Sensitivity Analysis of Goal Programming Model for Dietary Menu of Diabetes Mellitus Patients

Atmini Dhoruri, Dwi Lestari, and Eminugroho Ratnasari

Abstract—**Diabetes mellitus (DM) was a chronic metabolic disease characterized by higher than normal blood glucose level (normal blood glucose level = = 80 –120 mg/dl). In this study, type 2 DM which mostly caused by unhealthy eating habits would be investigated. Related to eating habit, DM patients needed dietary menu planning with an extra care regarding their nutrients intake (energy, protein, fat and carbohydrate). Dietary menu with appropriate amount of nutrients was organized by considering the amount of calories, proteins, fats and carbohydrates. Therefore, mathematical model of this problem is represented by linear programming with more goals. Those are to achieve the amount of calories, proteins, fats and carbohydrates and to minimize expenses. In this study, Goal Programming model was used to determine optimal dietary menu variations by minimizing the deviation that available each the goal function. From the data obtained from hospitals in Yogyakarta, optimum menu variations would be analyzed using Goal Programming model and also their sensitivity analysis.**

Index Terms—**Diabetes mellitus, goal programming model, sensitivity analysis.**

I. Introduction

Diabetes risk mostly came from messy eating habits and excessive consumption of sweet foods. However, according to recent studies, diabetes risk could also come from bad emotion management or from getting angered easily. "That was because high levels of stress might prompt the release of inflammatory hormones like cortisol that may mess with your body's ability to keep blood sugar levels in check. Therefore, people with high level of stress tend to have higher risks on suffering diabetes than people who don't get angered easily or patient," just as reported by menshealth, Tuesday (2/4/2013). Nevertheless, this diabetes risk also could not be avoided only by holding the anger or practicing patience either. Every individual was asked to implement appropriate eating habits and balance them with exercising or other physical activities to prevent themselves from this diabetes disease. World Health Organization(WHO) in 2000 expressed that there were 57 million deaths per year caused by non-contagious disease and it was estimated that 3.2 million of it was caused by diabetes mellitus. (Diabetes Care, 2004 in [1]).

Diabetes Mellitus was a metabolic disease included in hyperglycemia category or higher than normal blood glucose level (normal blood glucose level = 80 –120 mg/dl), therefore it was also referred as *penyakit gula* or *kencing manis*. Diabetes mellitus often referred as the great imitator because this disease could attack all organs and cause all kinds of complaints. Its symptoms really vary. Diabetes mellitus (DM) could emerge slowly and patients would not recognize the changes happened in them such as the urge to drink more, dropping urination or weight. Those symptoms could be suffered for a long time without noticed, until they went to the doctor and checked their blood glucose level. Nutrients could also show their roles in the occurring of Diabetes Mellitus in two opposite directions. More nutrients were common clues of increasing individuals' wellbeing, paving a way for bigger manifestation chance of DM, especially for those who were born with the trace. In that kind of condition, DM symptoms could be overcame by re-arranging their body's nutrients metabolism balance with nutrients input through the foods they consumed.

There were five basic DM treatments called DM Therapy Pentalogy, they are: (1) Diabetes Dietary, (2) Physical Practice, (3) Community Health Counseling, (4) Hypoglycemic Medicine (OAD and Insulin), and (5) Pancreas Transplant. DM treatment through dieting was one of the best methods to improve individuals' eating habit to enable them to avoid DM disease. Dietary menu arranging had to put extra attention on the nutrients contents to provide sufficient energy, carbohydrate, fat and protein. The optimal amounts of nutrients contents such as energy, carbohydrate, fat and protein had to be put attentively. Dietary menu arranging based on substitute foods sometimes could cause a deviation above or below the recommended amounts. Therefore, a dietary menu with smallest deviation possibility on the recommended amounts was arranged. The method that could be employed to solve this minimizing deviation problem was Goal Programming method.

