calculate the burnback analysis by using CAD software, and calculating the performance prediction, investigate that the 3D star grain will be better in internal ballistic results.

This paper improves the burnback analysis of the star grain geometry mainly. And to link the CAD software by the MATLAB program which calculate the performance prediction of the star grain 2D and 3D geometry. The objective of this work is to determine the burnback analysis of 3D star grain with different geometry for better

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performance of the solid rocket motor. We designed 3D star grain using 2 different star shapes from the aft and tail ends. The scale of the first shape differs from the scale of the second one in 0.8 times, as the result there is a slope between them to be burned as tapered grain. The grain will be burned as 3D star grain, and it will be important to use it as primary design of a solid rocket motor. Zero dimensional method is used to calculate the internal ballistic.

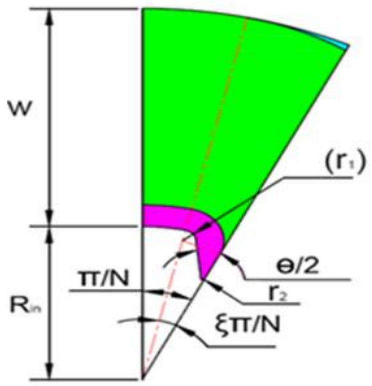


Fig. 1. Section of star grain geometry.

II. Burnback Analysis of Geometrical Model

A. Geometrical Modeling of Propellant Grain

Grain burnback analysis is the determination of the change in the grain geometry during the operation of the rocket motor. In this study using MATLAB and CAD software INVENTOR we will be able to calculate the burnback variation. As the burning surface changes chamber pressure and the thrust of the rocket motor changes, therefore performance of the motor is directly related to burnback steps of the propellant.

B. 2D Modeling

2D star grain model consists of Seven independent geometric variables defined in Figure (1) which characterize the star geometry: Rin, r1, r2, w, θ, ξ, N. where Rin is the inner radius, fr1 is the fillet radius, w is the web thickness, θ star point angle, ξ is the angular fraction, N number of star points.[14]

TABLE I: Values of Dimensions of Star Grain

Dimensions						
Rin (mm)	*r2 (mm)*	*r1 (mm)*	*W (mm)*	*Θ (degree)*	*ξ (degree)*	*N*
0.023	0	0.0016	0.0335	74	0.5058	7

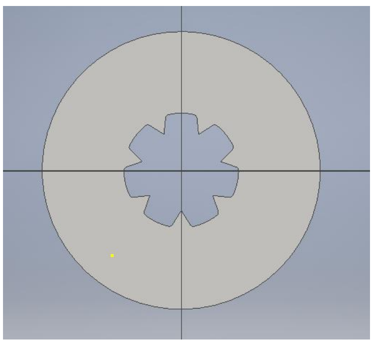


Fig. 2. Star grain cross section.

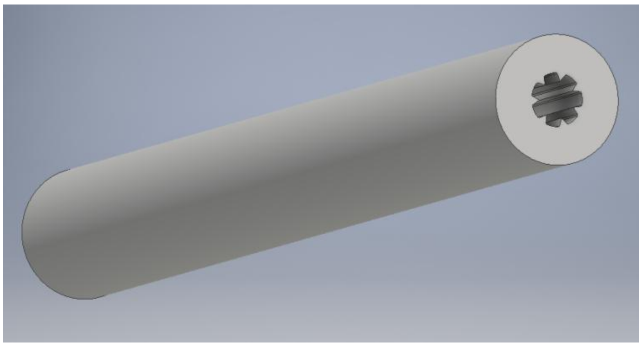


Fig. 3. 2D star grain.

