Optimizing Searchers that can Transport and Deploy another Searcher Using an Agent Based Model and Nonlinear Optimization Methods in a Maritime Domain

Jarrod K. Grewe and Igor Griva

Abstract—**This paper introduces a new methodology, nicknamed Pathfinder, for finding optimal search paths for searchers that can transport and deploy other searchers. The methodology applies an Agent-Based Model to model target movement, then uses nonlinear optimization methods to find optimal search plans. This methodology can optimize these search teams effectively and quickly. Pathfinder significantly increase probability of detection and decreases travel distance. In addition to advancing Search Theory, this methodology also has the potential to enhance current search and rescue (SAR) and anti-submarine warfare (ASW) operations.**

Index Terms—**Simulation, search theory, search and rescue, nonlinear optimization, agent based modeling.**

I. Introduction

Search theory started during World War 2 by B.O. Koopman to find optimal search strategies in hunting German U-boats [1]. In more recent times search theory has advanced to include various searcher and target types. For a review of the developed methods see, [2]. However, there are important questions in search theory that have not been addressed. One such question is how to optimize searchers that can transport and deploy other searchers. This is a critical question to address since many searchers can transport other searchers and optimizing where to deploy them may improve the chances of target detection. Such an approach may also save resources, since the search becomes more localized.

The methods that can organize a group of stationary searchers or sensors has been around for over a decade (see, for example [3]-[6]). These methods include adaptive fireworks algorithms, genetic algorithms, and multi-objective genetic optimization algorithms. Even though these algorithms can be effective at optimizing an array of stationary sensors, they cannot optimize a mobile searcher that deploys a sensor during a search, for example. Thus, the problem of how to optimize a searcher that can transport and deploy other searchers over a domain where a target is, needs to be addressed and a methodology that can address this problem needs to be developed.

One important application of optimizing searchers that can deploy other searchers is ASW, with airborne assets in particular, see [7] and [8]. A classic example of a searcher transporting and deploying another searcher would be an aircraft using an expendable sonar buoy while searching for submarines, which is a critical tool for ASW [9]. A methodology that can optimize these resources would be important for ASW. In addition, such an advanced planning methodology could also be used with Unmanned Aerial Vehicles (UAV) and Unmanned Surface Vehicles (USV) which can be transported by ships.

The goal of this research is to address the problem of optimizing search plans for search teams that consist of a carrier-passenger search pair. In this manuscript we describe how a new methodology, nicknamed Pathfinder, can optimize these search operations.

This paper also demonstrates how a search manager could use Pathfinder to search for a missing boat in the open ocean. We will present a search scenario to help describe this methodology. In this scenario, imagine being a search manager in Delaware USA and the operators of a boat sent a short distress call because of electrical issues a few moments before the search manager received the information. Attempts were made to contact the boat again, but the operator of the boat was incapable of communicating. The incomplete dataset includes the heading of the boat, an estimate of its current location, and relevant environmental information.

The paper is organized as follows. In the next section we will discuss in detail the Pathfinder methodology. In section three we will discuss the results from the considered scenario. We present our concluding remarks in section four. In section five we review future research areas and will examine why this new methodology is important to ASW and SAR operations.

II. Methodology

Pathfinder uses a two-dimensional domain to model the search area. In our example, this is a purely maritime domain.

$$\Omega \in R^2$$

Searchable subdomains are constructed to limit searchers from areas they are not allowed, such as foreign or restricted territories. We define this area. Ω_s, such that

Manuscript receivedMay 19, 2022; revised July 2022. This work was supported in part by (sponsor and financial support acknowledgment goes here).

J. K. Grewe is with George Mason University, Fairfax, VA 22030 USA (e-mail: jgrewe@gmu.edu).

I. Griva is with Department of Mathematical Sciences, George Mason University, Fairfax, VA 22030 USA (e-mail: igriva@gmu.edu).

