The Development of a Ball-screw Thermal Error Feedback System Based on FEM Simulations

M. Costea*, Şt. Cula and G. Jiga

***Abstract*—Thermal errors represent a design issue of numerically controlled axes, due to their negative influence on the working accuracy of shop floor devices. This paper proposes an approach for simulating the heat transfer and thermal expansion based on computer aided software. The results achieved are used to evaluate the s-domain transfer function (or the relationship between temperature and displacement). The research can be used for the design, simulation and validation of various control techniques.**

***Index Terms*—numerically controlled axis, thermal errors, transfer function, transient structural analysis**

I. INTRODUCTION

The present paper presents a theoretical research study performed on a system set-up for the analysis of thermal error feedback due to the changes in motion occurred in transformation mechanisms. This mechanism is represented by a screw and ball-nut assembly.

In numerical control machines, the thermal elongation of the ball screw has a great influence on the position accuracy.

The influence of the variation of rotational speeds leads to temperature changes at different regions in a ball screw system.

In paper [1] a new method is proposed to evaluate the temperature increase at different positions, when the ball screw is in a thermal equilibrium state. The thermal transmission of ball screws is analyzed and the heat generation and transfer coefficient are calculated based on the laws of thermodynamics. The depending function between the temperature rise and the position of the actuated mobile element is generated by solving the thermal equilibrium differential equations.

The thermal elongation is obtained after the temperature rise generated by the friction of rolling between the ball-type intermediate elements and the rolling paths of the screw. In order to prove the validity of this model, a series of detection tests have been carried out to obtain the temperature rise of a ball screw and the thermal elongation under different rotational velocities.

The experimental results show that the realistic temperature rise and the thermal elongation are corresponding to the theoretical values.

According to [2] the temperature and position measuring accuracy affect directly the thermal error compensate precision. Due to the interdependence of the measuring technique, the increasing of the compensation system cost and decreasing of the productivity level will be inevitable trends in machine tools.

In conclusion, the development of a ball screw thermal error compensation system working without any temperature or positioning feedback is absolutely necessary.

As a conclusion of paper [2], as parts of the thermal error compensation system, the component heat generator, the compensation method, the thermal and mathematic models as well as the calculation method were studied respectively. In order to verify the accruacy and the generalization abilites of the developed thermal model as well as the thermal error compensation system, a series of simulations were carried out in different kinds of working conditions.

Through the series of simulations with the thermal model, calculation method and simulation conditions, deformation characteristics and thermal behavior of the prototype ball screw system have been studied by the authors.

For any moving element involved in the translational motion, the final instrument in the kinematic chain is a mechanism of transformation of the movement nature (guide screw-ball nut), such one being found in the translation modules of industrial robots, called "Tracking module”.

In this case, the predominant reason for the increasing temperature is the existence of the rolling friction between the balls and the raceways of the screw.

As a result of the long-term high-speed motion of the screw, the temperature rise phenomenon will occur. Therefore, the thermal elongation of the screw will cause mechanical errors. By employing a thermal-expansion compensation function, the thermal-elongation variation could be dynamically predicted in order to limit its side effects.

Research on the factors influencing thermal expansion has also been carried out in the paper [3, 4] which states that the development of such a system for thermal compensating errors is based on several complex factors, which at their turn are governed by other variables making the stability of a theoretical transfer function quite difficult.

Also in paper [3, 4] essential guidelines are provided in order to calibrate the experimental and simulation thermal displacements of ball screw mechanisms. While the results achived prove a high level of accuracy, the computational demands are extensive. Therefore, it is almost impossible to use such approaches in real-time environments, being necessary to develop an estimator, representing in this case a transfer function.

II. INITIAL DATA

A. Data on the Studied Kinematic Chain

The kinematic chain is an electromechanical structure

Manuscript received May 20, 2022; revised July 15, 2022; accepted August 31, 2022; published March 21, 2023.

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DOI: 10.7763/IJMO.2023.V13.822

allowing the driving of a moving element in translational or rotating motion [5]. Fig. 1 depicts the kinematic chain of a Numerically Controlled axis.