Goal programming was an extension of linear programming to achieve desired objectives or goals. Basic approach of goal programming was determining an objective asserted by certain number for each objective, formulating an objective function, and finding the solution by minimizing total deviations of the objective function [2]. This mathematic model optimized the problem with multi objectives. Mathematically, in this method decision variables had to be defined first. Desired objectives had to be specified according to their importance level. Then the optimum solution would be found which minimized total objective deviations from the determined targets. Obtained optimum results would be used to run sensitivity analysis to determine how far the optimum solutions change is when the model parameter is also changed.

Sensitivity analysis was implemented to investigate how the effect of the parameter change in the Linear Programming

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problems is. Those changes could be implemented on: Cost coefficients of the objective functions, resource/fixed rate coefficients of the right hand side (RHS) of constraint, technical coefficients, additional constraint functions (the change of m), additional variables (the change of n). in this study, sensitivity analysis was discussed by seeing the change on resource/fixed rate coefficients of the right hand side of constraint.

Some studies about mathematics model relevant with nutrients and health were Pasic, et al [3] about "Goal Programming Nutrition Optimization Model". The result of this study was an optimum model about human nutrients need planning. In the next year was Darko, et al [4] about "Cost-minimizing foods budgets in Ghana". The result of this study was determination of optimal cost for communities' nutrients needs. In addition, Ujang Sumarwan, et al [5] studied about "Using Goal Programming method in diabetes mellitus dietary planning" based on diabetes mellitus patients' dietary menu in Cipto Mangunkusumo hospital (RSCM). In Ikeu Tanziha [6] about Goal Programming: Foods Optimization for Children Under Five of Fisher Families. In this study, menu variations in PKU Yogyakarta hospital were analyzed along with optimum cost calculation.

Based on the description above, Goal Programming model would be potential to be employed because it could solve the problem to be optimum with multi objectives. In this instance, the objective was meeting certain range amount of fat, and achieving certain range amount of carbohydrate. The discussion was started by Goal Programming model improvement from the study conducted by Atmini, et al [7]. There was a formula given by nutrient experts of the hospital to determine the amount of energy, carbohydrate, protein and fat needed by DM patients. Furthermore, dietary menu with substitute foods was discussed based on the guideline from the hospital, dietary menu according to the goal programming approach along with sensitivity analysis, conclusion.

II. Assumption of Mathematical Model for Dietary Menu

In the process of building the model, first there was a need to take assumptions. In this study, used assumptions were discussed dietary menu was for type II DM patients. Patients did not suffer any kind of complications or other chronic diseases. For a day the meal was divided into five stages (breakfast, morning snack, lunch, afternoon snack, and dinner) and given within three hours interval. Nutrient contents only discussed generally comprised of carbohydrate, protein, and fat. Discussed menu was only for normal weigh condition and patients were not undergoing any pregnancy process.

Then, one thing related to the dietary was weight criteria or condition, which was

To calculate the criteria above, the following formula was used:

$$BBR = \frac{BB}{TB - 100} \times 100\%$$

BB (Weight) was in kg and TB (Height) was in cm.

In Diah Krisnatuti, et al [8] DM dietary requirements without complications had to meet the following points:

1) Sufficient energy to maintain normal weight. Meals were divided into three big portions (morning, afternoon, and evening, each of them was 20%, 30& and 25%. Added with 2-3 smaller portion by 10-15%.
2) Carbohydrate requirement was 60 – 70% of total energy.
3) Normal protein requirement was 10 – 15%.
4) Moderate fat requirement was 20 – 25%.

TABLE I: Weight Criteria and Needed Amount of Calories

Criteria	BBR
Thin: BB x 40 – 60 calories	< 90%
Normal : BB x 30 calories	90-110%
Fat : BB x 20 calories	> 110- 120%
Obese : BB x 10 – 15 calories	> 120%

Therefore, arranging dietary menu model for DM patients needed to take note of those requirements. With the existence of required interval, it would give a wider tolerance in consuming the foods. However, they would still have to be inside the recommended range. According to the discussion with nutrient experts of the hospital where the research data was taken, there were some additional rules in determining DM dietary menu as the following:

1) Menu was served according to foods guideline with exchanger units.
2) Daily requirement of calorie amount calculation was based on the formula in Table 1.
3) Daily requirement of carbohydrate amount calculation was based on the formula:
Requirement amount of carbohydrate was 60%-70% of calorie amount, which was:
Total carbohydrate (in kg) = total % of calorie: 4
4) Daily requirement of protein amount calculation was based on the formula::
Requirement amount of protein was 10%-15% of calorie amount, which was:
Total protein (in kg) = total % of calorie: 4
5) Daily requirement of fat amount calculation was based on the formula:
Requirement amount of fat was 20%-25% of calorie amount, which was:
Total fat (in kg) = total % of calorie: 9

For example, there was a case of a patient with weight 56 kg and height 160 cm therefore he/she was in normal criteria. According to Table I, daily amount of required calorie for the patient was 1680 therefore 1700 calorie diet type was chosen.

The following table was a menu based on exchanger units for a day 1700 calorie diet

TABLE II: The Menu List Based on the Substitution Unit for A Day

	Day 1	
Time	Foods	Weight (gr)

07.00	bread	70
	egg	50
	lettuce	0
	tempeh	25
	oil	15
10.00	papaya	110
13.00	rice	200
	beef	35
	stir fried kale	100
	tofu + tempeh	80
	apple	85
	oil	30
16.00	banana	50
19.00	rice	200
	meatball	170
	broccoli	100
	tofu	110
	orange	110
	oil	15
TOTAL		1555

After that, calorie total carbohydrate, protein, fat and the cost per unit was calculated to obtain the following data:

1) Daily total calorie 1700 calories
2) Daily total carbohydrate 60%-70%
   Lower limit : 60% x 1700 : 4 = 255 kg
   Upper limit : 70% x 1700 :4 = 297.5 kg
3) Daily total protein 10%-15%
   Lower limit : 10% x 1700 : 4 = 42.5 kg
   Upper limit : 15% x 1700 :4 = 63.75 kg
4) Daily total fat
   Lower limit : 20% x 1700 : 9 = 37.77 kg
   Upper limit : 25% x 1700 :9 = 47.22 kg

For total carbohydrate, protein, fat and the cost per unit was provided in the Table III.

III. Goal Programming Model

The follow mathematics notation used in the model:

X_{ij} : daily consumed foods
E_{ij} : energy contents in 100 g of foods
K_{ij} : carbohydrate contents in 100 g of foods
P_{ij} : protein contents in 100 g of foods
F_{ij} : fat contents in 100 g of foods
b_1 : value of target energy
b_2 : value of target carbohydrate
b_3 : value of target protein
b_4 : value of target fat

According to the arranged menu based on the substitution unit, that model revised from [7] was obtained

Minimize

$$\sum_{m=1}^{4} d_m^- + d_m^+ + \sum_{\substack{j=1 \\ m=6}}^{4} d_{mj}^- + d_{mj}^+ + \sum_{\substack{j=1 \\ m=5,7}} d_{mj}^- + d_{mj}^+ \quad (1)$$

With constraints:

1) To meet energy amount attainment

$$\sum^{6}_{\substack{j=1 \\ i=1,3,5}} E_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4}} E_{ij}X_{ij} + d_1^- - d_1^+ = b_1. \quad (2)$$

2) To meet carbohydrate amount goal in specific range

$$\sum^{6}_{\substack{j=1 \\ i=1,3,5}} K_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4}} K_{ij}X_{ij} + d_2^- \geq {b_2}^-. \quad (3)$$

$$\sum^{6}_{\substack{j=1 \\ i=1,3,5}} K_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4}} K_{ij}X_{ij} - d_2^+ \leq {b_2}^+. \quad (4)$$

3) To achieve protein amount goal in specific range

$$\sum^{6}_{\substack{j=1 \\ i=1,3,5}} P_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4}} P_{ij}X_{ij} + d_3^- \geq {b_3}^-. \quad (5)$$

$$\sum^{6}_{\substack{j=1 \\ i=1,3,5}} P_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4}} P_{ij}X_{ij} - d_3^+ \leq {b_3}^+. \quad (6)$$

