

# Effectiveness of Conventional CUSUM Control Chart for Correlated Observations

D. R. Prajapati

**Abstract**—Control charts, one of the important tools of quality control, are also known as Shewhart charts or process behavior charts. Page (1954) was the first, who introduced the Cumulative Sum (CUSUM) control charts for detection of process shifts and claimed that these charts are more efficient compared to Shewhart chart to detect small shifts in the process average. Various schemes of the CUSUM chart for autocorrelated data for sample size of 4 are developed and compared with the schemes of the Shewhart  $\bar{X}$  chart for autocorrelated data. It is found that CUSUM chart outperforms the Shewhart  $\bar{X}$  chart for all the shifts and at all the levels of correlation ( $\Phi$ ) for sample size ( $n$ ) of four. So, the CUSUM control chart is much better option for faster detection in the process mean.

**Index Terms**—CUSUM chart, autocorrelated data, in-control ARLs and Out-Of control ARLs.

## NOMENCLATURE

Following symbols have been used in this paper:

$\mu_o$  = Target mean

$x_i$  = Observation,  $i$

$n$  = Sample size

ARL = Average Run Length

UCL = Upper control limit =  $Shi(i)$

LCL = Lower control limit =  $Slo(i)$

$\Phi$  = Level of correlation

$h$  = decision interval for CUSUM chart

$k$  = slack variable for CUSUM chart

## I. INTRODUCTION

Several types of control charts and their combinations are evaluated for their ability to detect changes in the process mean and variance, since two decades. The performance of a chart is usually measured in terms of the average run length (ARL), which is the average number of samples before getting an 'out-of-control' signal. The main effect of autocorrelation is reduction of 'in-control' ARL, leading to a higher false alarm rate. It has been studied by many researchers that autocorrelation among the observations has a significant effect on the performance of all the types of control charts.

Page [1] introduced the CUSUM control charts for detection of process shifts and claimed that CUSUM charts are more efficient compared to Shewhart chart to detect

small shifts in the process average. In CUSUM chart, the decision is based upon cumulative sum of a number of observations and in an implicit manner the process history for arriving at a final decision is considered. The ARLs of CUSUM chart depend upon the parameters  $h$  and  $k$ .

One of the assumptions in implementing the chart is that the process outputs must be independent and identically distributed (IID) but usually there is some correlation among the data. When this correlation builds up automatically in the entire process, this phenomenon is called autocorrelation.

The observations from the process output are usually positively correlated in most of the cases.

## II. THEORY OF CONVENTIONAL CUSUM CHART

In CUSUM chart, the cumulative sums of deviations of successive sample means from a target specification are plotted. So the permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, CUSUM chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the  $\bar{X}$  chart. The control limits for tabular method are calculated as:

$$Shi(i) = \max[(0, Shi(i-1) + xi - \mu_o - k)]$$

$$Slo(i) = \max[(0, Slo(i-1) + \mu_o - k - xi)]$$

where the  $h$  is called the decision interval and mostly its value is taken from 4 to 5. The parameter  $k$  is known as the reference value or slack variable and its value is taken to be half the delta shift. Initially the values of  $Shi(0)$  and  $Slo(0)$  are set to 0.0. Fig. 1 shows the sample CUSUM chart.

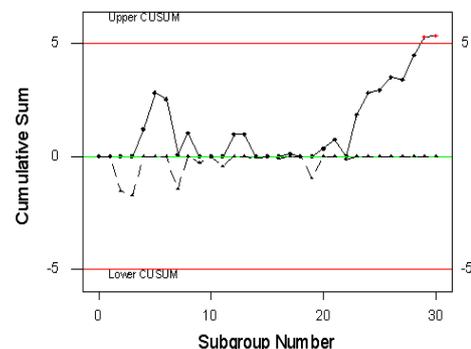


Fig. 1. Sample CUSUM chart.

For two sided CUSUM, when either  $Shi(i)$  or  $Slo(i)$  exceeds the value,  $h$ , the process is said to be out of control. If one sided or upper sided CUSUM is considered then only the  $Shi(i)$  is compared with  $h$ .

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When conventional CUSUM chart is applied to the autocorrelated data, it leads to more frequent false alarms.

### III. LITRETURE REVIEW

Various researchers examined the performance of CUSUM charts with and without autocorrelation as discussed in this section.