DOI: 10.7763/IJMO.2022.V12.814

$$\Omega_s \subseteq \Omega$$

We define our searcher paths $z_t^k, t = 1,..,T$ for searcher $k = 1, \ldots, K$ and target paths $u_t^g, t = 1, \ldots, T$ for target $g \in G$ of $|G|$ targets to satisfy the following.

$$u^g \in \Omega$$
$$z^k \in \Omega_s$$

To describe the position of the target before the search starts, we use its initial probability distribution $\theta(x)$, which could be based, in particular, on its last known location. To add more accuracy and flexibility, we use regions defined as $R_i \subseteq \Omega$ with probabilities a_i. This could be useful when there are several sources of information that need to be reconciled. These regions satisfy the probability that the target is in the domain M.

$$\sum_{i=0}^{m} a_i \int_{R_i} \theta(x)dx = M \text{ and } \sum_{i=0}^{m} a_i = 1 \qquad (1)$$

For example, we have two circular regions in our scenario. A large 40% region, where there is a 40% chance the target was in that region at. Then a smaller 50% region within it where there is a 50% chance the target was there at. The rest of the domain falls within a 10% region where there is a 10% chance that a target is there. The 40% and 50% regions will be towards the center of the search domain. The initial target types will be; boat with power at 60%, boat without power at 30%, raft at 5%, and person in the water at 5%.

We used this prior distribution in an agent-based model (ABM) to model target movement. This model uses numerous independent agents that are affected by environmental factors, behavioral factors, and hazards. First, environmental factors are wind and currents that are in the search area. The wind and currents in our example will push these agents north, then east. The ABM also can incorporate hazards such as rocks that agents navigate around or get stuck on. Second, there are behavioral factors. These behavioral factors depend on survival modes to model target movement. When people are lost, they rely on a survival strategy to survive or find their way home. These include overdue, travel aid, route finding, stay put, and wandering. In this scenario the overdue, travel aide, stay put, and wondering modes are present. When a target is overdue, it is not lost and is actually late getting home or their next waypoint. The travel aide mode is when a target has some travel aids and can self-rescue. This mode relies on the theory of “bounded rationality” [10]. According to this theory, rationality is bounded because of limited data and mental capabilities. Thus, a missing person’s idea of a path home is more accurate as they approach future waypoints. Wandering is when a target does not have travel aids or is incapable of rational behaviors and will wonder around the domain. Finally, the stay put mode is when a target stays where they are. With a boat, this could be implemented by using an anchor or beaching the boat. The ABM provides us with our estimate target paths $u_t^g, t = 1, \ldots, T$.

Next, we model the probability that a searcher at z_t^k will detect a target at u_t^g at time t. This is implemented using a detection function, which depends on several factors including time, distance, visibility, and properties of the target. Some previous methodologies use the idea of sweep widths, lateral ranges, etc. See [11]. In Pathfinder, we use a modification of the inverse Nth power law [12] below, because it provides flexibility.

$$\Gamma(u_t^g, z_t^k, \Delta t) = 1 - \exp\left(-\Delta t \frac{\alpha(z_t^k, \tau(z_t^k), u_t^g, v)}{|u_t^g - z_t^k|}^{n(z_t^k, \tau(z_t^k), u_t^g, v)}\right) \qquad (2)$$
$$n(*) > 0, \alpha(*) > 0, \Delta t > 0, z_t^k \in \Omega_s, u_t^g \in \Omega$$

This function depends on time step Δt, target type u_t^g, searcher type z_t^k, visibility v, terrain type $\tau(z_t^k)$, and the parameters $\alpha(*)$ and $n(*)$. These parameters are found using experimental data. For notational simplicity we also define the probability of not detecting a target as below:

$$\bar{\Gamma}(u_t^g, z_t^k, \Delta t) = 1 - \Gamma(u_t^g, z_t^k, \Delta t) \qquad (3)$$

The objective of the Pathfinder methodology is to find optimal searcher paths, $z_t^k, t = 1, \ldots, T$, that maximize the probability of detection function (POD). These paths depend on target paths from the ABM, $u_t^g, t = 1, \ldots, T$, and the detection function. We call a collection of searcher paths a search plan. This POD function is as follows:

$$F(z_t^k) = 1/|G| \sum_{t=1}^{\frac{T}{\Delta t}} \sum_{k=1}^{|K|} \sum_{g=1}^{|G|} \Gamma(u_t^g, z_t^k, \Delta t) \prod_{j=1}^{t-1} \bar{\boldsymbol{\Gamma}}(u_j^g, z_j^k, \Delta t) \qquad (4)$$

This objective function is a modification of the objective function found in [13] and follows the logic of [14].

