Analysis of Real Ocean Surface Photo Imaging by Using Weighted Least Square Method

Muhammad Sameer Sheikh, Qunsheng Cao, and Caiyun Wang.

Abstract—This paper is focused on an analysis of the real images of ocean surface, such as dark ocean surface, sunlight effect on ocean surface and reflected by boat, by using two-dimensional weighted least square method (2D-WLSM). We introduced three different factor values to achieve and get the maximum information of the ocean surface which contained some noisy signal. We determine the attributes of the real image using an image tool, then we convert the axes coordinates of the real image into the pixel coordinates and obtain the pixel information of the real images by the 2D-WLSM, and finally we compare the response factors of the measured values for the different cases. It is shown that the measured pixel response values are very close to the factor values and the errors between them are very small. Compared the error of the traditional LSM with WLSM we used, our method has shown better graphic results and the error are significantly reduced.

Index Terms—Image tool, ocean surface, real image, weighted least square method (WLSM).

I. INTRODUCTION

The research about ocean surface has started long time [1], ocean is difficult area for researchers of computer graphics. It combined together with all scale of waves [2]. The sea surface is with its complex interplay between the waves and with reflections of the sun and sky. There are several algorithms designed for modeling the surface of the sea, but it is highly complex problem. Evaluation and attributes of real images are involved by using image tool [3]. Weighted Least Square Method and Least Square Method is used to minimize the sum of the square of the error [4], [5].

Imaging radar is one kind of equipment to be used for imaging purpose is used when the radio waves reflect the object caused some changes in radio waves [6]. The return back wave is used to create images by collecting acquired data to create the 2D and 3D image. A problem occurs in many applications is how to eliminate noise from real image surface which contains some useful information. Here we have analyzed on real images pixel information by using the WLSM techniques to reduce the noise in real image surface and achieved the useful information from the real images. There is some application of weighted least square method to estimate the parameter of the surface on the digital image [7]. Some methods were proposed to de noising the digital image [8] and B-Spline shape representation [9]. Determination of radar image of ocean wave and surface current by least square curve fitting [10]. This paper have used the WLSM to get maximum information of the ocean surface from the photo image contained noise, by removing the noise effects in the real image of ocean surface. The image sizes and attributes have been evaluated from information of image, then converted the axes coordinates of all real images into the pixels coordinates of image. We processed pixel information which contain some noisy signal by applying 2D-WLSM, we emphasizes on real image having noisy information of image contains large errors. The error has been significantly reduced by our method and we achieved maximum useful information of images which containing some noise from the ocean surface by introducing three adjustable factor values.

II. MATHEMATICAL METHOD

Objective of the present work is to simulate and achieve the maximum information from photos image of the ocean surface that contain noisy information. For a general photo image, the main information processing steps are as follows:

- Using the image tool to calculate the real image attributes and information.
- Converting the axes coordinates of real images into the pixel coordinates of the images.
- Processing the pixel information of the real images used by 2D-WLSM.
- Comparing the errors of the LSM with error of the WLSM, evaluating the performance using the polynomial curve fitting.

In order to find the control parameter of the real image \( \{\phi_i, i = 1, 2, \ldots, n\} \), it is determined by the measured pixel values and the known values. There are two hypothesis to be considered. For the hypothesis 1, the measured pixel values are \( \{b_j, j = 1, 2, \ldots, m\} \) along the vertical direction, and known pixel values are \( \{a_i, i = 1, 2, \ldots, n\} \) along the horizontal direction. In contrast to the hypothesis 1, the measured pixel values are along the horizontal direction and known pixel values are \( \{b_j, j = 1, 2, \ldots, m\} \) along the vertical direction for the hypothesis 2.

The Least Square Method is used to evaluate the control parameter \( \{\phi_i\} \) and \( \{\phi_j\} \) of real image, the calculation errors are as follows,

\[
\epsilon_1 = \sum_{k=1}^{m} \left[ h\phi(a^k) - b^k \right]^2
\]

\[
\epsilon_2 = \sum_{k=1}^{m} \left[ h\phi(b^k) - a^k \right]^2
\]
where \( h\phi_i(a) \) and \( h\phi_j(b) \) are the predicted pixel values for two hypothesis. To get the control parameter value we do partial differential of Eq.(1) and (2), and then we obtain [9].

\[
\phi_i = (A^T A)^{-1} A^T B \\
\phi_j = (B^T B)^{-1} B^T A
\]

(3)

(4)

In the Eq. (3), \( B \) is the output pixel values of \( \{b_j\} \) and \( A \) is input sets of \( \{a_i\} \), similarly in Eq. (4), \( A \) is the output pixel value of \( \{a_i\} \) and \( B \) is the input set of \( \{b_j\} \).