Lucas [2] proposed parabolic CUSUM scheme by inserting a parabolic section in V mask. The parabolic section suggested by the author modifies the control action for a number of points until the parabolic section and V mask coincide. Westgard *et al.* [3] proposed modified CUSUM scheme and obtained the results by simulation. They simulated the probability of an out of control signal in series of N samples ( $1 \leq N \leq 30$ ). Tseng and Adams [4] proved that the traditional control charts such as the Shewhart chart, cumulative sum (CUSUM) chart and exponentially weighted moving average (EWMA) chart have the adverse effect on the presence of autocorrelation in data. Monitoring schemes which used these traditional control charts in conjunction with time series based forecasts are proposed and shown to have properties superior to schemes based on traditional charts alone. Walter and Peter [5] investigated that the cumulative sum (CUSUM) technique is well-established in theory and practice of process control. For a variant of the CUSUM technique, the cumulative score chart, authors investigated the effect of serial correlation on the in-control ARL. Using the fact that the cumulative score statistic is a correlated random walk with a reflecting and an absorbing barrier, authors derived an approximate but closed-form expression for the ARL of a control variable that follows a first-order autoregressive process with normally distributed disturbances. Apley and Shi [6] presented an on-line SPC technique, based on a Generalized Likelihood Ratio Test (GLRT), for detecting and estimating mean shifts in autocorrelated processes that followed a normally distributed autoregressive integrated moving average (ARIMA) model. The GLRT is applied to the uncorrelated residuals of the appropriate time-series model. Kim *et al.* [7] presented a new CUSUM chart for monitoring shifts in the mean of autocorrelated data. The monitoring statistic is the plain cumulative sum of differences between the observations and a target value, and the derivation of the control chart is based on an approach popular in the simulation literature rather than the classical CUSUM chart. They proposed a method which is completely model-free and valid asymptotically in the sense that it detects a shift and the direction of the shift correctly with a pre-specified average run length. Wu *et al.* [8] proposed a VSSI WLC scheme, which is a weighted-loss-function-based CUSUM (WLC) scheme using variable sample sizes and sampling intervals (VSSI). This scheme detects the two-sided mean shift  $\delta_\mu$  and increasing standard deviation shift  $\delta_\sigma$  based on a single statistic WL (the weighted loss function). Mertens *et al.* [9] monitored the livestock production processes by means of statistical control charts. The non-stationary and autocorrelated characteristics of most data originating from such processes impeded the direct introduction of these data into control charts. To deal with these characteristics

Engineering Process Control strategies can be applied. Stationary is achieved by modeling and subtracting the time dependent trend, using a non-linear model. The autocorrelation structure in the residual data is modeled and corrected for by means of an ARMA model. Liu and Wang [10] investigated the autoregressive process with the measurement error. For detecting the step shift of the autoregressive process mean with measurement error, a CUSUM chart based on the maximum log-likelihood ratio test is obtained. The simulation results showed that this new CUSUM scheme works well when the process is negatively autocorrelated. Lee *et al.* [11] formulated and evaluated distribution-free statistical process control (SPC) charts for monitoring shifts in the mean of an autocorrelated process when a training data set was used to estimate the marginal variance of the process and the variance parameter. Two alternative variance estimators were adapted for automated use in DFTC-VE, a distribution-free tabular CUSUM chart, based on the simulation-analysis methods of standardized time series and a simplified combination of autoregressive representation and non-overlapping batch means. Kiran *et al.* [12] carried out a study on training algorithms for control charts pattern recognition and selected the best two patterns with their optimal structure for both Type I and Type II errors for generalization with and without early stopping and proposed the best one. Chang and Wu [13] observed that it is difficult to find the run length distribution and the average run length. They developed a general and unified approach, based on the use of discretization and the finite Markov chain imbedding technique to investigate the run length properties for various control charts, when the process observations are autocorrelated.