To make the objective function produce realistic search trajectories, we incorporate three penalty terms for fuel, momentum, and center-of-mass. The fuel penalty below is used to make search plans look like more cost-effective trajectories and cut down on suboptimal waypoints.

$$P_F(z_t^k) = \sum_{k=1}^{|K|} \sum_{t=1}^{T/\Delta t} P_k^F \|z_t^k - z_{t-1}^k\|_2 \qquad (5)$$
$$where\ P_k^F \leq 0\ for\ searcher\ k$$

The following is the momentum penalty. This penalty reduces zig-zagging and generally smooths paths and make them easier to follow.

$$P_M(z_t^k) = \sum_{k=1}^{|K|} \sum_{t=1}^{T/\Delta t} M_k\, P_k^M \|z_{t+1}^k - 2z_t^k - z_{t-1}^k\|_2 / \Delta t \qquad (6)$$
$$where\ M_k > 0\ for\ searcher\ k$$
$$and\ P_k^M \leq 0\ for\ searcher\ k$$

Finally, the center-of-mass penalty eliminates erratic search trajectory and helps the nonlinear optimization model converge to a solution.

$$P_{CM}(z_t^k) = \sum_{k=1}^{|K|} \sum_{t=1}^{T/\Delta t} P_k^{CM} \|z_t^k - avg(u_t)\|_2 \qquad (7)$$
$$where\ P_k^{CM} \leq 0\ for\ searcher\ k$$
$$where\ avg(u_t) = \frac{1}{G} \sum_{g=1}^{G} u_t^g$$

With these 3 penalty terms we have the following objective function with the positive weights w_F, w_M, and w_{CM}.

$$F(z_t^k) + w_F P_F(z_t^k) + w_M P_M(z_t^k) + w_{CM} P_{CM}(z_t^k) \quad (8)$$

To model deployable searchers, we add the expressions $D(t,D_\varepsilon,D_k)$ and $D(t,D_P,D_k)$ that will transition from 1 to 0 depending on a variable and two parameters. A deployment variable D_k which will determine the time a carrier and passenger depart from each other. The other parameter, D_ε and D_P , which are unitless, determine how quickly the departure takes place.

$$D(t,D_\varepsilon,D_k) = \frac{1}{e^{D_\varepsilon (t-D_k)}+1} \quad (9)$$

$$D(t,D_P,D_k) = \frac{1}{e^{D_P (t-D_k)}+1} \quad (10)$$

Figure 1 shows how the dependence of $D(t,D_P,D_k)$ in equation (10) on parameter D_P with D_k=5min. The larger the value D_P, the quicker the deployment takes place. The similar dependence of $D(t,D_\varepsilon,D_k)$ on D_ε can be observed for equation (9).

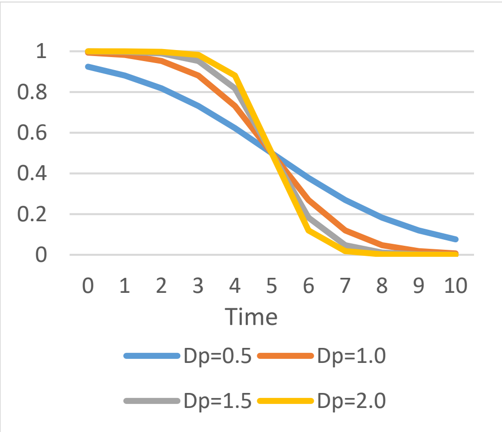

Fig. 1. The graph of equation (10) with various values of D_P. Note how a larger value models a faster deployment.

With equation (9) we create a max movement constraint. For the movement constraints to work each searcher will have two parameters to model maximum speed: $\varepsilon_1\left(s_k, \tau(z_{t-1}^k)\right)$ for before the deployment time and $\varepsilon_2\left(s_k, \tau(z_{t-1}^k)\right)$ for after the deployment time. For the searcher that is carrying another searcher both of these will be the same. For a searcher that is a passenger $\varepsilon_1\left(s_k, \tau(z_{t-1}^k)\right)$ will match that of its carrier and $\varepsilon_2\left(s_k, \tau(z_{t-1}^k)\right)$ will depend on the passenger searcher type. An easy example is a helicopter transporting a buoy. Pre-deployment, ε_1 would be the same for both of them and would be the max move limit of the helicopter. But post deployment, ε_2 of the helicopter would be that of a helicopter but ε_2 of the buoy would be that of a buoy.

With equation (10) we create a pairing constraint. This constraint controls the distance between the paired searchers. The pairing constraint will force the carrier and passenger to stay close to each other before the deployment time, but will allow them to separate after the deployment time. For example, if we are modeling a helicopter with a buoy, the distances between paired searchers would be small, a few meters, for pre-deployment. However, after deployment, the distances would become large, possibly a few thousand kilometers. This would allow deployed searchers to travel far from their carrier post deployment. With these modifications we derive the optimization model below.

