Using FGM Family to Predict the Materials Demand of Downstream Manufactures

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***Abstract*—In order to avoiding the materials shortage or price monopoly, the downstream manufactures will purchase materials not only from one upstream supplier. This paper focuses on the multi-source topic to provide a aterials demand model with Farlie-Gumbel-Morgenstern family of bivariate distributions. The empirical data is conduct to evaluate the RMSD of the proposed model and previous model. The results show that the model has better fitness than previous model.**

***Index Terms*—Farlie-gumbel-morgenstern family of bivariate distributions, multi-source ordering, materials demand quantity, recency, ordering time.**

I. Introduction

To predict the materials demand in the manufacturing process is an important issue and is also discussed in many previous researches [1]-[4]. In order to avoiding the materials shortage or price monopoly [3], the downstream manufactures will purchase materials not only from one upstream supplier [5], [7]. The multi-source ordering can help downstream manufactures to make sure the stable inventory[2]- [3], [6], [8]. Thus, this paper focuses on the multi-source topic to provide a new model which is different from previous researches and compare this new model with previous one.

Huang [9] proposes the materials demand model in which the total demand quantities consist of the quantity that the downstream orders in the past(is called “ordering quantity of past”) and the time interval between the last purchase and the end of observation time(is called “recency of ordering time”) [10]- [11]. In her research[9], the “ordering quantity of past” is considered as log normal distribution and “recency of ordering time” follows a renew process with exponential distribution. In this model[9], the “ordering quantity of past” is only from one source of upstream supplier. But in generally speaking, downstream manufactures want to keep the adequate supply of materials through ordering materials from different suppliers[7]- [8].

To portray this phenomenon, Huang[12] extend her model by computing the characteristic function to demonstrate multi-source ordering from various upstream suppliers. In this extended model[12], total materials demand quantities are still composed of “ordering quantity of past” and “recency of ordering time”. But the “ordering quantity of past” follows a characteristic function.

The goal of this research is to propose different distributions of the “ordering quantity of past” (different from characteristic function) to compare the validation with Huang’s[12] model. We also assume if the “recency of ordering time” is following exponential distribution, then the materials demand model will totally different from the previous one.

This paper is organized as follows: first we will introduce the concept of materials demand model. The probability distribution of “ordering quantity of past” and “recency of ordering time” will be demonstrated. Two types of assumption of probability distribution with “recency of ordering time” will also be compute. Scondly, cumulative distribution function (cdf) and the probability density function (pdf) of full materials demand quantity model will be derived. Thirdly, the empirical data will be used to estimate the parameters. We use the results of estimation to simulate data. Then the comparison of empirical data and simulation data will be calculated.The model validation are shown in this section. Finally, the conclusions are made.

II. The Model

A. The Materials Demand Model

Based on Huang’s [9], [12] model, we also consider the materials demand model which is to predict the total demand quantity (denoted as M) is composed by the ordering quantity of past(denoted as A) and the recency of ordering time(denoted as T).

$$M = T \cdot A \tag{1}$$

B. The Ordering Quantity

According Farlie-Gumbel-Morgenstern family of bivariate distributions[13]-[15], there are two inde-pendent univariate distributions and combine these two into a correlated bivariate distribution. We consider the ordering quantities (denoted as A) is from two different sources which are denoted as upstream supplier A_X and A_Y. Then the order quantities X and Y are the random variables”.The pdf of x is

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x{}^2}} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x{}^2}\right) \tag{2}$$

and its cdf is

$$F_x(x) = \frac{1}{2}\left[1 + erf\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x{}^2}}\right)\right] \tag{3}$$

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The pdf of y is

$$f_y(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \tag{4}$$

and its cdf is

$$F_y(y) = erf\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right) \tag{5}$$

Based on Farlie-Gumbel-Morgenstern family of bivariate distributions[13]- [15], the joint density of x and y is

$$\begin{aligned} & f_{xy}(x, y|\beta) \\ & = f_x(x) f_y(y)\{1 + \beta[1-2F_x(x)][1-2F_y(y)]\} \\ & = \frac{1}{\sqrt{2\pi\sigma_x^2}} exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \cdot \\ & \frac{1}{\sqrt{2\pi\sigma_y^2}} exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \cdot \\ & \left\{1 + \beta \cdot erf\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x^2}}\right) \cdot erf\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right)\right\} \\ & = \frac{1}{2\pi\sigma_x\sigma_y} exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \cdot \\ & \left\{1 + \beta \cdot erf\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x^2}}\right) \cdot erf\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right)\right\} \end{aligned} \tag{6}$$

