# Novel Method for Improving Motion Accuracy of a Large-Scale Industrial Robot to Perform Offline Teaching Based on Gaussian Process Regression 

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#### Abstract

Industrial robots that can respond to the current needs for variable-type and variable-volume production and that can play a variety of roles such as processing and transporting with a single robot to reduce time and cost. If realized, these robots will help save space in factories and increase production efficiency. However, this requires high positioning accuracy of the robot. In this study, we analyze the motion accuracy of industrial robots and their compensation method to construct this system. Here, we use a laser tracker to measure the coordinates of the hand tip of the robot when the robot is stationary. Subsequently, the error amount in an arbitrary posture is predicted using a Gaussian process. Furthermore, Bayesian optimization is used to efficiently search for points where the positioning error norm is likely to be large, which is then compensated for by a feedback method. This method successfully reduced the time cost of the experiment to approximately one-tenth of that required in the previous study and achieved a correction of approximately $66 \%$. However, because this method alone does not perform an exhaustive measurement, it is unclear whether all the points predicted to have small errors are so small that they do not require correction. Therefore, future studies, we will aim to verify this issue by considering the time efficiency.


Index Terms-Component, automation, industrial robots, positioning error, gaussian process, bayesian optiomization

## I. Introduction

The development of an automatic teaching system with offline teaching is essential to expand the range of applications of industrial robots. However, vertically articulated robots exhibit a complex overlap of kinematic and non-kinematic errors, and the absolute positioning error is approximately 100 times higher than that of machine tools. In addition, geometric error factors, such as offset and rotational angle errors of each joint significantly affect the error vector. In previous studies, the D-H (Denaviet-Hartenberg) method [1], which is a type of geometric approach, was used [2]. However, the relationship between these parameters and the positioning error of the robot is complicated and could not be formulated [3]. Therefore, machine learning was used to observe the relationship between these parameters and the

[^0]positioning error of the robot. However, the proposed correction method required 600 measurements, and so, was time-consuming and costly. In this study, we predicted the error vectors in each axis direction using a Gaussian process and Bayesian optimization.

## II. EXPERIMENTAL EQUIPMENTS AND BASIC THEORY

## A. Experimental Equipmemt

Fig. 1 shows a model diagram of the experimental apparatus, a large robot made by Fujikoshi, in its initial posture. $J_{i}(i=1 \sim 6)$ denotes each joint. A laser tracker manufactured by IHI Escube was used to measure the positioning error. The measurement accuracy was $\pm 0.25 \mathrm{~mm}$ $/ 2.0 \mathrm{~m}$ [4]. Fig. 2 shows the layout of the laser tracker.


Fig. 1. Robot model.


Fig. 2. Laser tracker layout.

## B. Coordinate Transform

First, if a point $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ in the laser tracker coordinates is $(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$ in the robot coordinate system, then using the parameters $a, b, c, d, e, f, g, h, i, j, k$

$$
\left\{\begin{array}{c}
X=a x+d y+g z+j  \tag{1}\\
Y=b x+e y+h z+k \\
Z=c x+f y+i z+l
\end{array}\right.
$$

The following is an example of the use of this expression. From this equation, 12 parameters $a \sim l$ can be obtained if the measurement is performed using four reference points for coordinate transformation [5]. However, because the error is zero at the point used for coordinate transformation, the question is whether an accurate coordinate transformation can be performed. Therefore, we used the least-squares method to perform a coordinate transformation with more than four reference points.