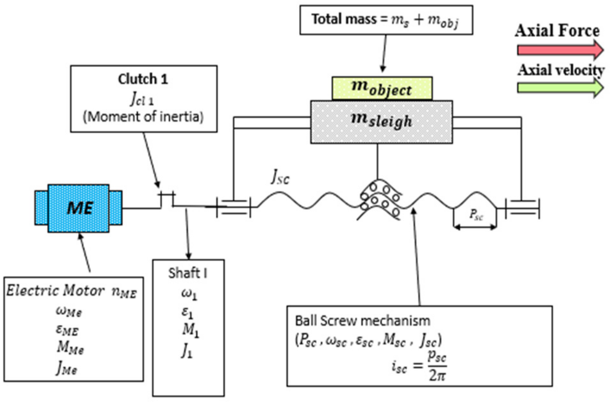

Fig. 1. Kinematic chain.

In the figure above:
- J_{cl1} is the moment of inertia for clutch 1;
- ω is the angular speed in [rad/s];
- ε is the angular acceleration [rad/s^2];
- p_{sc} represents the lead-screw pitch [mm/rot];
- M_1 is the twisting moment acting in shaft 1 [Nmm];
- M_{sc} is the twisting moment acting in the lead screw [Nmm].

1) The geometric characteristics of the ball screw

Table I depicts the main geometric characteristics of the studied ball screw assembly.

TABLE. I: GEOMETRIC CHARACTERISTICS OF THE SCREW

Characteristic	Value	Measurement unit
Length	400	mm
Diameter	16	mm
Pitch (P_{screw})	5	mm/rot
Nut mass (M_{nut})	0.25	kg
Screw mass (M_{screw})	0.52	kg
Moment of inertia for the guiding screw (J_{screw})	13.2	kg•cm^2

B. The Calculation of Heat Transfer Data

Through a conclusive set of input data, the calculation steps related to this kinematic chain are carried out to derive the data necessary to calculate the convective heat transfer coefficient. The steps are described below:

1) Conversion units of measurement

$$J_{EM} = 0.57 kg \cdot cm^2 = 5.7 \times 10^{-5} kg \cdot m^2 \quad (1)$$

$$J_{Screw} = 3.12 kg \cdot cm^2 = 3.12 \times 10^{-4} kg \cdot m^2 \quad (2)$$

$$J_c = 0.4 kg \cdot cm^2 = 0.4 \times 10^{-4} kg \cdot m^2 \quad (3)$$

$$M_{object} = 474 \ kg$$

$$M_{plateau} = 124 \ kg$$

$$M_{nut} = 6 \ kg$$

$$v_{\max} = 39.52 \frac{m}{\min} = 0.6586 \frac{m}{s} \quad (4)$$

$$P_{screw} = 5 \frac{mm}{rot} = 0.005 \frac{m}{rot} \quad (5)$$

2) Representation of the motion profile

The motion profile of the NC axis is depicted in Fig. 2, alongside with the gravitational, inertial and frictional forces.

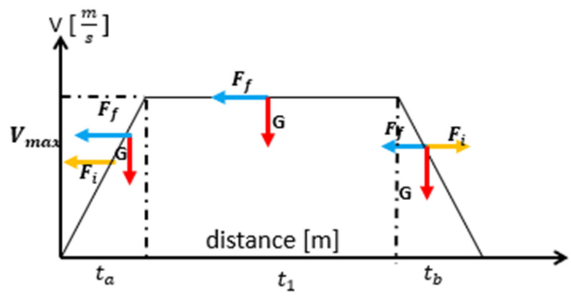

Fig. 2. Movement diagram.

3) Calculation of frictional, gravitational, and inertial forces

$$F_g = M_{total} \times g = \left(M_{plateau} + M_{object} + M_{nut}\right) \times g \quad (7)$$

$$F_f = \mu \times F_g \quad (8)$$

$$F_i = \frac{v_{\max}}{t_{\max}} \times m_{total} \quad (9)$$

4) Calculation of the kinematic criterion

The kinematic criterion is based on the pitch/lead of the ball screw and the desired linear velocity:

$$Y_{out} = Y_{in} \times i_{TLC} \quad (10)$$

$$v_{max} = \omega \times \frac{P_{SC}}{2\pi} \quad (11)$$

5) Evaluation of the heat transfer

The heat transfer in the ball screw mechanism occurs mainly due to the balls-raceway frictional torque. The resulting frictional heat is transferred to the entire assembly (by conduction). Cooling occurs due to the force convection that is induced by the rotational velocity (convection)—see Fig. 4.

Fig. 4. The steps required for evaluating the heat transfer in ball screws.