C. The Recency of Ordering Time

We consider two types of recency of ordering model. One is considering the recency of ordering time(denoted as T_1) as renew process[10]and the purchase interval time is a randomvariable which follows exponential distribution. The pdf and cdf in the recency of ordering time(denoted as t_1) are computing as

$$f_{T_1}(t_1|\gamma_1) = \frac{1}{\gamma_1^2} exp\left(-\frac{t_1}{\gamma_1}\right) \tag{7}$$

And

$$_{T_1}(t_1|\gamma_1) = \frac{1 - exp\left(-\frac{t_1}{\gamma_1}\right)}{\gamma_1} \tag{8}$$

Another one is considering the recency of ordering time (denoted as T2) as an exponential distribution. Its pdf and cdf are following as

$$f_{T_2}(t_2|\gamma_2) = \frac{1}{\gamma_2} exp\left(-\frac{t_2}{\gamma_2}\right) \tag{9}$$

And

$$F_{T_2}(t_2|\gamma_2) = 1 - exp\left(-\frac{t_2}{\gamma_2}\right) \tag{10}$$

D. The Full Model

According to equation (1), the materials demand model is composed by "the total ordering quantity (A)" multiplying "the recency of ordering time(T)" and we consider two types of the recency of ordering time(T) to demonstrate model 1 and model 2 . Thus we can compute two kinds of full model.

E. Model 1

In order to calculate the cdf of full model, we denote random variable K as demand quantity. Then, its cdf is

$$\begin{aligned} F_{M_1}(k) &= P(T_1 \cdot A < k) \\ &= \int_0^\infty P(T_1 \cdot A | T_1 = t_1, A = x, y) f_{T_1}(t_1|\gamma_1) dt_1 \\ &= \int_0^\infty P(A < \frac{k}{T_1}) f_{T_1}(t_1|\gamma_1) dt_1 \\ &= \int_0^\infty \int_0^{\frac{k}{t_1}} \left\{\frac{1}{2\pi\sigma_x\sigma_y} exp(C)\{G\}\right\} dxdy\, f_{T_1}(t_1|\gamma_1) dt_1 \\ &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_1^2} \int_0^\infty \int_0^{\frac{k}{t_1}} exp\left(C - \frac{t_1}{\gamma_1}\right)\{G\} dxdydt_1. \end{aligned}$$

where $C = -\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}$,

$$G = 1 + \beta \cdot erf\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x^2}}\right) \cdot erf\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right)$$

Based on equation (11), we can compute the pdf of the proposed model. It shows in equation (12).

$$\begin{aligned} & f_{M_1}(k) \\ & = \frac{d}{dk} F_{M_1}(k) \\ & = \frac{1}{2\pi\sigma_x\sigma_y\gamma_1^2} \frac{d}{dk} \int_0^\infty \int_0^{\frac{k}{t_1}} exp\left(C - \frac{t_1}{\gamma_1}\right)\{G\} \end{aligned} \tag{11}$$

F. Model 2

According to equation (1), the materials demand model is composed by "the total ordering quantity (A)" multiplying "the recency of ordering time(T)" and we consider two types of the recency of ordering time(T) to demonstrate model 1 and model 2 . Thus we can compute two kinds of full model.

$$\begin{aligned} F_{M_2}(s) &= P(T_2 \cdot A < s) \\ &= \int_0^\infty P(T_2 \cdot A | T_2 = t_2, A = x, y) f_{T_2}(t_2|\gamma_2) dt_2 \\ &= \int_0^\infty P(A < \frac{s}{T_2}) f_{T_2}(t_2|\gamma_2) dt_2 \\ &= \int_0^\infty \int_0^{\frac{s}{t_2}} \left\{\frac{1}{2\pi\sigma_x\sigma_y} exp(C)\{G\}\right\} dxdy\, f_{T_2}(t_2|\gamma_2) dt_2 \\ &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_2} \int_0^\infty \int_0^{\frac{s}{t_1}} exp\left(C - \frac{t_2}{\gamma_2}\right)\{G\} dxdydt_2 \end{aligned} \tag{12}$$

According to (13), we can compute pdf as

$$\begin{aligned} & f_{M_2}(s) \\ & = \frac{d}{ds} F_{M_2}(s) \\ & = \frac{1}{2\pi\sigma_x\sigma_y\gamma_2} \frac{d}{ds} \int_0^{\infty} \int_0^{\frac{s}{t_2}} exp\left(C - \frac{t_2}{\gamma_2} \right) \{G\} dxdydt_2 \end{aligned} \quad (13).$$

III. Math

We use 5328 samples which are from a downstream manufacture of solar energy generation. These data includes the information about the total demand quantity in each month, ordering quantity from two different upstream suppliers(company H and Q) and the recency of ordering time of this downstream manufacture.