$$
\begin{gathered}
A=\left(\begin{array}{cccc}
x_{1} & y_{1} & z_{1} & 1 \\
x_{2} & y_{2} & z_{2} & 1 \\
x_{3} & y_{3} & z_{3} & 1
\end{array}\right), X=\left(\begin{array}{ccc}
\Delta a & \Delta b & \Delta c \\
\Delta d & \Delta e & \Delta f \\
\Delta g & \Delta h & \Delta i \\
\Delta j & \Delta k & \Delta l
\end{array}\right) \\
\vdots \\
L=\left(\begin{array}{ccc}
X_{1}-f_{10} & Y_{1}-f_{20} & Z_{1}-f_{30} \\
X_{2}-f_{40} & Y_{2}-f_{50} & Z_{3}-f_{60}
\end{array}\right) \\
\vdots \vdots=\left(\begin{array}{ccc}
v_{1} & \vdots & v_{2} \\
v_{4} & v_{5} & v_{6} \\
\vdots & \vdots
\end{array}\right)
\end{gathered}
$$

If we assume that

$$
\begin{equation*}
A X=L+V \tag{2}
\end{equation*}
$$

and finding the modification $X$ such that the residual $V$ is minimized, we obtain

$$
\begin{equation*}
A^{T} A X=A^{T} L \tag{3}
\end{equation*}
$$

and the solution of this equation can obtain as Eq. (4).

$$
\begin{equation*}
X=\left(A^{T} A\right)^{-1} A^{T} L \tag{4}
\end{equation*}
$$

## C. Gaussian Process Regression

We used a Gaussian process, which is a type of statistical method, to predict the error value instead of intuition at the work site. The basic theory is that the distribution of the function value $f(x)$ obtained from any n points $X_{n}$ follows a multidimensional Gaussian distribution in a stochastic process defined by a function $\mu(x)$, which gives the mean value and a kernel function $k\left(x, x^{\prime}\right)$, which gives the covariance [6]. In this study, an RBF kernel was used as the kernel function. The formula is expressed as follows

$$
\begin{equation*}
k\left(x, x^{\prime}\right)=\exp \left(\frac{-\left|x-x^{\prime}\right|^{2}}{2 \sigma^{2}}\right) \tag{5}
\end{equation*}
$$

When standard deviation $\sigma$ is small, the individual training data are emphasized, and when $\sigma$ is large, the data are neglected. This is called Gaussian process regression and is also effective for estimating nonlinear functions that cannot be fitted well by linear functions. In addition, it is necessary to set hyperparameters for this Gaussian process. The SCG method was employed as the parameter optimization method [7]. This method has the advantages of being less dependent on the initial values and less prone to local solutions than
methods such as the gradient method.

## D. Bayesian Optimization

Bayesian optimization is a method for designing experiments in which the response function is estimated using Gaussian process regression on the experimental results, and the experimental setup that maximizes (or minimizes) the gain function is searched according to the objective. The acquisition function used in this study is the Upper Confidence Bound (UCB) [8], which plays two roles: one is to utilize data where the mean value is large, and the other is to search for the data where the number of data points is small. Although there are other acquisition functions such as Expected Improvements and Probability of Improvement [8] that use expected improvement or probability of improvement as indices, the purpose of this paper is to "efficiently find postures with large positioning errors of the large industrial robot to be tested, keeping in mind the subsequent error correction"; and therefore, the acquisition function that reduces the number of search points as much as possible was preferred, and the acquisition function UCB was determined to be suitable for this purpose. Using the expected value $\mu$ of $f(x)$ and standard deviation $\sigma$ of $f(x)$ indicated by the Gaussian process, the acquisition function $\alpha_{\mathrm{UCB}}(\mathrm{x})$ is as follows [9]:

$$
\begin{equation*}
\alpha_{\mathrm{UCB}}(\mathrm{x})=\mu+\sqrt{\frac{\log (n)}{n}} \sigma \tag{6}
\end{equation*}
$$

With this acquisition function, we can determine $x$ such that the expected value $\mu$ and standard deviation $\sigma$ of $f(x)$ are large.

## III. Experimental Method

## A. Measuring Range and Teaching Motion to The Robot

As shown in Fig. 3, the position and posture range of the robot is commanded in the cylindrical coordinate system $(R, \theta, Z)$, and the range is set to $1000 \leq R[\mathrm{~mm}] \leq 1800$, $700 \leq Z[\mathrm{~mm}] \leq 1700,0 \leq \theta[$ deg. $] \leq 60$.


