A New Approached Method for Solving the Four Index Transportation Problem with Fuzzy Parameters and Application Perspective in Industry

Hoa Pham, Fabien Escande, and Laurent Klupinski

Abstract—**The four index transportation problem (4ITP: origin, destination, goods type, vehicle type) is an extension of the Hitchcock problem, a problem best suited for the goods allocation planning in the case for using the shuttle type. Based on the research results in 1979, in 2011, we developed an exact method for fully solving the 4ITP with determinist parameters. However, because of the limitation of human in recognizing the imprecise natural events in the decision making science, it is necessary to use fuzzy numbers for uncertain condition. By extending the problem toward this direction, in this paper, we develop a method, an extension of fuzzy programming, solving the 4ITP when all parameters such as the cost coefficients, the supply and demand quantities, the goods type and vehicle type quantities are fuzzy numbers and obey the Gaussian law in order 2. We use Gauss method in the fuzzy programming to obtain the costs under form of point cloud. From these results, we use Pham's method, a method for the conventional 4ITP, to obtain a point cloud with the agglomerating costs. By using Kronechker's method, we extract the obtained minimal cost under form of an interval for a given solution. It allows us to obtain a goods allocation planning with the minimal total transportation cost.**

Index Terms—**Degeneration, point cloud, Pham's method, fuzzy programming, fuzzy four index transportation problem.**

I. Introduction

The transportation models play an important role in logistic and supply chain management for reducing cost and improving service. Thus, the transportation problem (TP) is one of particular problems in the linear programming, stemmed from a network structure consisting of a finite number of nodes and arcs attached to them. On the theoretical point of view, the research on TP was developed at the macro level with the *n* index problem.

In our research direction, we are interested in the 4ITP, a model best suited for transporting several goods types several vehicle types, from origins to destinations. In two years, 2011 and 2012, we proposed a new exact method, Pham's method, for fully solving the 4ITP when all parameters are determinist [1]-[3]. From the industrial database, we proposed a new algorithm determining the input database for this mathematical model [3].

In real-life, however, this condition may not be fulfilled because of many diverse situations resulting from uncertainty in judgment, lack of evidence, Sometimes it is not possible to get precise data which is relevant with the offer, demand, goods type and vehicle type quantities parameters. This data may be represented by the fuzzy number, therefore a fuzzy decision making method is required here.

In this paper, we elaborate the 4ITP in which all the parameters are presented by fuzzy numbers. Our problem will respond to the logistic planning requirement in an unstable economic conjunction with the difficulty for a precise prediction of industrial data.

II. Literature Review

Among many specific linear programming problems, the one called transportation problem is very popular. The first transportation problem, two index transportation problem (2ITP), was developed in 1941 by Hitchcock, based on the request of production enterprises: elaborate a planning for transporting the goods from one or several origins to one or several destinations with the criterion of minimizing the total transportation cost. The effective algorithms solving the 2ITP exist when the cost coefficients, supply and demand quantities are known exactly. However, there are cases where these parameters may not be presented in a precise manner. For example, the unit shipping cost may vary in a time frame; the supplies and demands may be uncertain due to some uncontrollable factors. Therefore, to deal quantitatively with imprecise information in making decision, the notion of fuzziness was introduced.

A. Two Index Transportation Problem (Fuzzy Parameters)

The first problem group researched in this branch is the transportation problems with fuzzy cost coefficients [4], and the problem in which all parameters (cost coefficients, supply and demand quantities) are represented by fuzzy numbers [5]. For solving it, one of important conditions is the ability to define and to determine the optimal solution of the problem. There are several algorithms, in the literature, for solving the transportation problems in fuzzy environment but in all the proposed algorithms the parameters are represented by normal fuzzy numbers. Moreover, in several researches, the generalized fuzzy numbers are used for solving real-life problems but until now, no one has used generalized fuzzy

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numbers for solving the transportation problems. So, a new algorithm was proposed for solving a special type in this domain: transportation problem with the cost coefficients is represented by generalized trapezoidal fuzzy numbers [6]. Achieving this result, in 2011, two new methods, based on fuzzy linear programming formulation and classical transportation methods, were proposed to find a fuzzy optimal solution for fuzzy transportation problems with all parameters represented by trapezoidal fuzzy numbers [7].