Maximize:

$$F(z_t^k) + w_F P_F(z_t^k) + w_M P_M(z_t^k) + w_{CM} P_{CM}(z_t^k)$$

Subject to:

1) *Movement constraints on the searchers*

$$\|z_t^k - z_{t-1}^k\|_2 \le \varepsilon_1\left(s_k, \tau(z_{t-1}^k)\right)\left(D(t, D_\varepsilon, D_k)\right) + \varepsilon_2\left(s_k, \tau(z_{t-1}^k)\right)\left(1 - D(t, D_\varepsilon, D_k)\right)$$

$$\mathit{for\ k\ searcher.}$$

2) *Distance between paired searchers*

$$\|z_t^{kc} - z_t^k\|_2^2 \le v_L\left(1 - D(t, D_P, D_k)\right)$$

$$\mathit{for\ searcher\ k\ and\ its\ carrier}$$

$$\mathit{kc\ and\ sufficiently\ large\ number}\ v_L$$

3) *Initial locations constraints on the searchers*

$$z_0^k = Z_0^k \quad \mathit{for\ searcher\ k}$$

4) *Final locations constraints on the searchers*

$$z_T^k = Z_T^k \quad \mathit{for\ searcher\ k}$$

This is the optimization model we will use to optimize searchers that can transport and deploy other searchers. The final step in the methodology is a post-processor that fixes unrealistic movements.

III. Results

In this section we describe the results obtained using described methodology. The prototype uses Netlogo [15] for the ABM and the nonlinear solver MINOS [16] with AMPL [17] for the optimization model. The computer is a Dell Alienware M17 with an Intel i7-9750H. The timestep for the following models are 5min.

For our example, the objective is to find optimal search plans for a 5-hour search operation over a maritime domain. This domain is modeled after the maritime domain east of Delaware USA. The search domain is 1,000 km^2 with the searchable sub-domain Ω_s being the same as the whole domain Ω. The area to the north contains a strong surface current that flows west to east. The missing object is a boat that most likely has power and is moving towards the northeast of the map. The last known location is near the center of the map one hour before search assets can be deployed. To create a prior distribution two circular regions were used: one has a 50% probability region of radius 1.6 km and is surrounded by a 40% region which has a radius of 6.5 km with the rest of the domain having a 10% chance the target is there. The initial target agents are; boat with power at 60%, boat without power at 30%, raft at 5%, and person in the water at 5%. The ABM will use 501 agents. The weather is clear skies with variable winds at 10 knots.

We will examine three search pairs. A cutter modeled on the Legend Class cutter [18] transporting a UAV modeled on the ScanEagle [19]. This will represent the situation where the passenger is faster than its carrier. In fact, the United States Coast Guard (USCG) produced a draft request for proposal (RFP) for this capability [20]. A helicopter modeled on the HH-60 "Jayhawk" [21] transporting a theoretical "smart" buoy. This represents the case where the passenger is stationary. The third search team is a helicopter on the HH-60 "Jayhawk" transporting a theoretical USV with the performance of the 29 Defiant [22]. This is the case where the passenger is slower than its carrier but is still mobile. We excluded the case where a passenger is the same speed as its carrier since it is trivial. We focus our experiments on using these three search teams and we use USCG documentation [23] and [24] to create our initial search plans.

We will also perform some preliminary analysis on D_P and D_ε for each case. This variable determines how quickly $D(t,D_P,D_k)$ and $D(t,D_\varepsilon,D_k)$, will transition from 1 to 0. This function is critical since it determines how quickly a passenger can deploy and get to full speed. We will examine the range of 0.001 to 2 for both D_P and D_ε. This range was chosen because the value of 2 represents a transition within a timestep of this model and 0.001 represents a transition time that is too long to be realistic.

A. Cutter Transporting a UAV

The initial search plan is the cutter and UAV traveling to near the target center-of-mass and the cutter deploying the UAV at $D_k = 95$ min. The UAV searches the target-center-of-mass with a ladder pattern while the cutter searches the area south of it with a ladder pattern. The cutter has a destination to the east while the UAV does not have a destination. figure 2 is the visualization of this search plan. This plan also has a POD of 8.48%.

This search pair was successfully optimized and we found several optimal search plans. With the initial search plan Fig. 2 we get the optimal search plan in Fig. 3. with $D_k = 129.5$ min $D_\varepsilon = 0.5$, and $D_P = 1.5$ plus a POD of 9.19% and a runtime of 22 minutes. This is an 8.4% increase in POD.

