Forecasting Taiwan's GDP by the Novel Weighted Average Nonlinear Grey Bernoulli Model

Pei-Han Hsin

***Abstract*—In grey forecasting model, the average relative percentage error is an important criterion for assessing the forecasting precision. However, it seldom considers in-sample and out-of-sample tests at the same time. Besides, overestimation or underestimation usually exists when the original data has a rising or falling trend. Thus, this study proposes weighted average nonlinear grey Bernoulli model to solve above two problems. A weighted average moderates the effect of overestimation or underestimation. A new criterion which considers in-sample and out-of-sample tests at the same time indicates that the weighted average nonlinear grey Bernoulli model has the smallest modeling error. Finally, the proposed method is used to forecast Taiwan's GDP. The results show that Taiwan's GDP is steadily growing. It may serve as valuable information for policy makers and investors.**

***Index Terms*—Grey forecasting, nonlinear grey bernoulli model, GDP, weighted average.**

I. INTRODUCTION

Grey forecasting [1] has been developing for more than 20 years and its evolution never stops. Deng [2] proposes grey forecasting which only needs four data and is suitable for short-term forecasts. Grey forecasting has been successfully applied in many fields, including finance [3], agriculture [4], transportation [5], electric power load [6], semiconductor industry [7], economy [8] and so on.

The traditional grey forecasting is sometimes with its unsatisfied prediction precision. Thus, the researchers develop the hybrid grey model for improving the forecasting ability, for example, Grey-Markov model [9], Grey-Fuzzy model [10], Grey-Taguchi model [11] and so on. Besides, some researchers try to change the stucture of grey forecasting models, including Grey Verhulst model [12], the nonlinear grey Bernoulli model (NGBM) [13-14], and Nash NGBM [15]. On the other hands, some researchers use particle swarm optimization algorithm [16-17] or genetic algorithm [18] to optimize grey model .

In NGBM [13], [14], the coefficient p is always set as 0.5. In fact, the coefficient p may be between 0 and 1. Besides, overestimation or underestimation are common in forecasting practics. For this reason, this study tries to find out all forecast values as p changes and then adopts arithmetic average to moderate the effect of overestimation and underestimation.

This study also develops a new criterion which considers in-sample error and out-of-sample tests. Because those two types of error sometimes are trade-off, it is not enough to judge a forecasting model only by in-sample error, such as in Chen's articles [14], [15].

This paper is organized as follows. Section 2 introduces the mathematics of weighted average NGBM**,** and defines the forecasting relative percentage error. Section 3 forecasts Taiwan's GDP by the proposed method. Finally, section 4 presents conclusion.

II. MATHEMATICAL METHODOLOGY

The procedures for obtaining NGBM(1,1) are elaborated below:

Step 1: Assume that the original series of data with *m* entries is:

$$X^{(0)}(1,m)=\left\{x^{(0)}(k)\mid x^{(0)}(k)\geq 0, k=1,2,...,m\right\} \qquad (1)$$

where raw matrix $X^{(0)}$ represents the non-negative original time series data.

Step 2: Construct $X^{(1)}(1,m)$ by one time accumulated generation operation (1-AGO). Thus, $X^{(1)}(1,m)$ is

$$X^{(1)}(1,m)=\left\{x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(k), \mid x^{(1)}(k)\geq 0, k=1,2,...,m\right\} \qquad (2)$$

Step 3: The grey differential equation of NGBM(1,1) has following form [13]-[15].

$$\frac{d\hat{x}^{(1)}}{dt}+\alpha\hat{x}^{(1)}=\beta[\hat{x}^{(1)}]^{n}, \qquad (3)$$

where *n* is any real number but 1.

The background value is $\hat{x}^{(1)}(t)\cong px^{(1)}(k)+(1-p)x^{(1)}(k+1)=z^{(1)}(k)$ and $p\in[0,1]$.

Step 4: A discrete form of (3) is described as

$$x^{(0)}(k)+\alpha\, z^{(1)}(k)=\beta\,[z^{(1)}(k)]^{n}, k=2,3,4,... \qquad (4)$$

By the least square method, the parameters α and β become

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=(Z^{T}Z)^{-1}Z^{T}X \ , \qquad (5)$$

where Z and X are defined as follows.

Manuscript received October 10, 2015; revised December 10, 2015.