In the classical transportation problem with integer demand and supply values there is always an integer solution. However, this property (the possibility of finding an integer solution) is not preserved in the fuzzy problem with fuzzy demands and supplies, even if the characteristics of fuzzy numbers occurring in the problem are integer. Therefore, an exact algorithm has been proposed which solves the problem with fuzzy supply and demand values and the integrality condition imposed on the solution [8].

The third branch consists of the multi-objective transportation problem, in which the concept of optimal solution takes place of that of non-dominated solutions. For solving it, the researches often used the theory based on the foundation of mono-objective problems with fuzzy parameters. In 2010, a method was proposed to minimize the transportation cost-time with fuzzy demand, supply quantities, and cost coefficients. The problem was modeled as multi-objective linear programming problem with imprecise parameters. Chakraborty used the fuzzy parametric programming to handle impreciseness and then, to solve this problem by the prioritized goal programming approach [9], [10]. Following this extension, in 2011, a new method was proposed to find the solution of a linear multi-objective transportation problem by representing all the parameters as interval-valued fuzzy numbers [11].

B. Three Index Transportation Problem (Fuzzy Parameters)

By increasing the index numbers, the three index transportation problem (3ITP), called the solid transportation problem (STP), was developed and solved by an extension of the potential method, under the geometric form. These properties are source, destination and type of products or mode of transportation (conveyance). The STP was put forward by Shell in 1955 [12]. The Fuzzy Solid Transportation Problem (FSTP) appears when the nature of the data problem has been expressed as imprecise values by the decision maker.

In 1998, for solving FSTP in the case in which the fuzziness affects the constraints set, a fuzzy solution to the problem is required, and an arbitrary linear or nonlinear objective function is considered, Jimenez and Verdegay proposed an evolutionary algorithm based parametric approach to obtain an auxiliary Parametric Solid Transportation Problem (PSTP) associated to the original problem, which can finally be applied to find a "good" fuzzy solution [12]. In 2006, based on the extension principle, Liu developed a method that is able to derive the fuzzy objective value of the FSTP when all parameters are fuzzy numbers [13]. Firstly, the FSTP has been transformed into a pair of mathematical programs employed to calculate the lower and upper bounds of the fuzzy total transportation cost at possibility level α. Then, from the obtained different values, the membership function of the objective value was approximated. Finally, since the objective value was fuzzy, the values of the decision variables derived are fuzzy as well.

The bi-criteria STP is the basic in the processing multi-objective problems. Until now, many researches also have great interest in this problem and some methods have been presented for solving it. All these method used their special techniques in finding the solutions for two objective functions approximately approaching to the ideal solution. In 1995, Gen and his colleagues elaborated the bi-criteria FSTP and presented an implementation of genetic algorithm for solving it [14]. They used the ranked fuzzy numbers with integral values to create the chromosomes initialization and to calculate the evaluation and selection, also find the non-dominated points in the criterion space based on the decision maker degree of optimism. Taking advantage from this result, after two years, in 1997, they continued proposing an improved genetic algorithm, incorporated with the problem-specific knowledge, for solving the multi-objective STP in which the fuzzy numbers were used in the objective function [15].

C. Four Index Transportation Problem

Among the transportation problems, the four index transportation problem (4ITP) is a model that concerns business organizations and attracts researches. It fits the transportation model in which several goods types are transported from several production hub (origin) to several deposits (destination) by several vehicles. This problem was first elaborated by Ninh in 1979 [16],[17].

Determine the variables

$$x_{ijkl} \geq 0 \ (i = 1 \dots m, j = 1 \dots n, k = 1 \dots p, l = 1 \dots . q)$$

$$for \quad minL(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{q} c_{ijkl}.x_{ijkl} \quad (1)$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{q} x_{ijkl} = \alpha_i \ (i = 1 \dots m) \quad (2)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{p}\sum_{l=1}^{q} x_{ijkl} = \beta_j \ (j = 1 \dots n) \quad (3)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{q} x_{ijkl} = \gamma_k \ (k = 1 \dots p) \quad (4)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijkl} = \delta_l \ (l = 1 \dots q) \quad (5)$$

$\alpha_i, \beta_j, \gamma_k, \delta_l > 0;\ c_{ijkl} \geq 0$ are known and determinist.