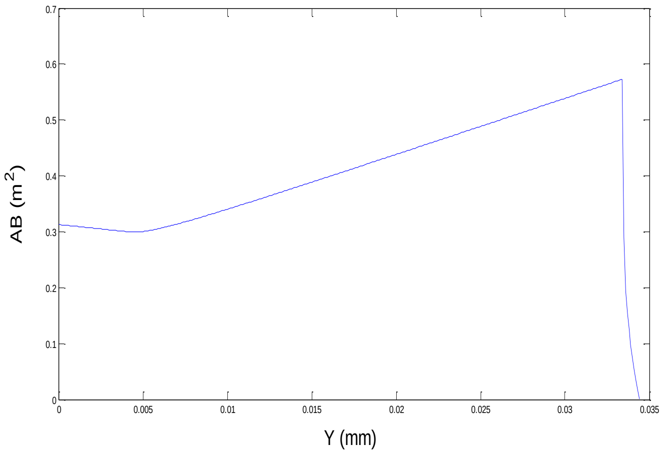


Fig. 4. The curve of burned area to distance.

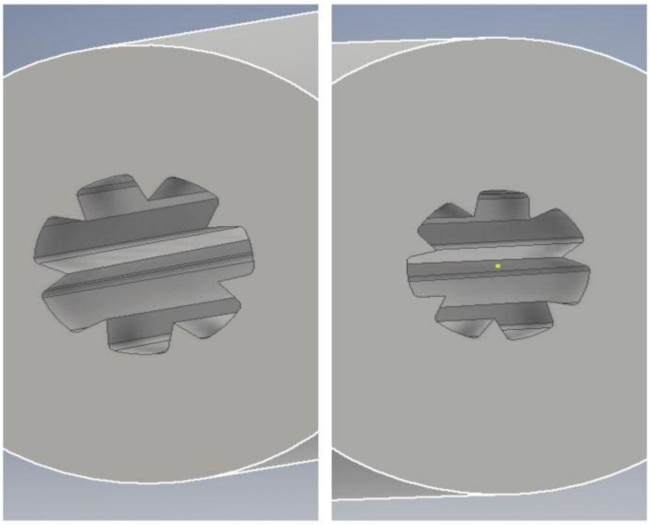
Fig. 5 shows the relation between distance change (YYT) and the change of area burning (ABT). We developed a MATLAB program capable to calculate the area burning of the star grain and give the same values using the simple mathematical equations to calculate the 3 zones of the star grain burning area, which are the same values used by CAD software.

C. 3D Modeling

We created and modify 2D star grain to be burned as 3D star grain by drawing 2 different star shapes from the aft and tail ends. The scale of the first shape differs from the scale of the second one in 0.8 times. The burning characteristics were applied to three dimensional grain, which was afterwards burnt step by step with web distance in order to get the burnback analysis in each step and then calculate performance prediction of the grain. After calculating the internal ballistic of the solid rocket motor it will become obvious to indicate the pressure time curve and thrust time curve of 3D star grain geometry [15]

The 3D model is preferred as we can get the accurate pressure and better volumetric loading ratio at step station of the combustion chamber

3D model is preferred as burnback analysis of the grain will be accurate which will lead to less error in the estimation of pressure along the combustion chamber. And for better volumetric loading fraction. As an example the first shape from the aft will differ from the second shape in the tail in 0.8 times.



(a) Forward (b) End

Fig. 5. 3D star grain with 0.8 scale between two ends.

III. 2D Experimental Model

It is very important to conduct laboratory test of individual parts up to the final product, this test is used to measure and fulfill all the parameters of the internal ballistic of the rocket motor and calculate the burning law of the propellant used, including the erosive burning. The rocket motor lies on a horizontal test stand, the propellant used is composite propellant, and the main output of the experiment is the pressure time curve. With the help of pressure transducer which is fixed to the tested rocket motor it is possible to investigate the pressure time curve, which investigates the internal ballistic of rocket motor.

Erosive burning will take part in this experiment during the factors of erosive burning. (Cross flow velocity, pressure, burning rate, motor size, etc …) where there is a big ratio between L/D which is a factor of the erosive burning [16]. The results from the experiment will be compared with the data from CAD and MATLAB software [17], [18].