For two hypothesis,

\[
h\phi(a) = \sum_{i=0}^{n-1} \phi_i a_i = \phi^T a
\]

(5)

\[
h\phi_j(b) = \sum_{i=0}^{n-1} \phi_j b_j = \phi^T b
\]

(6)

where the ranges of the input parameter ranges \( a \) and \( b \) 0 to \( n-1 \).

Assuming \( \epsilon \) is the minimized parameter, which is the square of the error between the predicted pixel values \( h\phi_i(a) \) or \( h\phi_j(b) \) and all the output \( b_j \) or \( a_i \) for all \( K \) value for the hypothesis.

\[
\epsilon = \frac{1}{2} \sum_{k=1}^{m} w^k [h\phi(a^k) - b^k]^2
\]

(7)

\[
= \frac{1}{2} (A\phi - B)^T W (A\phi - B)
\]

Table I is listed the matrix size of input \( A(i, j) \) and predicted \( B(j) \) for hypothesis 1, and input \( B(i, j) \) and predicted \( A(i) \) for hypothesis 2, in which the index \( n \) is the number of pixel elements and dimension is shown below.

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( A(i, j)[n \times n] )</td>
<td>( B(j)[n \times 1] )</td>
</tr>
<tr>
<td>2</td>
<td>( B(i, j)[n \times n] )</td>
<td>( A(i)[n \times 1] )</td>
</tr>
</tbody>
</table>

Similarly to the LSM, the minimized squared error of the real image ocean surface representation in the WLSM are as given as:

\[
\epsilon_1 = \sum_{k=1}^{m} w^k [h\phi(a^k) - b^k]^2
\]

(8)

\[
\epsilon_2 = \sum_{k=1}^{m} w^k [h\phi_j(b^k) - a^k]^2
\]

(9)

where the parameter \( w \) is the weight.

Based on the Eq. (8) and (9), the total error yields.

\[
\epsilon = \sum_{k=1}^{m} w^k [h\phi(a^k) - b^k]^2 + \sum_{k=1}^{m} w^k [h\phi_j(b^k) - a^k]^2
\]

(10)

In order to obtain better response of the system, the weights are chosen as follow,

\[
w^k = e^{-\frac{(a^k-a)^2}{2\tau^2}}
\]

(11)

\[
w^k = e^{-\frac{(b^k-b)^2}{2\tau^2}}
\]

(12)

where the parameter \( a \) and \( b \) are the input pixel values, \( k \) is the number of the pixels element, and \( \tau \) is the adjustable factor. The weight \( w \) is very important in the WLSM, depends on the adjustable factor value \( \tau \).

Partial differentiate to \( \epsilon \) with respect to \( \phi_i \) minimize the weighted least square error, so

\[
\frac{\partial \epsilon}{\partial \phi_i} = (A^T W A\phi - A^T W B)
\]

(13)

It leads finally,

\[
\phi_i = (A^T W A)^{-1} A^T W B
\]

(14)

\[
\phi_j = (B^T W B)^{-1} B^T W A
\]

(15)

### III. PROCESS BLOCK DIAGRAM

The process diagram contains of several block and each block has its own process and functionality, consider the real image of oceans surfaces such as dark ocean surface, sunlight effect on a surface and reflected by boat, captured by radar, the real image we get is noisy image in terms of dark ocean, sunlight and reflected by boat, after getting the noisy real image we find the attributes of the real image ocean’s surface, secondly we convert the axes coordinate to the pixel coordinates of the image, though we get pixel information of the real image which contain some noise, and finally we apply weighted least square method (2D) by defining three different factor values (see Fig. 1).

### IV. SIMULATION RESULTS

First, we consider an ocean surface image, the all image information is obtained by the image tool, and the attributes are shown in Table II, which the size of height and the width of the image are 360 and 480 pixels, respectively. Fig. 2 is the original dark ocean surface in the real image.

Similarly, we can obtain successively the image information of Sun light effect on ocean and reflected by boat on ocean. The sizes of images both are 177×284 pixels.

For any real image, we get pixel information of all real images by converting from the axes coordinates of the real images and then processed this pixel information using by the 2D-WLSM fit to exponential by defining three factor values to achieve the more information of the real image which contain some noisy information. We compare the response between the measured pixel information and adjust factor values (see Fig. 1).
Eq. (11) and (12) are used to determine the weight of the system, Eq. (14) and (15) are used to find value of $\Phi_i$ and $\Phi_j$ which are used for the both hypothesis to minimize the error between the measured pixel information and $\tau$ values.

Fig. 3 are the comparisons of the response of the dark ocean surface real image for different factor $\tau$ values and measured pixel values by simulating. It has shown that the two responses are nearly same, which means that our measured pixel values are very close to all factor values. From Fig. 3(a), when factor $\tau$ increased, the measured pixel response is nearly the same of factor values.