Being different from the precedent thinking, Ninh did not use the extension of potential method in space, he proposed an extension of this method on the plan, an exact method, by coordinating the resolution of the primal and dual problems. He also presented and demonstrated the theorem of the necessary and sufficient condition so that the problem has had the solution (solution existence condition-SEC).

Theorem 1 (SEC): The necessary and sufficient condition so that the 4ITP has a solution:

$$\sum_{i=1}^{m}\alpha_i = \sum_{j=1}^{n}\beta_j = \sum_{k=1}^{p}\gamma_k = \sum_{l=1}^{q}\delta_l \quad (6)$$

If the 4ITP has solution, this problem will certainly have the optimal solution.

If the problem (1…5) responds to the condition (6), the constraint system (2…5) can transform into a system consisting of (m+n+p+q-3) equations.

Based on this result, Ninh solved the 4ITP when the SEC was satisfied and the problem was not degenerated (basic case). In 1994, Zitouni elaborated the capacitated 4ITP

(C4ITP) by adding a condition of limited transportation capacity and proposed an exact method, extension of method Ninh, for solving it [18], [19]. In 2010, based on a large database, Djamel realized a comparative numerical study between two classical methods (simplex method and interior point method) and Zitouni's method on this basic case of 4ITP. Following the criterion of execution time, the numerical result demonstrated that the method Zitouni is the most favorable [20].

In 2011, Pham and her colleagues proposed and demonstrated the degeneration theorem: the sufficient condition so that the 4ITP is degenerated with a constraint where the SEC is satisfied [1].

Theorem 2 (degeneration): If there are numbers $m_1, n_1, p_1, q_1 (0 < m_1 < m, 0 < n_1 < n, 0 < p_1 < p, 0 < q_1 < q)$ that:

$$\sum_{i=1}^{m_1} \alpha_i = \sum_{j=1}^{n_1} \beta_j = \sum_{k=1}^{p_1} \gamma_k = \sum_{l=1}^{q_1} \delta_l \qquad (7)$$

Then, the 4ITP will be degenerated.

Based on this condition, Pham demonstrated that the cause of degeneration of 4ITP is: the initial problem can be divided into two independent sub-problems and each sub-problem responds to its SEC. It allowed Pham to propose an exact method for recognizing a degenerated problem and eliminating the degeneration in the cases: the degeneration appears in the elaboration process of the first solution and the degeneration appears in the process of solution improvement to obtain the better solution [1]-[3]. The algorithm (or Pham's method) was introduced and programmed in FORTRAN 95 language with Gfortran compiler running on a computer equipped with a CPU Intel Core 2 at 1,97Ghz.

The two methods (Pham's method and simplex method) were tested with using the real series which are not degenerated and divided into two groups: SEC is verified and SEC is not verified [1], [2]. In the case of verified SEC, both methods get the optimal value of the objective function: the minimum transportation cost. However, the execution time of Pham's method is fast enough for middle sized problems and large ones. In the case where the SEC is not satisfied, Pham's method always gets the optimal solution and the execution time is very fast (100,000 variables treated in 3.8 seconds) while the simplex stops immediately without solution [21].

Pham and her research team also found the degeneration series and tested them by two methods: Pham's method and simplex method. The simplex and its options in Cplex cannot cross the degeneration for all cases in the 4ITP resolution field. The results show that, for the case where the simplex cannot find the optimal solution, Pham's method eliminated the degeneration and easily obtained the optimal solution [2], [3], [21].

Hence, under the angle of numerical calculation, the results are pertinent because the calculation errors are eliminated. Extending toward this direction, in 2012 and 2013, they proposed an approached method, coupled with the principal program of 4ITP, for solving the problem with integer variables and developed a mix model for the transportation system of weight unit (real variable) and piece unit (integer variable) [22]. This result allowed them to propose a new algorithm, coupling between the production management principle and an extension of the ant colony method, determining the input database for the model 4ITP from the industrial database. In addition, an algorithm has been proposed, extension of particle swarm optimization and coupling the full advantages of basic 4ITP, for solving the 4ITP with interval cost parameter [23].