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DOI: 10.7763/IJMO.2015.V5.492

$$Z=\begin{bmatrix} -z^{(1)}(2) & [z^{(1)}(2)]^n \\ -z^{(1)}(3) & [z^{(1)}(3)]^n \\ \vdots & \vdots \\ -z^{(1)}(m) & [z^{(1)}(m)]^n \end{bmatrix},\ X=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(m) \end{bmatrix} \quad (6)$$

Step 5: The corresponding particular solution of (4) is

$$\hat{x}^{(1)}(k+1)=\left[\left(x^{(0)}(1)^{1-n}-\frac{\beta}{\alpha}\right)e^{-\alpha k}+\frac{\beta}{\alpha}\right]^{1/(1-n)},k=1,2,3,\ldots \quad (7)$$

Step 6: Calculate $\hat{x}^{(0)}(k+1)$ which is defined as

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) \quad (8)$$

Step 7: In the gray model, the main criterion for assessing forecasting accuracy is relative percentage error which compare the fitted and actual values. The relative percentage error (RPE) is defined as

$$RPE=\xi(k)=\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\times 100\% \quad (9)$$

and the average relative percentage error (ARPE) is

$$ARPE=\zeta(k)=\frac{1}{m-1}\sum_{k=2}^{m}|\xi(k)| \quad (10)$$

Generally speaking, the gery forecasting modes with the smallest ARPE is regarded as the best forecasting model.

Step 8: Consider in-sample and out-of-sample tests.

This study uses in-sample and out-of-sample tests to assess the performance of a full model.

Suppose that the ARPE of in-sample test is ζ_{in}, while the ARPE of out-of-sample test is ζ_{out}.The full model error (ζ_{model}) is defined as

$$\zeta_{model}=Max\{\zeta_{in},\zeta_{out}\} \quad (11)$$

Those grey forecasting models which have the smallest in-sample error don't guarantee to have the smallest in-sample error. Sometimes, there is trade-off relationship between ζ_{in} and ζ_{out}. Therefore, the criterion of (11) become useful to evaluate the forecast performance. The forecasting model which has the smallest ζ_{model} is called the best forecasting model.

Step 9: Calculate the optimal *n*.

Consider the following optimization problem.

$$Min\ \zeta(n\mid p,X^{(0)}(1,m)) \quad (12)$$

$p\in[0,1], n\in R$. The original series is $X^{(0)}(1,m)$ .The solution of above optimization problem is

$$n_j^*=\underset{\{n\}}{Arg\,Min}\ \zeta(n\mid p_j,X^{(0)}(1,m)) \quad (13)$$

where $p_j\in[0,1]$.

Step 10: Seek for all n_j^* for a given p_j.

Set $p\in\{p_1...p_j...p_J\}, j=1,2,...,J$. For a given p_j, figure out the forecast value $\hat{x}_{p_j}(k+1)$.

Step 11: Calculate the weighted average for the forecast value $x(k+1)$ to moderate the effect overestimation and underestimation . Thus,

$$\tilde{x}^{(0)}(k+1)=\{\sum_{j=1}^{j=J}\hat{x}_{p_j}^{(0)}(k+1)\}/J \quad (14)$$

III. Forecasting Taiwan's GDP

Forecasting Twain's GDP is selected as a case study. GDP annual data is obtained from the website of the Ministry of Economic Affairs of Taiwan. The data period is from 2009 to 2014. The unit of GDP is million US dollars. There are two stages to finish forecasting GDP.

In the first stage, the author develops candidate models and uses in-sample and out-of-sample tests to assess models. To develop candidate models, NGBM(1,1) appears with different p. This study assumes that p can be 0.1, 0.3, 0.5, 0.7 and 0.9. Besides, this study proposes weighted average NGBM(1,1) as a new candidate model.

The data from 2009 to 2013 are used to construct NGBM(1,1) with in-sample error. The data in 2014 acts as out-of-sample test. All empirical results are showed in Table I. The fitted value is overestimation as $\xi(k)\%$ is negative. Underestimation exists as $\xi(k)\%$ is positive. For example, as p equals to 0.7, the forecast values are 453905.74, 487781.13, 503250.38, 508853.24 and 508340.42 from 2010 to 2014, respectively. The signs of $\xi(k)\%$ are $-,-,-,+$ and $+$. It means that the forecast values are with overestimation in 2010, 2011 and 2012, and with underestimation in 2013 and 2014. As p equals to 0.5, overestimation exist only in 2012. Thus, the method of weighted average balances the effect of underestimation and overestimation. Using (11) as a criterion to assess grey forecasting models, we find that ARPE of the six candidate modes are 1.8583%, 1.1464%, 3.1960%, 4.0119%, 3.5868% and 1.0407%. The weighted average NGBM(1,1) has the smallest model error.

Moreover, Chen's NGBM [14-15] has the smallest in-sample error (0.6163%) as p is 0.5, but out-of-sample error is not the smallest (3.1960%). It demonstrates that those grey forecasting models with the smallest in-sample error don't guarantee to have the smallest in-sample error.

The results indicate that weighted average NGBM(1,1) is the most suitable for acting as a forecasting model.