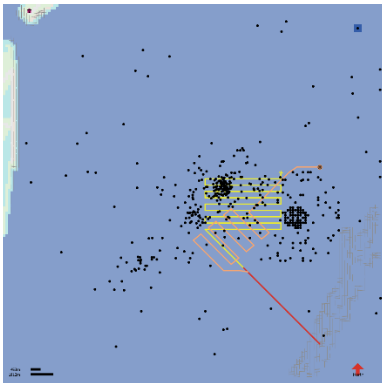

Fig. 2. The initial search plan of a cutter (orange) transporting a UAV (yellow) with $D_k = 95$ min and POD of 8.48%. The red line is the path of the search team before deployment.

This optimal plan is reasonable since the UAV is faster than the cutter. The UAV searches the target-center-of-mass with a ladder pattern while the cutter uses more of a wave pattern. The optimization model increases the POD primarily by slowing down the searchers while they are near agents and by removing suboptimal waypoints. In this experiment Pathfinder shortened the path of the cutter by 68% and shortened the distance of the UAV by 22%. Thus, decreasing fuel costs while increasing the POD.

After several experiments appears that the optimization model is sensitive to D_ε and D_P. For example, a small D_P results in more challenging optimization problem. This can be explained since D_P determines the distance between the search pair. A larger D_P will result in the pair distance constant increasing faster after the passenger is deployed. Thus, an easier model for the optimization application to solve.

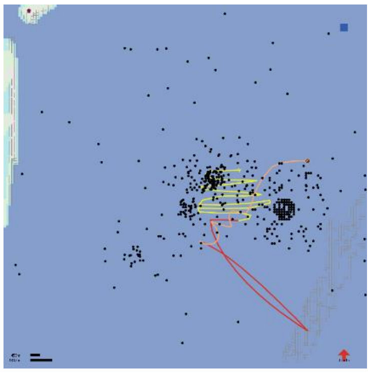

Fig. 3. Optimal search plan of a cutter *(orange)* transporting a UAV (yellow), with a destination, $D_k = 129.5\ min$, $D_\varepsilon = 0.5$, and $D_P = 1.5$. The POD was 9.19% and 22 minute runtime. The red line is the path of the search team before deployment. Note how the paths of the searchers diverged slightly before the deployment time, this is due to the values of D_ε and D_P and the performance of the UAV.

As we leave a more detailed analysis of sensitivity of optimization problems to D_ε and D_P for future research, here we report the results of a few experimental runs.

B. Helicopter transporting a Smart Buoy

The second search team is a helicopter transporting a "smart" buoy. Figure 4. shows the initial search plan used to experiment with this team. The helicopter travels to the target center-of-mass and searches it with a ladder pattern. At deploy time $D_k = 225$ min the helicopter deploys the buoy then continues following the ladder pattern. This smart buoy is intentionally deployed very close to an area of primary concern. Finally, it travels to its destination location to the east. This initial search plan has a POD of 7.99%

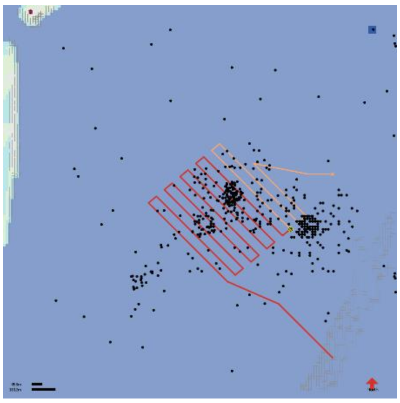

Fig. 4. The initial search plan of a helicopter (*orange*) transporting a smart buoy (yellow) with $D_k = 225$ min and POD at 7.99%. The red line is the path of the search team before deployment.

With this initial search plan, we get the optimal search plans in Fig. 5. The constraints were $D_k = 225$ min, $D_\varepsilon = 0.25$ and $D_P = 1.5$. This plan has a POD of 11.85% and a runtime of 19 minutes and 50 seconds. This is an 48.3% increase in POD. This search plan also shows how pathfinder uses nontraditional search paths including loops and wave patterns to condense the ladder pattern into only areas of high concern.