The table 1 extracts the research results on the fuzzy transportation problems, and also the 4ITP. It presents the possible extensions, methods and illustrated sizes.

TABLE I: RESEARCH RESULT

Model	Reference	Method	Size
2ITP (fuzzy parameters)			
Fuzzy cost	[3]	Approached	2x3
Fuzzy integer supply and demand	[4]	Exact (several 2ITP)	2x3
Fuzzy all parameters	[12]	Approached on extension principle	2x3
Generalized trapezoidal fuzzy cost	[8]	Approached	2x3
Trapezoidal fuzzy all parameters	[9]	Approached	2x3
Cost-time minimization with fuzzy all parameters	[1]	Approached	Criterion k=6 5x8
Linear multi-objective with fuzzy parameters	[86	Approached	Criterion k=2 3x4
3ITP (fuzzy parameters)			
Fuzzy all parameters	[9]	Approached	2x2x2
Fuzzy objective value with fuzzy all parameters	[13]	Approached	2x3x2
Bicriteria Problem with fuzzy all parameters	[5]	Genetic algorithm	4x4x3
Multi-objective problem with fuzzy all parameters	[12]	Improved Genetic algorithm	Criterion k=3 3x3x3
4ITP			
Real variables, basic case	[6,23]	Exact Ninh's method	2x2x2x2
Real variable, basic case, limited transportation capacities	[15,21,23]	Exact, extension of Ninh's method	2x2x2x2
Real variables	[14,16,17,19]	Exact, Pham's method	10x10x100x10
Integer variable	[20]	Extension of Pham's method	10x10x100x10
Industrial implementation	[14]	Approached coupling with Pham's method	10x10x100x10
Interval cost parameter	[18]	Approached extension PSO method	10x10x100x10

Thus, the research results on two index and three index transportation problem group are significant. The methods used in this case are the fuzzy programming and its extensions. Concerning the 4ITP, Pham and her colleagues fully solved it when all parameters are given in the precise way. This result allows us to open the door of uncertain environment by solving the 4ITP with interval cost parameters. Hence, how can the 4ITP with fuzzy parameters (F4ITP) present the resolution?

III. Problem Presentation

The formalization of 4ITP with fuzzy parameters is based on the 4ITP with determinist parameters and the 2ITP with fuzzy parameters [4]. We consider the following data:

- m origin nodes $O_i (i = 1 \dots m)$. The goods delivery capacity of this node is $\tilde{\alpha}_i (\tilde{\alpha}_i > 0\ i = 1 \dots m)$.
- p goods types. The total quantity of goods type S_k $(k = 1 \dots p)$ at all the nodes is $\tilde{\gamma}_k (\tilde{\gamma}_k > 0\ k = 1 \dots p)$.
- n destination nodes $D_j (j = 1 \dots n)$. The reception capacity of goods at this node is $\tilde{\beta}_j$ $(\tilde{\beta}_j > 0\ j = 1 \dots n)$.
- q vehicle types. The total quantity of goods that the vehicle type $H_l (l = 1 \dots q)$ can transport is $\tilde{\delta}_l$ $(\tilde{\delta}_l > 0\ l = 1 \dots q)$.

The hypothesis must be fixed by inhibition for the transport of goods in the direction from destination nodes to origin nodes.

The following parameters must be known to begin solving the 4ITP when all parameter are fuzzy numbers:

- $\tilde{\alpha}_i, \tilde{\beta}_j, \tilde{\gamma}_k, \tilde{\delta}_l$ are previous defined total quantity; these parameters are fuzzy numbers.
- $\tilde{c}_{ijkl} \geq 0 (i = 1 \dots m, j = 1 \dots n, k = 1 \dots p, l = 1 \dots q)$ transportation cost for a unit of goods type S_k that is transported from origin node O_i to destination node D_j using the vehicle type H_l ; these cost coefficients are represented by fuzzy numbers.

The variables of fuzzy 4ITP are

$\tilde{x}_{ijkl} \geq 0$ $(i = 1 \dots m, j = 1 \dots n, k = 1 \dots p, l = 1 \dots q)$ is a not-negative fuzzy number;

$\tilde{x}_{ijkl} \geq 0$ is the quantity of goods type S_k transported from node O_i to node D_j by vehicle type H_l in the established solution.