In the second stage, we use GDP data from 2010 to 2014 to predict GDP from 2015 to 2017. To obtain the forecast values of weighted average NGBM(1,1), it is necessary to calculate the forecast values of NGBM(1,1) with different *p*. All results show in Table II. It's worth noting that ARPE of the proposed model is 0.1699% which is less than any other NGBM(1,1).

The predicted values of Taiwan's GDP are 547271.91, 567295.77 and 588638.33 from 2015 to 2017, respectively. The evidences show that Taiwan's GDP is steadily growing.

TABLE I: ASSESSING THE FORECAST EFFICIENCY OF WEIGHTED AVERAGE NGBM(1,1) USING IN-SAMPLE AND OUT-OF-EXAMPLE TESTS

	Year 2009	Year 2010	Year 2011	Year 2012	Year 2013	$\zeta_{in}(k)\%$	Year 2014 $\zeta_{out}(k)\%$	$\zeta_{model}(k)\%$
Actual GDP	392065	446105	485653	495845	511293		529587	
Forecast GDP	392065	446816.99	465700.88	485382.87	505896.67	1.8583	527277.46	1.8583
$\xi(k)\%$ (p=0.1, *n*=0.0)	0	-0.1596	4.1083	2.1099	1.0554		0.4361	
Forecast GDP	392065	443562.57	472104.74	492673.98	509288.28	1.0978	523515.31	1.1464
$\xi(k)\%$ (p=0.3, *n*=0.1)	0	0.5699	2.7896	0.6395	0.3920		1.1464	
Forecast GDP	392065	444792.11	480964.36	499820.93	509228.91	0.6163	512661.36	3.1960
$\xi(k)\%$ (p=0.5, *n*=0.2)	0	0.2943	0.9654	-0.8018	0.4036		3.1960	
Forecast GDP	392065	453905.74	487781.13	503250.38	508853.24	1.0393	508340.42	4.0119
$\xi(k)\%$ (p=0.7, *n*=0.2)	0	-1.7486	-0.4382	-1.4934	0.4771		4.0119	
Forecast GDP	392065	462534.65	482690.47	503724.62	525675.37	2.1737	548582.67	3.5868
$\xi(k)\%$ (p=0.9, *n*=0.0)	0	-3.6829	0.6100	-1.5891	-2.8129		-3.5868	
Forecast GDP by weighted average NGBM	392065	450322.41	477848.32	496970.55	511788.49	0.7190	524075.45	1.0407
$\xi(k)\%$	0	0.9454	1.6070	0.2270	0.0969		1.0407	

TABLE II: THE FORECAST VALUE OF TAIWAN'S GDP USING THE WEIGHTED AVERAGE MGBM (1,1)

Year	2010	2011	2012	2013	2014	2015	2016	2017	$\zeta(k)\%$
Actual GDP	446105	485653	495845	511293	529587				
Forecast GDP	446105	487567.30	490823.53	503767.90	521569.99	542700.32	566517.95	592725.42	1.0981
$\xi(k)\%$(p=0.1, n=-0.1)	0	-0.3942	1.0127	1.4718	1.5138				
Forecast GDP	446105	480770.30	494939.05	509525.36	524541.55	540000.27	555914.58	572297.90	0.6216
$\xi(k)\%$(p=0.3, n= 0.0)	0	1.0054	0.1827	0.3457	0.9527				
Forecast GDP	446105	488338.13	494343.81	510174.30	531096.92	555633.18	583182.25	613483.88	0.3398
$\xi(k)\%$(p=0.5, n=-0.1)	0	-0.5529	0.3028	0.2188	-0.2851				
Forecast GDP	446105	486507.81	501014.54	515953.83	531338.58	547182.07	563497.99	580300.41	0.6152
$\xi(k)\%$(p=0.7, n= 0.0)	0	-0.1760	-1.0426	-0.9116	-0.3307				
Forecast GDP	446105	489427.57	504107.80	519228.36	534802.45	550843.69	567366.08	584384.05	1.2451
$\xi(k)\%$(p=0.9, n= 0.0)	0	-0.7772	-1.6664	-1.5520	-0.9848				
Forecast GDP by weighted average NGBM	446105	486522.22	497045.75	511729.95	528669.90	547271.91	567295.77	588638.33	0.1699
$\xi(k)\%$	0	-0.1790	-0.2422	-0.0855	0.1732				

IV. CONCLUSION

Overestimation or underestimation often occurs in forecasting practices. This study adopts weighted average NGBM(1,1) to balance the effect. Besides, a criterion proposed for assessing grey forecasting models includes in-sample and out-of-sample error. This study uses the weighted average NGBM(1,1) to forecast Taiwan's GDP. The results indicate that the proposed method performs better than any other NGBM(1,1) when in-sample and out-of-sample tests are taken into account. Besides, the forecast outcomes show that Taiwan's GDP is steadily growing. It may serve as valuable information for policy makers and investors.

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