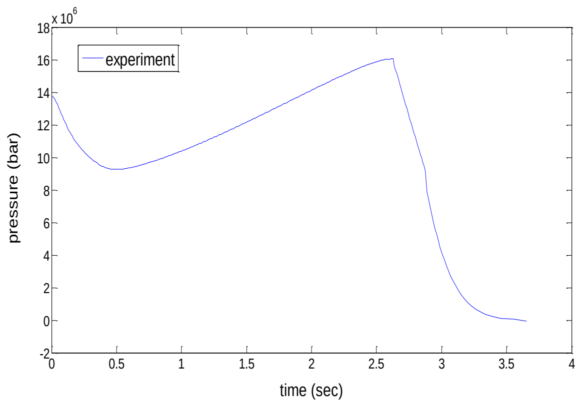


Fig. 6. Pressure time curve due to experimental data.

The results from the pressure time curve show that in the beginning of burning there is a pick due to the igniter and the erosive burning. This pressure depends on the area allowable inside the rocket motor in the beginning of burning related to the shape of the grain and the flow velocity. Then the pressure starts to decrease rapidly due to the burning of the first phase of the star grain geometry, then again it starts to increase due to the burning of the second phase, so it will be a progressive burning. After finishing of the burning phase the sliver phase will start to burn. At this section the pressure starts to decrease and the burning area also decreases in a short time.

IV. Performance Prediction

The performance prediction is calculated using simplified ballistic model; the pressure time curve is characterized by the rapid change of pressure inside the rocket motor during burning, the governing equations of the pressure are driver from equations of mass conversation, ideal gas formulation and ideal gas velocity [19].

With the help of a coding program the pressure and thrust time curves with the inputs of thermo chemical properties of the propellant, nozzle dimensions and performance efficiencies are calculated.

A. MATLAB Simulation

The computer program is capable of calculating performance of star grain and draw pressure time curve. The program computes in SI units which are used to calculate the main parameters and performance of solid propellant rocket motor. The program may be used for rockets of all sizes. The main task of the program is to draw the P-T curve and to calculate the phenomena of the erosive burning.

We will divide the web thickness into equal segment for easy calculation and determination of the output with minimum error.

The internal ballistic analysis follow zero dimensional frictionless compressible flow theory with the flow taken as isentropic in the nozzle, equilibrium pressures are established by iteration to find the correct chamber pressure which balance the continuity equation. By dividing the web thickness into segments, solution of the governing equation for each segment will calculated, and make a comparison of the total amount of mass generated with the mass discharge through the nozzle, if the check fails a new assumption is required to obtain a solution for one instant of time. The average burning rate is taken to be liner between the head and nozzle end.

Determination of head and nozzle end burning rate take into account erosive burning effect the burning surface at tail off is reduced in proportion to the length of the propellant that has experienced burn out according to the linear model of the regression surface. The computation are performed in two sections at the head and nozzle end of the grain, it is useful to for computing pressure, temperature, and velocity of gases of the nozzle end of the grain, the web thickness divided into equal segments to determine the output with minimum error.

We assume that the burning rate and pressure at head and nozzle end is linear along combustion

B. Numerical Solution Procedure

Prediction of the pressure variation with time is performed according to the following steps; it is assumed that all the parameters are known at every increment of burning.

1) Compute the Mach number at nozzle end (mn); this step is solved by iteration, the first estimated value is mn = 0.6, the solution is accepted when the difference between two successive value becomes less than 0.00000001.
2) Compute the velocity of chamber gases at nozzle end (Un).
3) The web thickness is divided into segments.
4) Predict the stagnation pressure and the mass discharge rate.
5) Compute the pressure and burning rate at head and nozzle ends respectively (ph, rh, pn, rn).
6) To find the total burning rate, we use the Lenoir and Robillard equation to estimate the total burning rate which includes the erosive burning term. The equation of the burning rate at nozzle end is solved by using Newton Raphson method. The solution is accepted when the difference between two successive values becomes less than 0.000001.
7) Calculate mas flow rate generation and dp/dy.
8) Calculate the new stagnation pressure from the new mass discharge under the assumption that the mass generated is not equal to the mass discharge. Then repeat steps from 3 to 7 until the difference between two successive values of masses becomes in the range of 1%
9) Determine the time of the total burning rate at this section
10) Repeat steps from 1 to 8 until the web thickness from the nozzle side touches the motor case.
11) Determine the length, area, pressure, burning rate and time at the first section of tail off until the sliver ends.
12) Calculate the free volume of chamber, then calculate the pressure in this section until the pressure becomes less than 2 bar.
13) Calculate the total burning time.
14) Draw the pressure time curve from the output data.