The problem becomes: Determine the variables

$$\tilde{x}_{ijkl} \geq 0\ (i = 1 \dots m, j = 1 \dots n, k = 1 \dots p, l = 1 \dots . q)$$

$$for \quad minL(\tilde{X}) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \sum_{l=1}^{q} \tilde{c}_{ijkl} . \tilde{x}_{ijkl} \quad (8)$$

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{p} \sum_{l=1}^{q} \tilde{x}_{ijkl} = \tilde{\alpha}_i\ (i = 1 \dots m) \quad (9)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{p} \sum_{l=1}^{q} \tilde{x}_{ijkl} = \tilde{\beta}_j\ (j = 1 \dots n) \quad (10)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{q} \tilde{x}_{ijkl} = \tilde{\gamma}_k\ (k = 1 \dots p) \quad (11)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} \tilde{x}_{ijkl} = \tilde{\delta}_l\ (l = 1 \dots q) \quad (12)$$

The constraint (9) shows that all types of goods transported from origin node O_i to all destination nodes by all vehicle types are equal to the offer $\tilde{\alpha}_i$.

The constraint (10) shows that all types of goods transported from all origin nodes to destination node D_j by all vehicle types are equal to the demand $\tilde{\beta}_j$.

The constraint (11) shows that the type of goods S_k transported from all origin nodes to all destination nodes by all vehicle types is equal to the quantity $\tilde{\gamma}_k$.

The constraint (12) shows that all types of goods transported from all origin nodes to all destination nodes by vehicle type H_l are equal to the quantity $\tilde{\delta}_l$.

In the next section, we present a new algorithm, based on the fuzzy programming, for solving this problem.

IV. Resolution Method

Firstly, we present the general resolution principle, which is an extension of fuzzy programming. Based on this principle, we propose an algorithm for solving the 4ITP when all parameters are represented by fuzzy numbers. This algorithm will be coupled with Pham's method to obtain an optimal solution from a set of optimal costs under the interval form. In the end, we develop a program in FORTRAN 95 language and present the obtained results on artificial database, before testing in Selecom OpenPro ERP system.

A. General Principle

The fuzzy programming involving the issues on fuzzy parameters is a used technique in artificial intelligence. It was formalized by Lorfi Zadeh in 1965, and applied in various fields such as automations, robotics (form recognition), production management (decision support). It is based on the mathematical theory of fuzzy parameter sets and fuzzy functions. Therefore, this theory is an extension of the classical fuzzy parameter set theory for the consideration of parameter sets defined as interval. This is a formal mathematical theory in the sense where Zadeh, starting from the concept of membership function to model the definition of a sub-set for a given universe, has developed a full model on property and formal definition criterions. He also showed that this fuzzy sub-set theory is effectively restricted to the classical sub-set theory in the case where the membership functions considered as the ones of linear coefficient family of some programs, which are insufficient because the interactions are unique, provide the very limited solutions and it is difficult to use them.

The programming with fuzzy parameters was formalized and their theorem allows making a bridge between classical parameters programming and fuzzy parameters programming. In the particular case where the treated parameters of programming (proposition) are not fuzzy, the fuzzy logic is restricted to the Fuzzy-Logic classical logic. The general theorems are often difficult; therefore, we introduce a set of simplifying assumptions for our fuzzy problem.

The capacity of fuzzy sub-sets to model as well as the gradual properties, flexible constraints, incomplete information, makes them suitable for facilitating the resolution of a large number in the case of our problem as critical cycling.

The simplification rules are imposed:

- We may represent the state of a fuzzy parameter by a Gaussian probability in order 2.
- All used tools to make the decision will be either isomorphic to the probability theory, or inconsistent.