Fig. 3. The range of measurement.
Even if the same command coordinates are used, there is a possibility that the posture may change owing to a change in path. Therefore, the teaching motion is performed such that the robot returns to the initial posture one after another and then moves to the target command coordinates again.

## B. Initial Data and Evaluation Indicators

To reduce the possibility of bias in the initial data used for the Gaussian process, 108 reference points were set evenly in
$(R, \theta, Z)$. The program used in this study rescales the range of $\mathrm{R}, \theta$, and Z from 0.0 to 1.0 , and forms a grid by dividing it into 50 equal parts. The positioning error vector is defined by Eq. (7), where the taught posture is $(X, Y, Z)$, and the measured posture is $\left(X^{\prime}, Y^{\prime}, Z^{\prime}\right)$.

$$
\left\{\begin{array}{l}
\Delta L_{x}=X-X^{\prime}  \tag{7}\\
\Delta L_{y}=Y-Y^{\prime} \\
\Delta L_{z}=Z-Z^{\prime}
\end{array}\right.
$$

The positioning error norm $|\Delta L|$ is the square root of the sum of the squares of $\Delta L_{x}, \Delta L_{y}$ and $\Delta L_{z}$.

## C. Experimental Method

The next posture to be measured, obtained from the initial data by Bayesian optimization, was used as the next command posture until the measurement with the laser tracker was performed. Subsequently, the measured data were added to the initial data, the next measurement posture was determined again by Bayesian optimization, and the experiment proceeded iteratively. After a certain amount of data was collected, the results of Bayesian optimization begin to converge to the same measurement posture, and the experiment is terminated when the results showed the same measurement posture twice in a row.

## IV. ReSUlts and Consideration

## A. Rotational Angle Errors Generated from Each Joints

Offset and rotational angle errors are present at each joint. We rotated the joints and focused on the changes in the errors. As an example, a conceptual diagram of the errors caused by the angle of Joint 2 is shown in Fig. 4. Let $\mathrm{R}, \mathrm{P}_{c}, \mathrm{P}_{a}$, and $\Delta L_{x=y}$. the radius of rotation, the command coordinates, the measured coordinates, and the measurement error on the $\mathrm{X}=$ Y plane, respectively. This is shown in Eq. (8).

$$
\begin{equation*}
\Delta L_{x=y}=\left(\Delta L_{x}+\Delta L_{y}\right) / \sqrt{2} \tag{8}
\end{equation*}
$$

As shown in Fig. 4, if $\Delta \theta$ is sufficiently small, the error vectors $R \Delta \theta$ of $\mathrm{P}_{c}$ and $\mathrm{P}_{a}$ can be approximated as straight lines, and each component of this vector is expressed as $R \Delta \theta=(-(R \Delta \theta) \sin \theta,(R \Delta \theta) \cos \theta)$. Therefore, the relationship between $\mathrm{X}=\mathrm{Y}$, the Z -axis directional error $\Delta L_{x=y}$, and $\Delta L_{Z}$ is as follows:

$$
\begin{equation*}
\Delta L_{z}=-\frac{1}{\tan \theta} \Delta L_{x=y} \tag{9}
\end{equation*}
$$



Fig. 4. $\mathrm{J}_{2}$ joint angle error.
Table I shows the correlation coefficients calculated for
joints $\mathrm{J}_{1}, \mathrm{~J}_{2}$ and $\mathrm{J}_{3}$, by comparing $\Delta L_{Z}$ derived using Eq. (9) and $\Delta L_{Z}$ measured.

TABLE I. CORrelation Coefficient Between

| Joint 1 | Joint 2 | Joint 3 |
| :--- | :--- | :--- |
| -0.83 | -0.32 | -0.47 |

From this table, it can be concluded that the joint angle error arising from Joint 1 has the greatest influence on the error vector. Based on the above, we incorporated the joint angle of $\mathrm{J}_{1}$ into the evaluation index used for the Gaussian process.