In Fig. 3(b) there is slightly difference between two responses. Because the measured pixel value and factor values were primarily caused by the weighting function and the input parameter when it approaching $\tau = 25$ the measured pixel response has some resemblance of factor response, and the calculated error is less when applied the WLSM by defining three different factor values.

Now consider the sunlight effect on the ocean surface in real image shown in Fig. 4, the size of image has been calculated using by the image tool.

Eq. (11) and (12) are used to determine the weight of the system, Eq. (14) and (15) are used to find value of $\Phi_i$ and $\Phi_j$ which are used for the both hypothesis to minimize the error between the measured pixel information and $\tau$ values.

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Now consider the sunlight effect on the ocean surface in real image shown in Fig. 4, the size of image has been calculated using by the image tool.

Fig. 5 are the comparisons of the response of the ocean surface with sunlight to the real image corresponding different factor $\tau$ values and measured pixel values by simulating.

Fig. 5(a) is presented that in the early time region there has slightly difference between the measured pixel value and the factor values because of the sun light effect and the factor value, when the factor is increased to 10 the measured
pixel response is quite closer to the factor response and smoothly, while in Fig. 5(b) is slightly differ at initially when the response started because of initial weights and factor value but when it is approaching toward $\tau=25$ the measured pixel response of sunlight image is close to the factor value response and the error between them is less.

![Image](image1.png)

![Image](image2.png)

![Image](image3.png)

Finally, we consider the reflected image by a boat on the ocean surface shown in Fig. 6.

Fig. 7 is the comparisons of the response of the reflected image by a boat for different factor $\tau$ values and the measured pixel value by simulating.

Fig. 7(a) is shown the measured pixel values at the starting stage are nearly identical to the factor value, at the middle stage the measured pixel values response are slightly differ from factor response because of the reflected object and the increased weights. When the factor value $\tau$ chosen as 25, the measured pixel values response of the boat are nearly same the factor response and the error between them is less.

Fig. 7(b) shows that the measured pixel value and the factor value are identical when the weight is low. At the middle stage the measured responses are slightly differ from the factor value because of reflected object and increasing weight. When $\tau=25$, the measured response is closely identical to the factor response and the calculated error between them is minimized.

For two hypothesis in order to compare the errors between the LSM and the WLSM, consider 10 pixel values ($k=10$) for each image such as dark image, sunlight image and reflected image. For hypothesis 1 and hypothesis 2, the errors are given in Table III and Table IV, respectively, the weighted LSM has less error than the LSM.

### Table III: Approximation Error of Hypothesis 1

<table>
<thead>
<tr>
<th>Images</th>
<th>LSM</th>
<th>WLSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark Image</td>
<td>6.8</td>
<td>0.087</td>
</tr>
<tr>
<td>Sun Light Image</td>
<td>49.38</td>
<td>9.43</td>
</tr>
<tr>
<td>Reflected Boat</td>
<td>30.3</td>
<td>10.88</td>
</tr>
</tbody>
</table>

### Table IV: Approximation Error of Hypothesis 2

<table>
<thead>
<tr>
<th>Images</th>
<th>LSM</th>
<th>WLSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark Image</td>
<td>7.2</td>
<td>0.763</td>
</tr>
<tr>
<td>Sun Light Image</td>
<td>27.2</td>
<td>2.59</td>
</tr>
<tr>
<td>Reflected Boat</td>
<td>49.95</td>
<td>15.9</td>
</tr>
</tbody>
</table>

Fig. 8 shows that evaluation of the performance of the method, the curve of the simulation result by the WLSM and the curve of the second-order polynomial are fitting. In Fig. 8 at the beginning stage there is little difference between the measured value and the curve response, but at the middle
and final stage there is some resemblance between two curves. It is noted that our measured values are close to the curve response and the errors between them are less. Fig. 9 displays the error comparison between the LSM and the WLSM techniques by considering the image reflected by boat for hypothesis 1 evaluate the error values response between the LSM and the WLSM. It has found that the error is reduces significantly in the WLSM, at the same time, the spikes in the error curve have been reduced obviously by using our proposed method.

![Fig. 8. Comparison of the simulation result and second-order polynomial.](image)

![Fig. 9. Error comparison between LSM and WLSM.](image)

V. CONCLUSIONS

In this paper, the ocean surface real images have been studied firstly based on the WLSM for two assumed hypothesis. Three adjustable factor values have been introduced for both hypothesis and the responses of the real image ocean surface have been evaluated. It has found that the measured pixels values response are very close to the evaluated response, we have achieved the maximum information of the ocean surface which contain noisy information’s and the ocean surface errors have been minimized. The error comparison between the LSM and the WLSM has been done and as well the comparison of our simulated data with polynomial curve fitting.

ACKNOWLEDGMENT

This work is supported by Key Laboratory of Radar Imaging and Microwave Photonics Ministry of Education, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016 People Republic China.

REFERENCES


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