1) Managing the creation of database

For the supply, demand, goods type and vehicle type, the function generating a database under form of a set of points is used.

$$\Gamma_{O,D,G,V} = \frac{1{,}25}{\beta . \sqrt{2\pi}} e^{-(\frac{S(c)}{2.\beta^2}} \quad (13)$$

$\Gamma_{(s)}$ (s is the transportation sequence) is the function that determines the probability density. The coefficient β depends on the database being generated. *O*: offer; *D*: demand; *G*: goods; *V*: vehicles

$S(c)$ is a function which gives the interaction constraints between each actor of the base. For example: on 16 offers, 3 are identical: they are eliminated from the base. This function is found with a great regularity in the estimation questions. The calculation of entropy variation between the old and the new distribution allows quantifying exactly the objective function. Thus, based on the obtained results, the application of fuzzy parameters is optimally executed for our problem.

It shows that for the application of the fuzzy method to particle swarm method, the coefficient β should be very low because the probability of degeneration cycling increases and the problem is irresolvable.

The principle of choice β is:

- Take any value to β. Each value β gives a series.
- If this series is oriented → accept;
- On the contrary, if it is too random → eliminate. For example, 50 vehicles (total loads: 1500 tons) used to transport 2 goods types (total quantities: 7 tons).

Continue choosing the value β until we obtain an adaptive database series.

2) *Application of fuzzy method to the 4ITP*

Our resolution method is based on the calculation of standard deviation of the closest median-vectors that are contented in a cloud of points in the fuzzy linear programming.

For defining the 4ITP with fuzzy parameters (F4ITP), we use Bayles's theorem resulting from the probability theory, and we try to determine that is currently called the *p* probability distribution of a binominal law. Assuming a uniform distribution of binominal parameter *p* and an observation *m* of a binominal law, where *m* is the number of observed positive outcomes and *n* is the number of observed failures, the probability *p* will be a value $\in [a, b]$ knowing that *m* takes the lowest probability for a logistic event which arrives.

As we have pointed out that the membership function which defines the fuzzification can receive only one parameter. It is the abscissa value (the actual value) which must calculate the ordinate, i.e. this is the function defining the database for the supply (offer), demand, goods type and vehicle type (quantities).

Based on the degeneration condition (7) of classical 4ITP, we propose the elaboration function of point cloud and their interactions:

$$\sum_{i=1}^{m_1} \lambda\tilde{\alpha}_i = \sum_{j=1}^{n_1} \lambda\tilde{\beta}_j = \sum_{k=1}^{p_1} \lambda\tilde{\gamma}_k = \sum_{l=1}^{q_1} \lambda\tilde{\delta}_l \quad (14)$$

We associate for the supply, demand, vehicle type as well as the function λ which corresponds to the vector field of event set associated with their tendency *s(x)*. This tendency is an expression that gives a representative database of reality from the production activities in the enterprises with their mutual interaction. This function uses a Gaussian base in order 2.

$$\oiint_x^{\infty} \lambda(x)dx = \frac{\sum_1^{\infty} q_{n-1}}{\sigma_2 - \sigma_1} \quad (15)$$

- *s(x)*: interaction tendency
- *x*: convergence radius in order 1, probability density of a Gaussian
- $\oiint_x^{\infty} s(x)$: vector field
- q: standard of vector σ contained in the vector field. q_{n-1}: q [Amplitude, arguments]
- σ_2: factor considered the errors of non-linear covariance for a maximal cost $0 < \sigma_2 < 1$
- σ_1: factor considered the errors of non-linear for the medium cost. $k(\sigma_2 - \sigma_1) < 1$ for $n > 4$

For *k*=1, the problem falls into the degeneration because the fields of generated costs cannot be separated. This shortcoming is treated by Pham's method, which would be impossible with the simplex.

Offer, demand, goods type and vehicle type vectors interfere in one another by conforming to the method of 4ITP. It is very interesting that, for the 4ITP, their interactions produce only very few errors on the calculation of costs, contrary to the other methods (kangaroo,…).

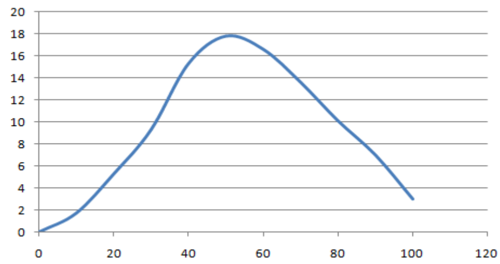

Fig.1. Gradient-variance curve on the cost.