We compare the proposed model (model 1 and model 2) with Huang's model. Thus we use these empirical data respectively to estimate the parameters of the proposed model 1, model 2 and Huang's model [12].

The analysis process follows these steps:

Step1: The parameters are estimated by these empirical data.

Step2: We use the results of parameter estimation of each model (model 1, model 2 and Huang's model[12]) to poll the simulation data .

Step3: In this step, we use root-mean-square deviation (RMSD) to calculate the difference between empirical data and simulation data.

A. The Parameters Estimation

We use MLE (maximum likelihood estimate) to estimate the parameters.

Model 1. Let kj denote the materials demand quantities by upstream j, that is:

$$\begin{aligned} & L_{M_1}(\mu_x, \sigma_x^{\ 2}, \mu_y, \sigma_y^{\ 2}, \gamma_1, \beta) \\ & = \prod_{j=1}^{n} f_{M_1}(k_j) \\ & = \frac{1}{2\pi\sigma_x\sigma_y\gamma_1^{\ 2}} \frac{d}{dk} \int_0^{\infty} \int_0^{\frac{k}{t_1}} exp\left(C - \frac{t_1}{\gamma_1} \right) \{G\} dxdydt_1 t \end{aligned} \quad (14)$$

Model 2. Let s_k denote the materials demand quantities by upstreamp, that is:

$$\begin{aligned} & L_{M_2}(\mu_x, \sigma_x^{\ 2}, \mu_y, \sigma_y^{\ 2}, \gamma_2, \beta) \\ & = \prod_{p=1}^{m} f_{M_2}(s_p) \\ & = \frac{1}{2\pi\sigma_x\sigma_y\gamma_2} \frac{d}{ds} \int_0^{\infty} \int_0^{\frac{s}{t_2}} exp\left(C - \frac{t_2}{\gamma_2} \right) \cdot \{G\} dxdydt \end{aligned} \quad (15)$$

IV. Results and Discussion

The estimation results are shown in Table I.

We can find that the parameter" β" which in the Farlie-Gumbel-Morgenstern family of bivariate distributions can reflect the relationship between ordering quantities of upstream supplier H and Q. They are negative relations. If downstream manufacture orders more materials quantities from supplier H, then the less ordering will happen in supplier Q.

TABLE I: The Results of Parameters estimation

Parameters	Huang's Model[12]	Parameters	New Model 1	New Model 2
μ_u	435	μ_1	490	488
σ_u^2	5.11	σ^2_1	4.97	5.01
μ_w	550	μ_2	534	545
$\sigma_w^{2}{}_2$	7.56	σ^2_2	6.61	7.03
Θ	0.69	γ_1	4.56	---
P	3.31	γ_2	---	4.87
		β	-0.31	-0.24

We compute the root-mean-square deviation (RMSD) to make comparison between empirical data and simulation data in Huang's model [12], the proposed model 1 and model 2. The results show that new model 2 has best fitness than new model 1 and Huang's model [12].It means the new model 2 have predictive power to forecast the total materials demand. The results of prediction is most close to the real data.

TABLE II: The Results of RMSD

	RMSD*
Huang's model[12]	0.6743
New model 1	0.8913
New model 2	0.7655

V. Conclusion

This paper considers multi-source from different upstream suppliers and demonstrates more complicate phoneme when the recency of ordering time are both in renew process or exponential distribution. It shows the combination of exponential distribution(as recency of ordering time) and Farlie-Gumbel-Morgenstern family of bivariate distributions(as ordering quantity of past) are best than that of renew process and Farlie-Gumbel-Morgenstern family of bivariate distributions or renew process and characteristic function. The proposed model provides the information to detect the relationship between upstream suppliers. This can reflect not only different source ordering but also the competitive situation on the supplier side. Thus, these two proposed models are more fitness with real data. In the future, the researchers can use different data to test the relations between upstream supplier and extend these models in different industries.

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