The calculation of the frictional torque is carried out by employing the formula [1]

$$M_{BS} = \frac{2f \times F \times \left(r_m + r_b \cos\alpha\right)}{\sin\alpha} \quad (12)$$

where:
- M_{BS} is frictional torque [Nm];
- f is friction coefficient;
- F is axial force [N];

- r_m is screw radius [m];
- r_b is ball radius;
- α is contact angle lubrication;
- f_{BS} is lubrication coefficient.

On the other hand, the forced convection heat transfer coefficient is derived by employing [1]:

$$h = 0.664 \times \lambda \times \left(\frac{n}{60 \times \nu}\right)^{\frac{1}{2}} \times \Pr^{\frac{1}{3}} \quad (13)$$

where:

- P_r is the Prandtl number (0.7 J/kg·K) for the air;
- h is heat transfer coefficient [W/m^2];
- n is the rotational velocity [rot/min];
- ν_s is the kinematic viscosity of the air [m^2/s].

III. Transient Coupled Analysis

ANSYS Workbench is used to capture the transient temperature gradients occurring at the ball screw level. Only the passage of the rolling elements is considered. Accordingly, to these aspects, it is necessary to follow some guidelines: choosing the optimal modeling approach, deciding the total simulation time, establishing the definition of the simulated sequence and the material used (Fig. 4).

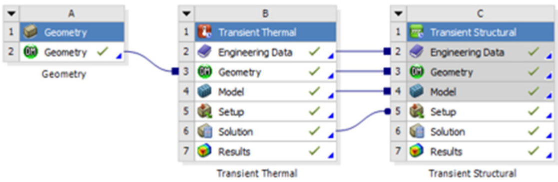

Fig. 4. Transient coupled analysis ANSYS.

Two types of analysis are carried out:

- **Transient thermal analysis**

For studying the time vs. temperature profile of the ball screw. The required input data is described in Table II.

TABLE. II: Input Data for the Transient Thermal Analysis

Input data	Characteristic
Modeling approach	2D Axial symmetrical
Total simulation time	67 s
Simulated sequence	Segment 2
Material used	Structural steel

The results of the thermal analysis are used as input data for the structural one. In this regard, the temperature profile of the assembly is depicted in Fig. 5.

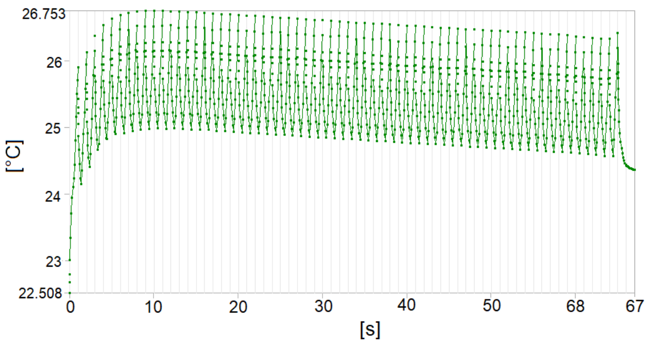

Fig. 5. Temperature variation.

For information, the temperature distribution of the screw body is emphasized in Fig. 6 (the maximum over time temperature).

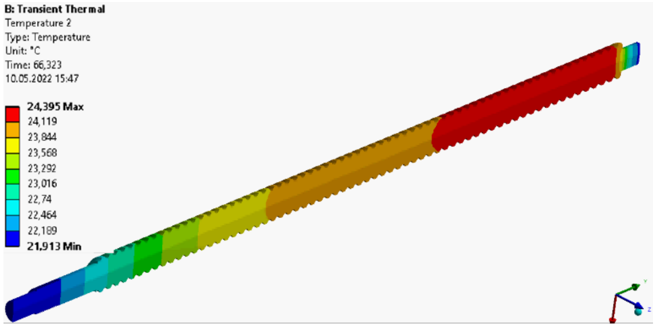

Fig. 6. Temperature variation on screw body.

The figure above shows a maximum temperature of 26.752 ˚C (considering an ambient temperature of 22 ˚C).

- **Transient structural analysis**

Transient structural analysis is completed to evaluate the time vs. temperature displacements that occur at the level of the screw body after coupling with thermal analysis, i.e., the raceway of the balls at the level of the screw produces a radial and axial displacement.

The simulations prove a total deformation of 0.2682 µm (Fig. 7).