This model is based on fundamental gas dynamic and thermodynamic relationship with proper consideration for conservation of mass, energy and momentum. Pressure time history of most rocket motors is evaluated in steps that treat separately each of three phases of motor operation, ignition, transient, quasi steady and tail off transient.

The internal ballistic analysis follows one dimensional frictionless compressible flow theory with the flow taken as isentropic in the nozzle, equilibrium pressures are established by iteration to find the correct chamber pressure which balances the continuity equation.

By dividing the web thickness into segments, solution of the governing equations for each segment will be calculated. The total amount of mass generated will be compared with the mass discharge through the nozzle; if the check fails- a new assumption is required for obtaining the solution for one instant of time.

The average burning rate is taken to be liner between the head and nozzle ends. Determination of head and nozzle ends burning rate takes into account erosive burning effect. The burning surface at tail off is reduced in proportion to the length of the propellant that has experienced burn out according to the linear model of the regression surface.

The computation is performed in two sections: at the head and nozzle end of the grain. It is useful for computing pressure, temperature, and velocity of gases of the nozzle end of the grain. The web thickness is divided into equal segments to determine the output with minimum error. We assume that the burning rate and pressure at head and nozzle ends is linear along combustion, starting from the Mach number:

$$Mn = \frac{A_{cr}}{A_p}\{\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}Mn^2\right)\}^{\frac{\gamma+1}{2(\gamma-1)}} \tag{1}$$

where Mn is the Mach number, Acr is the critical area, Ap *is the port area*, γ is the specific heat ratio.

$$P_{on} = (a\,\rho_p\,C^*\,\frac{A_b}{A_{cr}})^{\frac{1}{1-n}}\,(1+\frac{\Gamma^2 J^2}{2})^{\frac{n}{1-n}} \tag{2}$$

where pon is the stagnation pressure, a is the burning rate coefficient, ρ_p is the density of the propellant, C^* is the characteristic velocity, Ab is the burning area, Γ is the function of specific heat ratio, n is the combustion index.

$$P_n = P_{on}\,(1+\frac{\gamma-1}{2}{M_n}^2)^{\frac{-\gamma}{\gamma-1}} \tag{3}$$

where pn is pressure at the nozzle.

$$r_n = a{p_n}^n \tag{4}$$

where rn is the burning rate at nozzle.

$$p_h = p_n + \frac{m_d\,u_n}{a_p} \tag{5}$$

where ph is the pressure at head, md is the mass discharge, un is velocity of gases, ap is the port area.

$$r_h = a\,{p_h}^n \tag{6}$$

where rh is the burning rate at head.

$$r_t = a{p_n}^n + \alpha\,G^{0.8}\,L^{-0.2}\,e^{\left(\frac{-\beta r_t\,\rho p}{G}\right)} \tag{7}$$

where rt is the total burning rate, α is erosive burning coefficient, G is the mass flux, L is the length of the grain, β is erosive burning pressure coefficient,

$$r_{av} = \frac{r_h+r_t}{2} \tag{8}$$

where r_{av} is the average burning rate.

$$\mathrm{t} = \frac{\mathrm{dy}}{\mathrm{r}_{av}} \tag{9}$$

where *t* is the time, *dy* is the change of burning depth.

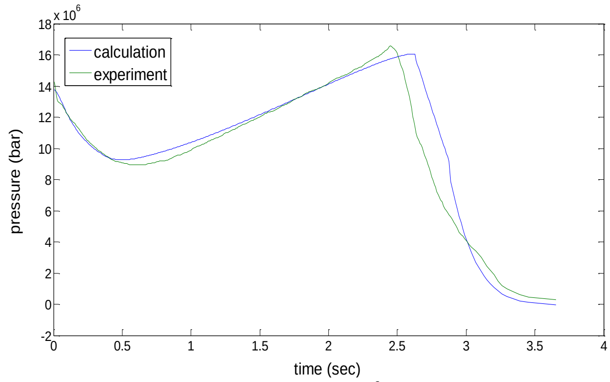


Fig. 7. Pressure time curve of 2D star grain.