4) To achieve fat amount goal in specific range

$$\sum^{6}_{\substack{j=1 \\ i=1,3,5}} F_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4}} F_{ij}X_{ij} + d_4^- \geq b_4^-. \quad (7)$$

$$\sum^{6}_{\substack{j=1 \\ i=1,3,5}} F_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4}} F_{ij}X_{ij} - d_4^+ \leq b_4^+. \quad (8)$$

5) To minimize expenses.

$$R = \sum^{6}_{\substack{j=1 \\ i=1,3,5}} c_{ij}X_{ij} + \sum_{\substack{j=1 \\ i=2,4,}} c_{ij}X_{ij}. \quad (9)$$

Constraint functions of food amounts:

$$X_{ij} + {d_{mj}}^- \geq l_{ij} \quad (10)$$

$$X_{ij} - {d_{mj}}^+ \leq u_{ij} \quad (11)$$

For $m = 5, \ldots, 7$.

TABLE III: VALUE PER UNIT OF TOTAL CARBOHYDRATE, PROTEIN, FAT AND THE COST PER UNIT (IN MILLION)

Day 1											
Time	Foods	Weight (gr)	Energy (kal) aij		Carbohydrate (gr) kij		Protein (gr) pij		Fat (gr) lij		Cost
07.00	bread	70	175	2.5	40	0.57	4	0.057	0	0	15
	egg	50	75	1.5	0	0	7	0.14	5	0.1	20
	lettuce	0	0	0	0	0	0	0	0	0	
	tempeh	25	37.5	1.5	3.5	0.14	2.5	0.1	1.5	-	25
	oil	15	50	3.33	0	0	0	0	5	0.33	12
10.00	papaya	110	50	0.45	12	0.109	0	0	0	0	9
13.00	rice	200	350	1.75	80	0.4	8	0.04	0	0	10
	beef	35	75	2.142	0	0	7	0.2	5	0.142	120
	Stir fried kale	100	25	0.25	5	0.05	1	0.01	0	0	3
	Tofu + tempeh	80	75	0.9375	7	0.0875	5	0.0625	3	0.0375	25
	apple	85	50	0.588	12	0.14	0	0	0	0	40
	oil	30	100	3.33	0	0	0	0	10	0.33	12
16.00	banana	50	50	1	12	0\.24	0	0	0	0	15
19.00	rice	200	350	1.75	80	0.4	8	0.04	0	0	10
	meatball	170	75	0.44	0	0	7	0.041	5	0.029	2.5
	broccoli	100	25	0.25	0	0	5	0.05	1	0.01	15
	tofu	110	75	0.681	7	0.063	5	0.045	3	0.027	25
	orange	110	50	0.454	12	0.109	0	0	0	0	15
	oil	15	50	3.33	0	0	0	0	5	0.33	12
Total		1555	1737.5		270.5		59.5		43.5		

TABLE V: MENU BASED ON GOAL PROGRAMMING MODEL

Day 1						
Time	Foods	Weight (gr)	Energy (kal) aij	Cost (million)	output LINGO	Energy (kal) aij
07.00	bread	70	175	15	60	150
	egg	50	75	20	40	60
	lettuce	0	0		0	0
	tempeh	25	37.5	25	15	22.5
	oil	15	50	12	25	83.25
10.00	papaya	110	50	9	100	45.45
13.00	rice	200	350	10	210	367.5
	beef	35	75	120	25	53.55
	Stir fried kale	100	25	3	90	22.5
	Tofu + tempeh	80	75	25	90	84.375
	apple	85	50	40	75	44.1
	oil	30	100	12	21.05856	70.1250048
16.00	banana	50	50	15	40	40
19.00	rice	200	350	10	210	367.5
	meatball	170	75	2.5	160	70.4
	broccoli	100	25	15	90	22.5
	tofu	110	75	25	100	68
	orange	110	50	15	100	45
	oil	15	50	12	25	83.25
Total		**1555**	**1737.5**	**22897.7**		**1700**

d^-: lower deviation

d^+: upper deviation

c_{ij}: average price of foods

l_{ij} : lower limit of foods amount

u_{ij}: upper limit of foods amount

R : daily expenses.

for $i = 1,2, \dots ,5; j = 1,2, \dots , n$

The menu of the day is divided into 5 stages (breakfast, morning snack, lunch, afternoon snack, and evening meal) is given at intervals of three hours, so indexes i describe about number of shifts. While indexes j describe about number of various food consumed.