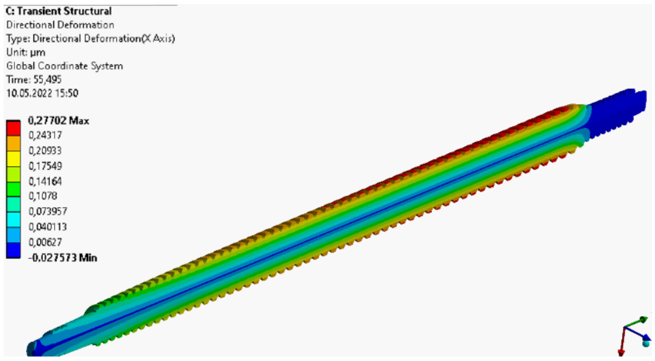

Fig. 7. Directional deformation.

Due to the isotropic nature of the material employed for manufacturing the screw body, the relationship between radial expansion and the temperature increase has a linear variation (Fig. 8).

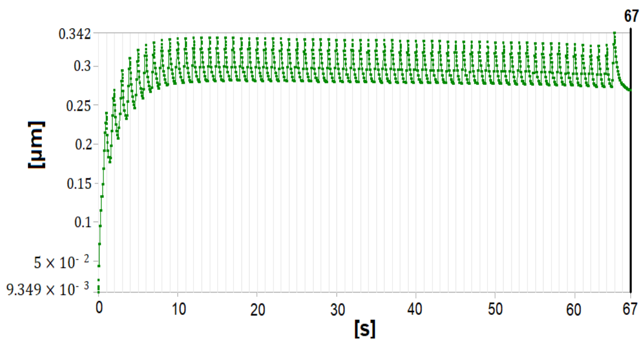

Fig. 8. Radial expansion vs temperature increase.

On the other hand, the axial displacement, on a work shape sequence, is depicted in Fig. 9. This representation is only for one work cycle (acceleration – constant acceleration –

deceleration).

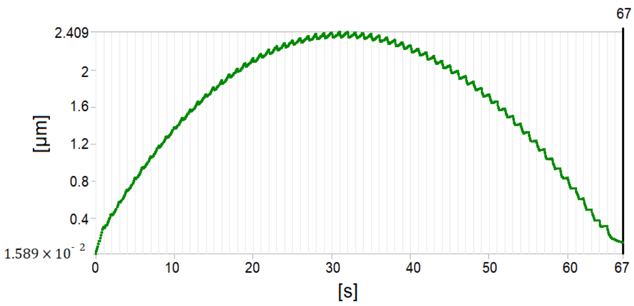

Fig. 9. Axial expansion variation in time.

IV. Evaluation of the Transfer Function

In engineering, a transfer function (also known as the system function or network function) of an electronic system or component control system is a mathematical function that theoretically models the output of the device for each possible input. In its simplest form, this function is a two-dimensional graph of a scalar input independent of the dependent scalar output, called the transfer curve or characteristic curve. Component transfer functions are used to design and analyze assembled systems from components, especially using block diagram technique, electronics, and control theory.

In many applications, it is sufficient to define the transfer function, which reduces Laplace transforms with complex arguments to Fourier transforms with the real argument ω.

The Laplace transform is a mathematical tool which is used to convert the differential equations representing a linear time invariant system in time domain into algebraic equations in the frequency domain.

Mathematically, the Laplace transform of a time domain function $x(t)$ is defined as:

$$L\left[x(t)\right] = X(s) = \int_{-\infty}^{\infty} x(t)\cdot e^{-st}dt \qquad (14)$$

where s is a complex variable being given by:

$$s = \sigma + j\cdot\omega\sigma \qquad (15)$$

The Laplace transform of a signal x(t) is equivalent to the Fourier transform of the signal $x(t)e^{-\sigma t}$.

The operator L is called the Laplace transform operator which transforms the time domain function x(t) into the frequency domain function X(s).

The Fourier transform represents a transformation technique which transforms a signal from continuous time domain to the corresponding frequency domain. Mathematically the Fourier transform of a continuous-time signal *x(t)* is defined as:

$$F\left[x(t)\right] = X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}dt \qquad (16)$$

The applications in which this is common are those in which there is interest only for the response to the steady state of an LTI system, not for transient start and stop behaviors or stability issues. This is usually the case with signal processing and communication theory.

To derive the transfer function, the time vs. temperature displacements must be linearized. This objective can be achieved by using an FFT filter [6]. Filtering is a process of selecting the frequency components of a signal. FFT is a filter that performs filtering by using Fourier transformations to analyze the frequency components in the input.