Comparing the theoretical and experimental data from the shown curve, we notice that the error between the two curves is small, which means that the program is capable of investigating the pressure time curve with minimum error, and can be used to solve the internal ballistic of 2D star grain with different geometry.

The phenomenon of erosive burning is used during the theoretical calculation. Lenoir and Robillard [20] were the first to develop a model based on the heat transfer. They made a mathematical model to predict the effects of erosive burning in solid propellant rocket motor, two mechanisms of gas to solid heat transfer were proposed:

1-from the primary burning zone which is independent of core of gas velocity and is function only of pressure

2-from the core of the hot combustion gases which depend on the gas velocity using this approach total burning rate is expressed as :

$$r_t = r_o + r_e \tag{12}$$

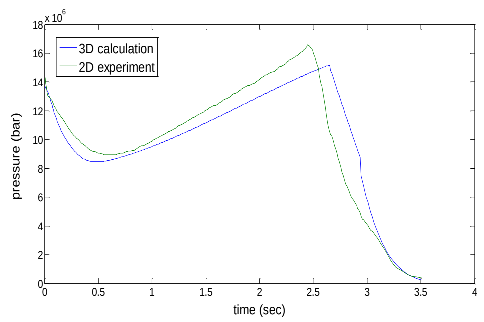


Fig. 8. Comparison of 2D experiment and 3D calculation.

The curve above shows the relation between the experimental 2D and the theoretical 3D star grain, which describes that by using 3D grain the pressure will decrease due to the change in the burning area which gives better performance inside the combustion chamber.

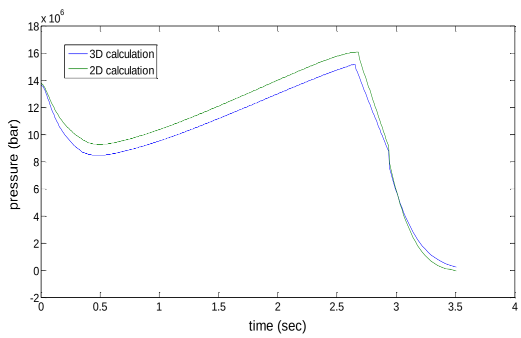


Fig. 9. 2D and 3D calculated pressure time curve.

The pressure time curve between 2D and 3D star grain shows that by using a 3D star grain the pressure decreases which provides better performance of the rocket motor. At the second phase of burning the decrease in pressure is clearly observed, at the sliver phase it remains approximately the same and the time of burning will finish early due to the erosive burning and high speed of velocity gases.

In the 2D model we assume that the pressure is constant along the combustion chamber which is not true, so the 3D model is preferred as we can get the accurate pressure at each station of the combustion chamber then estimate the average pressure for the whole motor.

In the presence of erosive burning, the burning rate in the grain end will be more than at its head so any 2D model will be suddenly changed into 3D model. For that reason if we use 2D model our calculations will not be accurate, except at the beginning only, so 3D model is preferred as burnback analysis of the grain will be accurate which will lead to less error in the estimation of pressure along the combustion chamber.

V. Conclusion

The grain burnback analysis is one of the most important steps in the solid rocket motor calculation and design. According to this work we offered a simple method of three dimensional burnback analysis, the results and data obtained from this work can be compared by some experimental methods.

The experiment shows the pressure time curve of 2D star grain geometry which is compared by the MATLAB program to verify the theoretical solution. The error between them is acceptable due to the assumption taken into theoretical solution for simplicity calculation. CAD drawing is used to investigate the burnback analysis and determine the area burning in bo th 2D and 3D star grain geometries. The erosive burning takes place during the theoretical method.

The 3D model used has a better performance for the pressure time curve which decreases the pressure inside the combustion chamber during burning. 3D model is preferred as burnback analysis of the grain, which will give accurate results lead to less error in the estimation of pressure along the combustion chamber.

This MATLAB program can calculate and investigate the pressure time curve and internal ballistic of different geometry of 2D and 3D star grain geometry

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