Lee and Apley [14] investigated that the residual-based control charts for autocorrelated processes are sensitive to time series modeling errors, which can seriously inflate the false alarm rate. They proposed a design approach for a residual-based EWMA chart that mitigates this problem by modifying the control limits based on the level of model uncertainty. Using a Bayesian analysis, they derived the approximate expected variance of the EWMA statistic, where the expectation is with respect to the posterior distribution of the unknown model parameters. They compared their approach to two other approaches for designing robust residual-based EWMA charts and claimed that their approach generally results in a more appropriate widening of the control limits. Snoussi [15] discussed the development of a multivariate control charting technique for short-run autocorrelated data manufacturing environment. They proposed an approach which is a combination of the multivariate residual charts for autocorrelated data and the multivariate transformation technique for IID process observations of short lengths. The proposed approach consists in fitting adequate multivariate time-series model of various process outputs and computes the residuals, transforming them into standard normal  $N(0, 1)$  data and then using standardized data as inputs to plot conventional uni-variate IID control charts. Lin *et al.* [16] stated that the presence of autocorrelation in the process data can result in significant effect on the statistical performance of control charts. They presented the economic design of ARMA (autoregressive moving average) control chart to determine

the expected total cost per hour. Singh and Prajapati [17] studied the effect of correlation on the performance of CUSUM and EWMA charts for the positively correlated data. The ARLs at various set of parameters of the CUSUM and EWMA charts are computed, using MATLAB software. The behavior of the CUSUM and EWMA charts at the various shifts in the process mean is studied, analyzed and compared at different levels of correlation ( $\Phi$ ). Mahadik [18] presented Hotelling's  $T^2$  charts with variable sampling interval and warning limit (VSIWL) and concluded that VSIWL  $T^2$  chart yields different out-of-control performances for the same in-control statistical performance depending on the choices of values of its warning limit.

IV. FORMULATION OF SERIES OF AUTO CORRELATED OBSERVATIONS

The series of normal distributed IID numbers is generated using the MATLAB. The positively correlated series of observations is obtained using the coefficient of correlation ( $\Phi$ ) in the series. Assuming N pairs of observations on two variables, x and y. The correlation coefficient between x and y is given by equation (1). Some authors use coefficient of correlation ( $\Phi$ ) instead of “r”.

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left[ \sum (x_i - \bar{x})^2 \right]^{1/2} \left[ \sum (y_i - \bar{y})^2 \right]^{1/2}} \quad (1)$$

where, the summations are over the N observations.

The series generated are positively correlated in nature. For each level of correlation ( $\Phi$ ), various schemes of chart are developed, using MATLAB. The procedure to implement the CUSUM chart is given in the following section.

V. PROCEDURE TO IMPLEMENT THE CUSUM CHARTS

In this paper, data are simulated with the help of MATLAB software. The autocorrelated numbers are generated from the series of data taken from the standard normal distribution with mean of zero and standard deviation of one. Initially the process is assumed to be under control, at the given mean and standard deviation. The following procedure may be adopted to calculate the ARLs of the CUSUM charts at various shifts in the mean.

**Step 1.** Take the observations from industry at random basis.

**Step 2.** Observations may also be generated randomly at given mean and standard deviation.

**Step 3.** For simulation, 10,000 observations are generated with a sample size of 4.

**Step 4.** The observations are generated in such a way that there should be positive correlation with their previous data.

**Step 5.** Those sets of parameters of the control chart which gives the Average Run Lengths (ARL) of approximately 400 are considered for comparison.

**Step 6.** For the selected combinations of CUSUM schemes, the ARLs are calculated at various shifts in process mean, at different values of h and k = 0.5, for

different level of correlation ( $\Phi$ ).

Next section deals with the computation of ARLs of the conventional CUSUM chart at different levels of correlation ( $\Phi$ ) for sample size of 4.

VI. VARIOUS SCHEMES OF CUSUM CHART

The performance of the CUSUM chart is measured in terms of Average Run Lengths (ARLs). The ARLs at different levels of correlation ( $\Phi$ ) are calculated by keeping the in-control ARL of approximately 371. Control limits are adjusted in such a way that the in-control ARL of approximately 371 may be maintained. Table I shows the various schemes of the CUSUM chart at the level of correlation ( $\Phi$ ) of 0.00 and 0.20.