The Fig. 1 shows the gradient-variance curve on the cost. The advantage of this method is:

- Precision gain on the calculation of costs.
- Limitation of cases being able to lead the program to a degeneration situation with the method of quadratic discrimination.

B. *Result*

1) *Database*

- We take and test three values β:

  $$\beta_1 = 0.0452;\ \beta_2 = 7.25;\ \beta_3 = 1.56$$

  β_1 and β_2 give too random series → eliminated;

  β_3 gives an oriented series → accepted and used for creating the database.
- We get string on *m*=2, *n*=4, *p*=5, *q*=4 and the SEC is not satisfied.

2) *Executive steps*

- We begin by transferring the coefficients given by the function (λ) in the first line in order of decreasing exponents.
- Given that there are *n* generation possibilities of numbers on the random series, we must consider the *n*, which little vary, as simple permutations being stored in all possible ways by using the following method:
  - The lowest value is placed in the boxes indexed *m*(1,1) to *m*(1,*k*).
  - Transferring the first coefficient in the first box of the third line (this is the line dedicating the costs). Then, the following actions are repeated to go to the last box.
  - Multiplying the number of the last line by the correction coefficient which depends on the precision given by epsilon (epsilon is readjusted).
  - The obtained results are transferred in the processing chain of classical 4ITP.

3) *Optimal transportation planning*

We give a numerical example on small artificial database to test the proposed method. The studied case is a F4ITP degenerated with SEC not satisfied and the degeneration appears on the solution modification process.

TABLE II: OPTIMAL TRANSPORTATION PLANNING

Origin	Destination	References	Vehicle Load (*t*)	Quantity (*t*)
1	1	4	1.5	1.2985
1	3	5	5	4.4239
1	1	1	0.65	0.0156
1	1	3	5	0.1894
1	2	3	3	1.5204
2	1	4	0.65	0.1074
2	1	3	3	0.1136
2	4	2	5	0.0945
2	2	3	0.65	0.0983
2	3	1	3	0.3219
2	4	2	1.5	0.1640
2	4	5	5	1.3484

In this simple example, the hypothesis is composed of 2 origins, 4 destinations, 5 products and 4 vehicle types (0.65t, 1.5t, 3t and 5t). The optimal transportation planning obtained is presented in the table 2.

- Transportation cost: 453.8761€
- Iteration number: 328 (gotos of module);
- Index goto number: 6498
- Index goto number with degeneration elimination: 7

Then, some performances of our program are presented in the table 3, the figures 2 and 3.

TABLE III: EVOLUTION OF VALUE λ DURING CYCLING

Storage Index	Ite number i_t	λ	Degeneration number
(1,1)	67	0.054	Yes (1)
(1,2)	18	0.324	Yes(10)
(1,3)	53	0.785	Not
(2,1)	7	0.845	Not
(2,2)	68	0.004	Not
(2,3)	34	0.006	Not
(3,1)	73	0.001	Yes(2)
(3,2)	90	0.001	Not
(3,3)	50	0.001	Not
(4,1)	275	0.001	Yes(2)
(4,2)	734	0.006	Not
(4,3)	2	1.128	Not

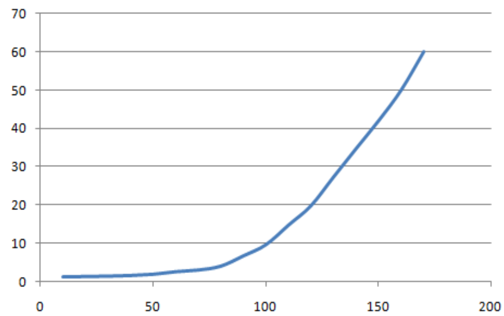

Fig. 2. Program activity: iteration number in terms of λ

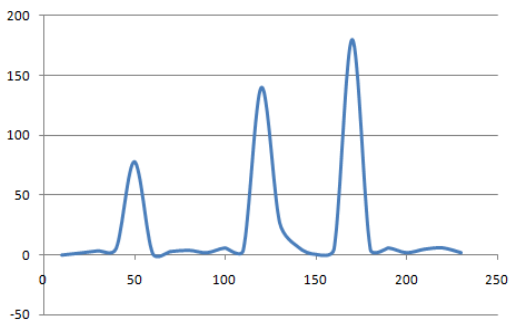

Fig. 3. Program activity in the degeneration case

C. Discussion: Toward a Probabilistic Modeling Approach

Modelling and solving the transportation problem with fuzzy parameters has brought out the characteristics and the new algorithmic technique. The treatment algorithm by fuzzy parameters uses the Viterbi coefficients for the adjustment of parameters. The method consists of calling the main program to solve the problem. The database is sorted so that the interaction between the operator and the data is positive interval. The data is calculated by series of *n* packets and the difference is squared.