Fig. 10 emphasizes the linearized variation of the displacements, corresponding to the definition of a second order system.

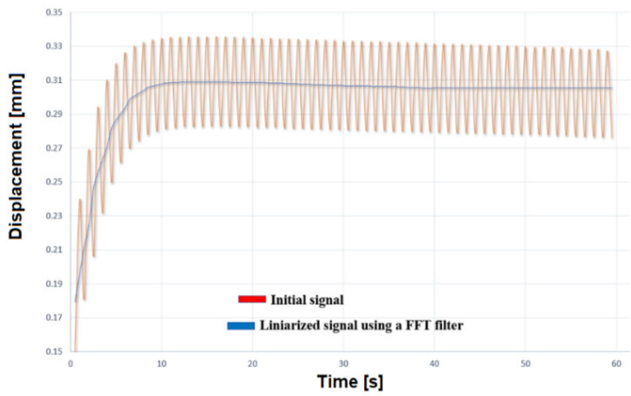

Fig. 10. Linearized variation of displacements.

The System Identification toolbox from MATLAB was used to identify the corresponding transfer function model [7]. Statistical methods are used to evaluate the required constants. Following the previous results, the best match was achieved for a function with two poles and one zero. A good match can be noticed between the outputs of the transfer function vs. the linearized function during the step-response simulation.

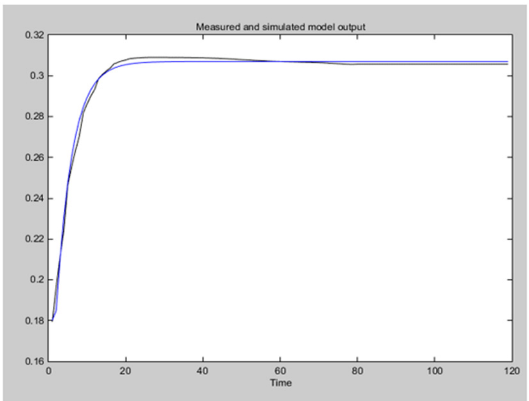

Fig. 11. Output of the transfer function.

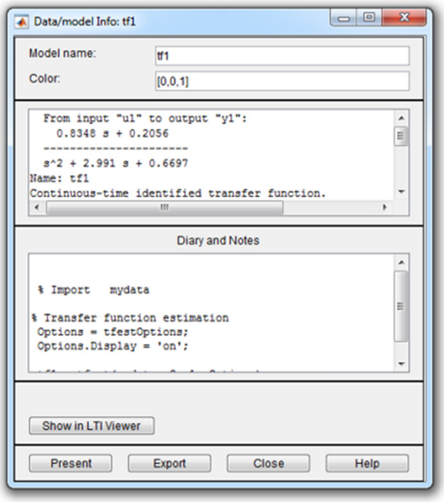

Fig. 12. Transfer function.

Fig. 12 depicts the results of the transfer function.

V. Conclusion

The aim of this paper was to simulate the thermal errors that occur at the level of a ball screw by using coupled transient analysis. The initial model was simplified to enhance the computational efficiency. The results were linearized, and the resulting function was imported into MATLAB to identify a transfer function. The optimal one has two poles and one zero. A good match was noticed between the linearized curve and the step-response of the transfer function. The mathematical description of the relationship between input and output of a system can be further used to implement a controller or other error feedback procedures.

The value of 0.2 μm represents a value obtained after only one work sequence (including acceleration-constant acceleration- deceleration). The slope of the resulting curve can be employed to anticipate the behavior of the system's output for any given working cycle. This objective is achieved by estimating an s-domain transfer function. Since the processing on a larger period of time requires more than 10 hours we can conclude that it's enough to analyze a period of 67 seconds, in order to obtain, the variation of radial and axial deformations for a working phase, represented in Figs 8–9.

These considerations will be taken into account in a future edition.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Şt. Cula decided the parameters of the ball screw by referencing specialized catalogues, simplified the geometry and carried out the structural analysis as well as the linearization of the resulting displacements. M. Costea has performed the analytical calculations for identifying the friction torque, convection heat transfer coefficient, geometry pre-processing, performing transient thermal analysis, transfer function evaluation as well as the underlying literature survey. G. Jiga performed the FEM analyses and check the final paper content. All authors had approved the final version.

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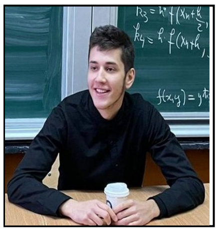

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