TABLE I: ARLs CUSUM CHART FOR SAMPLE SIZE OF FOUR AT  $\Phi = 0.00$  AND 0.20

ARLs of CUSUM chart for n = 4						
Shift (in mean)	$\Phi = 0.00$			$\Phi = 0.20$		
	h = 5.0 and k=0.5	h =4.9 and k=0.5	h = 5.1 and k=0.5	h=6.9 and k= 0.5	h=6.8 and k= 0.5	h=7.0 and k=0.5
0.0	371.0	370.4	372	372	372.4	371
0.5	36.1	38.8	41.5	54.9	52.2	48.9
1.0	10.4	11.5	12.3	12.3	12.4	12.0
1.5	4.6	4.6	4.7	5.1	5.3	5.1
2.0	3.2	3.3	3.5	4.3	4.5	3.8
2.5	2.5	2.6	3.0	3.0	3.0	2.8
3.0	1.8	1.8	1.9	1.9	2.1	1.9

TABLE II: ARLs CUSUM CHART FOR SAMPLE SIZE OF FOUR AT  $\Phi = 0.05$  AND 0.70

ARLs of CUSUM chart for n = 4						
Shift (in mean)	$\Phi = 0.50$			$\Phi = 0.70$		
	h = 8.0 and k=0.5	h = 8.1 and k=0.5	h = 8.2 and k=0.5	h=8.9 and k= 0.5	h=9.0 and k=0.5	h=9.1 and k=0.5
0.0	372	372	371	372	372.4	371
0.5	58.2	68.8	70.5	73.2	65.2	75.2
1.0	15.2	18.5	19.3	35.1	30.2	37.2
1.5	5.9	6.6	7.4	7.7	7.6	8.5
2.0	4.6	5.3	5.5	5.7	5.5	6.3
2.5	3.3	3.6	3.5	3.6	3.5	4.2
3.0	2.1	2.1	2.1	2.3	2.3	2.4

The ARLs of various schemes of CUSUM chart for

sample size of four at the levels of correlation ( $\Phi$ ) of 0.50 and 0.70 are shown in Table II.

The ARLs of various schemes of CUSUM for sample size of two at the levels of correlation ( $\Phi$ ) of 1.00 are shown in Table III.

TABLE III: ARLS OF CUSUM CHART FOR SAMPLE SIZE OF FOUR AT  $\Phi = 1.00$

Schemes of conventional CUSUM chart			
Shift (in mean)	$\Phi = 1.0$		
	$h=9.2$ and $k=0.5$	$h=9.3$ and $k=0.5$	$h=9.4$ and $k=0.5$
0.0	370.6	371	372
0.5	80.9	85.6	89.4
1.0	35.1	39.5	41.2
1.5	8.5	8.5	8.7
2.0	6.9	7.0	7.3
2.5	3.6	3.7	3.9
3.0	2.5	2.5	2.6

Out of above suggested schemes, those schemes which are having the in-control ARLs of approximately 370 and having the out-of-control ARLs consistently lower than other schemes are selected as the optimal schemes. Other possible schemes of the CUSUM chart have also been tried but their in-control ARLs are not near to 370, that's why those schemes have not been included in this paper. Table IV shows the optimal schemes of the conventional CUSUM chart for  $n = 4$  for various levels of correlation ( $\Phi$ ).

TABLE IV: ARLS OF OPTIMAL SCHEMES OF CUSUM CONTROL CHART FOR SAMPLE SIZE OF FOUR

Shift (in mean)	ARLs of CUSUM chart at $n=4$				
	$\Phi=0.00$	$\Phi=0.20$	$\Phi=0.50$	$\Phi=0.70$	$\Phi=1.00$
	$h=5.0$ and $k=0.5$	$h=7.0$ and $k=0.5$	$h=8.0$ and $k=0.5$	$h=9.0$ and $k=0.5$	$h=9.2$ and $k=0.5$
0.00	371.0	371	372	372.4	370.6
0.50	36.1	48.9	58.2	65.2	80.9
1.0	10.4	12.0	15.2	30.2	35.1
1.5	4.6	5.1	5.9	7.6	8.5
2.0	3.2	3.8	4.6	5.5	6.9
2.5	2.5	2.8	3.3	3.5	3.6
3.0	1.8	1.9	2.1	2.3	2.5

Following facts are summarized from the Tables I to V:

- 1) The false alarm rate (in-control ARL) of approximately 370 is maintained, for all the optimal schemes of conventional CUSUM chart.
- 2) When the level of correlation ( $\Phi$ ) increases, the sensitivity of the conventional CUSUM chart to detect

shift in the process mean decreases. For sample size of four and level of correlation ( $\Phi$ ) of zero, the CUSUM chart detects  $0.5\sigma$  shift in the process mean after about 10 samples, whereas; at level of correlation ( $\Phi$ ) of one, the same shift in the process mean is detected after 35 samples.