## B. Search and Acquisition Function Transition by Bayesian Optimization

Fig. 5 shows the points explored using Bayesian optimization.


Fig. 5. Measurement number and $|\Delta L|$ in those coordinates.
The size of each point is the size of $|\Delta L|$, and the numbers $1-5$ are the order of the search. From this figure, we can observe that the size of $|\Delta L|$ increases as the search progresses. Fig. 6 shows the distribution of the acquisition function in 2-dimensional space for the first and fifth searches, respectively, when R and Z are fixed.


Fig. 6. The acquisition functions each time.
This figure shows the distribution of the acquisition function for the posture with the largest acquisition function at each time point. From this figure, we can see that the acquisition function changes each time and that we can search for the posture with the largest value of $|\Delta L|$.

## C. Error Vector Prediction and Correction Using Gaussian Processes

As described in the previous section, the error vector arising from Joint 1 was found to be large. Taking this into account, we adopted a cylindrical coordinate system of $R, \theta$
and $Z$ instead of $\mathrm{X}, \mathrm{Y}$, and Z coordinates, as the coordinates used in the Gaussian process. In addition, we predicted the error vectors for coordinates within the range shown in Fig. 3. In this case, we gave the objective function $f(x)$ only $\Delta L$, $\Delta L_{x}, \Delta L_{y}, \Delta L_{z}$, and $\Delta \mathrm{R}, \Delta \theta, \Delta L_{z}$ and performed a Gaussian process for each of them. The results are shown in Fig. 7.
$\Delta \mathrm{R}, \Delta \theta$ are polar coordinate transformations of $\Delta L_{x}, \Delta L_{y}$ in the XY plane. It was found that only (c) takes a different form from the other two. One possible reason for this is that regression analysis does not work well because $\Delta \theta$ is so small that it falls within the measurement error of the laser tracker. Therefore, we adopted $\Delta L_{x}, \Delta L_{y}, \Delta L_{z}$ as objective functions and predicted the error vectors in each axis direction using Gaussian process regression.


Fig. 7. Gaussian process of each objective function.
And we defined $\Delta L^{\prime}, \Delta L^{\prime \prime}$ as Eq. (10).

$$
\left\{\begin{array}{l}
\Delta L^{\prime}=\sqrt{\Delta L_{x}{ }^{2}+\Delta L_{y}{ }^{2}+\Delta L_{z}{ }^{2}}  \tag{10}\\
\Delta L^{\prime \prime}=\sqrt{\Delta \mathrm{R}^{2}+\mathrm{R} \Delta \theta^{2}+\Delta L_{z}{ }^{2}}
\end{array}\right.
$$

## D. Comparison of Error Correction and Efficiency

For measured postures with an error of 1.5 mm or more, the measurement error vector was fed back for correction. As a result, all postures were corrected to an error of 1.5 mm or less, with an average correction rate of approximately $66 \%$. This is approximately $1 / 6$ of the training data and approximately $2 / 3$ of the correction rate compared to the study by Y. Jiang et al. [10], which corrected an average error norm of $94.2 \%$ with 600 training data points using machine learning. Therefore, when correction is necessary only for points with error norms above a certain level, as in this study, the search by Bayesian optimization is effective from the viewpoint of time efficiency.

## V. Summary

The positioning error correction for offline teaching of a large industrial robot was examined using a Gaussian process. The results indicate that the error vector generated by joint $\mathrm{J}_{1}$ is larger than that at other joints. Furthermore, Bayesian optimization based on Gaussian process regression is effective when error prediction is performed only for postures with large errors over a wide range. However, because this method alone does not perform exhaustive measurements, it is unclear whether all the points predicted to have small errors are so small that they do not require correction. Therefore, our future goal is to guarantee the certainty of the distribution based on the geometric factors of the robot.

## Conflict of Interest

The authors declare no conflict of interest.

## Author Contributions

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