IV. Output Lingo Discussion

For the next step, calculation would be conducted by using LINGO to obtain the following menu, see Table V.

According to the calculation, it resulted in zero objective function value (deviation). It means that all of the objective functions were achieved and therefore the optimum solution was showed in the table. For the expenses, according to the goal programming model it would cost IDR 22.897,7. Meanwhile, sensitivity analysis of model was presented on Table V. Sensitivity analysis is a systematic study of how sensitive (duh) solutions are to (small) changes in the data. The basic idea is to be able to give answers to questions of the form:

1) If the objective function changes, how does the solution change?
2) If resources available change, how does the solution change?
3) If a constraint is added to the problem, how does the solution change?

In this research, we discussed about changing a right hand side (RHS) constant. When changed the amount of resource in a non-binding constraint, then increases never changed your solution. Small decreases also did not change anything, but if we decreased the amount of resource enough to make the constraint binding, our solution could change. (Note the similarity between this analysis and the case of changing the coefficient of a non-basic variable in the objective function.

TABLE V: Sensitivity Analysis of Model

Foods	RHS	Increase Limit	Decrease Limit
Bread	70	1.41	14.396
		inf	20
Egg	50	2.35	40
		inf	20
lettuce	0	0	Inf
		0	Inf
Tempe	25	2.35	15
		inf	20
Oil	15	20	Inf
		1.058	18.941
papaya	110	7.756	79.188
		inf	20
Rice	200	20	Inf
		2.014	20
Beef	35	1.646	25
		inf	20
Stir fried kale	100	14.1	90
		inf	20
tofu + tempe	80	20	Inf
		3.76	20
Apple	85	5.995	61.209
		inf	20
Oil	30	1.059	Inf
		inf	18.941
Banana	50	3.525	35.991
		inf	20
Rice	200	20	Inf
		2.014	20
Meatball	170	8.011	143.352
		inf	20
Broccoli	100	14.1	90
		inf	20
Tofu	110	5.184	88.312
		inf	20
Orange	110	7.833	79.98
		inf	20
Oil	15	20	Inf
		1.059	18.941

Table V showed the tolerance value of the allowed parameter change so that the optimum value could still be reached. For example, food type bread with right hand side value 70, therefore if it enters the mathematical model as a constraint, then the allowed decrease limit for the bread rises until 1.41 unit and downs until 14.396 unit. Meanwhile, for the increase limit interval for food type bread consumed in a day is allowed to rise until unlimited and down until 20 unit.

V. Conclusion

Dietary menu planning for Diabetes Mellitus (DM) had to pay extra attention on the nutrients to get the sufficient energy, carbohydrate, fat and protein. The optimal amount of nutrients contents such as energy, carbohydrate, fat and protein had to be observed. Dietary menu planning based on Goal Programming model revised from [7] aimed to minimize the deviation above or below the recommended amount. From the calculation result using LINGO, recommended menu variations with appropriate total calorie and minimum expenses was obtained. From one day variation, obtained zero deviation which means that all the objective functions are achieved and optimum solution are obtained. For the expenses, the menu costs IDR 22.897.7.

Sensitivity analysis model also discussed to determine the advised value limit so that the change of the right hand side (RHS) constraints will not affect the optimum value.

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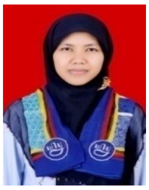

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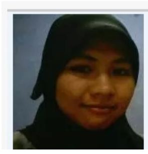

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