- 3) The in-control and out-of-control ARLs of the optimal schemes of the conventional CUSUM chart also depends on the values of parameters, 'h' and 'k'.

### VII. COMPARISON WITH SHEWHART $\bar{X}$ CHART

Shewhart [19] developed basic  $\bar{X}$  chart for independent and identically distributed (IID) data. Various schemes of the Shewhart  $\bar{X}$  chart for sample size of four at various levels of correlation ( $\Phi$ ) are developed by Singh and Prajapati [20]. Values of width of control limits (L) are adjusted in such a way that they generate the in-control average run lengths (ARLs) of approximately 370 in all the presented schemes. They also calculated the ARLs for sample size of four at various levels of correlation. The performance of the  $\bar{X}$  chart is compared with performance of CUSUM chart in terms of ARLs at various levels of correlation. The comparison of ARLs of optimal schemes of Shewhart  $\bar{X}$  chart with the ARLs of optimal schemes of the CUSUM chart for sample size ( $n$ ) of four at the levels of correlation ( $\Phi$ ) of 0.00, 0.25 is presented in Table V.

TABLE V: ARLS COMPARISON FOR SAMPLE SIZE OF FOUR AT  $\Phi=0.00, 0.25$

Shift (in mean)	$\Phi = 0.00$		$\Phi = 0.50$	
	Shewhart $\bar{X}$ chart	CUSUM Chart	Shewhart $\bar{X}$ chart	CUSUM Chart
0.00	370.4	371.0	370.4	372
0.50	155.2	36.1	188.7	58.2
1.0	44.0	10.4	51.5	15.2
1.5	14.9	4.6	20.1	5.9
2.0	6.3	3.2	8.4	4.6
2.5	3.2	2.5	4.0	3.3
3.0	2.0	1.8	2.1	2.1

Similarly, the comparison of ARLs of optimal schemes of Shewhart  $\bar{X}$  chart with the ARLs of optimal schemes of the CUSUM chart for sample size ( $n$ ) of four at the levels of correlation ( $\Phi$ ) of 1.00 is presented in Table VI.

- Following facts are observed from the Tables V and VI:
- All the schemes have the in-control Average run length (ARL) of approximately 370.
- When autocorrelation exist in the observations, false alarm rate increases in all the shifts in the process mean.
- CUSUM control chart outperforms the Shewhart  $\bar{X}$  chart for all the shifts and at all the levels of correlation ( $\Phi$ ) for sample size ( $n$ ) of four.

- The in-control and out of control ARLs also depend on the parameters of the Shewhart  $\bar{X}$  and CUSUM charts.

TABLE VI: ARLS COMPARISON FOR SAMPLE SIZE OF FOUR AT  $\Phi=1.00$ 

Shift (in mean)	$\Phi = 1.00$	
	Shewhart $\bar{X}$ chart	CUSUM Chart
0.00	370.4	370.6
0.50	207.2	80.9
1.0	75.0	35.1
1.5	27.8	8.5
2.0	11.8	6.9
2.5	6.7	3.6
3.0	3.3	2.5

### VIII. CONCLUSIONS

Autocorrelation among the observations can have significant effect on the performance of a control chart. The detection of special cause/s in the process may become very difficult in such situations. A process is said to be in a state of statistical control, if it operates under common causes of variation and the probability distribution representing the quality characteristic is constant over time. If there are some changes over time in this distribution, the process is said to be out-of-control.

In this paper, the performance of CUSUM chart has been studied at the various levels of correlation ( $\Phi$ ). Since CUSUM chart consider the past history of the data; that's why its performance deteriorates for correlated data. The performance of the charts is measured in terms of the Average Run Lengths (ARLs). It is found that the CUSUM schemes show faster signals than the Shewhart schemes for various shifts in process mean at each level of correlation ( $\Phi$ ). The limitation of the CUSUM chart is that they can catch small shifts in the process mean more efficiently than Shewhart  $\bar{X}$  chart only when there is a single and sustained shift. If samples are not taken from same stream, these charts may not be able to catch the process shift quickly. Other limitation of CUSUM and EWMA charts is that they are not efficient to catch a larger shift in the process mean; that's why various industries prefer to use the standard Shewhart  $\bar{X}$  chart because of its inherent simplicity.

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