The figure 4 shows the number of operations performed by the algorithm according to the size of parameter bounds. We see that when the difference between two parameters from 25 to 45, the number of operations performed is proportional to the deviation of bound: It is called the bounds of first species. Between 45 and 65, the number of operations performed varies with the deviation of bound very slowly because the algorithm uses the results of previous calculations: the bound of 2nd species, beyond the bounds possessing a deviation 65, the number of operations performed decreases.

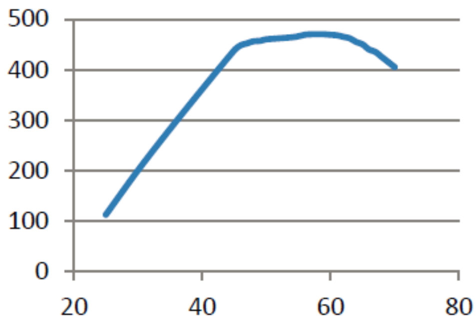

Fig.4. Concurrence number according to the deviation of bound

V. CONCLUSION

In this paper, a fuzzy four index transportation problem (F4ITP), in which all parameters (offer, demand, goods type, vehicle type quantities, and cost coefficients) are represented by fuzzy numbers, has been elaborated. A new approached method, coupling with the strengths of Pham's method in classical 4ITP, for finding the optimal solution of F4ITP. The lots of advantages of the proposed method are discussed and a numerical example is solved to illustrate the proposed algorithm. It is easy to apply in the real life situation because of the optimal solution represented by real numbers, not fuzzy, and high precise.

Based on this interested result, we envisaged implementing F4ITP model in transportation module for third period of our full traceability project at Selecom's platform. This model will be running on our OpenPro ERP system with testing databases, a database created like as an additional option of OpenPro system that allows us to approach decision making support tool for transportation planning.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Fabien Escande conducted the research. All authors have contributed in determining idea and implementing paper; all authors had approved the final version.

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In 2003, he decided to join Selecom, a group specializing in the cables and electrical equipment distribution in Auvergne. With his management director status, he was in charge of the Logistics, R&D, Purchasing and Management Departments.

From 2006, he was the General Director and in 2010, he was appointed as President of Selecom Group, Clermont Ferrand, France.

He is the Head of several projects for continuous improvement in supply chain and quality management. Currently, he is leading a large platform optimization and renovation project, considered as a technological "liver" at Selecom, which allows his group to enter the industrial revolution 4.0. In addition, on a social level, he is gradually approaching and implementing the CSR system (Corporate Social Responsibility) at Selecom. His ambition is to lead Selecom toward sustainable development in a green economy.

Laurent Klupinski was born in 1970, Clermont Ferrand, France. He has been working for Selecom Group since he graduated. For ten years, he acquired his solid experiences by occupying different operational positions in four main areas: logistics, purchasing, management and trade. Due to his brilliant skills, he then successively took on the responsibility and piloting of these different activities so as to become Trade Director and Co-Leader in 2014. In 2018, he has been known as Operation and Trade Director of Selecom Group, Clermont Ferrand, France.

He has made a lot of contribution in the development process of Selecom Group. His rich experience has helped him succeeded in applying theory into supply chain management as well as transportation modeling and optimization: changing the warehouse structure to optimize the internal transportation system, proposing the shuttle transportation system, improving the internal good flows, and developing modules in the ERP system.

Along with Supply Chain activities, he is also the Head of several files in the others domains: prospecting, fixing cable selling prices based on complex forecast method, developing and correcting the database for the current ERP system at Selecom. Being a member in the Selecom's Organization and Direction Committee, one of his most primordial current objectives is to renovate and modernize the platform on the basis of high technology.