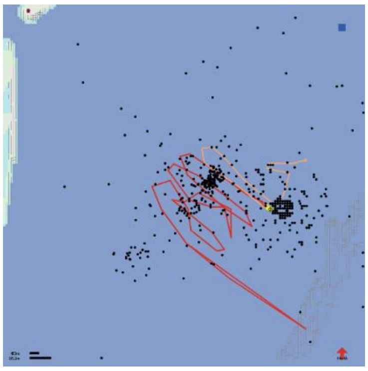

Fig. 5. Optimal search plan of a helicopter *(orange)* transporting a “smart” buoy (yellow), $D_k = 225\, min$, $D_\varepsilon = 0.25$ and $D_P = 1.5$. This plan has a POD of 11.85% and a runtime of 19 minutes and 50 seconds. The red line is the path of the search team before deployment.

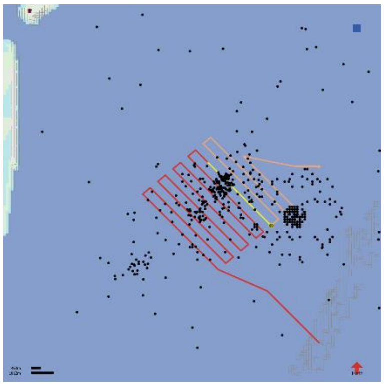

Fig. 6. Initial search plan of a helicopter (*orange*) transporting a USV (yellow), $\mathrm{D_k} = 155$ min and POD of 6.07%. The red line is the path of the search team before deployment.

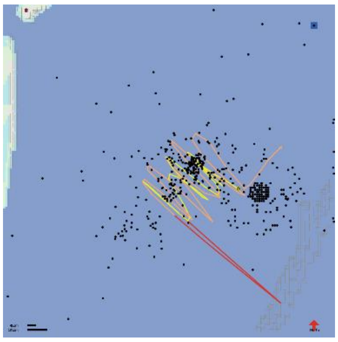

Fig. 7. Optimal search plan of a helicopter *(orange)* transporting a USV (yellow), without a destination, $D_k = 75\, min$, $D_\varepsilon = 1$ and $D_P = 1$. This plan also has a POD of 11.01% and a runtime of 17 minutes and 48 seconds. The red line is the path of the search team before deployment.

C. Helicopter Transporting a USV

The final search pair we will examine is a helicopter transporting a USV. In this scenario a helicopter travels to the target center-of-mass and searches it with a ladder pattern. At $D_k = 155$ min the helicopter deploys the USV. The USV continues searching while the helicopter finishes its search and heads to its destination. The POD of this initial search plan is 6.07%. The visualization of this initial search plan is in Fig. 6.

Again, Pathfinder was successful in finding optimal search plans. The initial search plan gave us the optimal search plan in Fig. 7 with $D_\varepsilon = 1$ and $D_P = 1$. The deploy time of the USV was $D_k = 75$ min. The new POD is 11.01% and the runtime is 17 minutes and 48 seconds. Pathfinder increased the POD by 81.4% for this search pair.

To increase the POD the optimization model condensed the search paths and slowed down the searchers. In this experiment Pathfinder shortened the path of the helicopter by 84.4% and shortened the distance of the USV by 41%.

IV. Concluding Remarks

The results demonstrate that Pathfinder can find optimal search plans for search pairs consisting of a carrier and passenger. This is important to SAR operations and particularly ASW operations. In ASW there are several searcher types that can transport and deploy other searcher types. For example, surface ships can transport helicopters, aircraft can transport sonar buoys, and so on. Therefore, the proposed methodology could potentially benefit ASW and naval operations.

All the calculated runtimes are less than 25 minutes as expected for the performed computational experiments. The increase in POD and decrease in travel times over initial search plans are significant. In particular, the increase of the POD is more than 46% comparing to traditional ladder pattern search plans with optimal search plans that incorporate wave patterns.

Preliminary analysis of D_ε and D_P also gave us some important insight. The resulting optimal search plans depend on the values of D_ε and D_P. In some cases, we can use large values for of D_ε and D_P, which results in having realistic scenarios that Pathfinder can address. We also note that the analysis of sensitivity of the optimal search path to D_ε indicate that there may be an optimal range of values for this important parameter. Finally, we note that the analysis of the sensitivity of the optimal search paths to D_P indicates that larger values may be optimal.

V. Future Research and Applications

There are several research directions worth pursuing in the future. First, further analysis of the effect of D_ε and D_P on the model will be necessary. In some search pairs a higher D_ε and D_P may be more realistic but optimization software may not be able to solve the model quickly or at all. It appears that larger values of D_P increase the chance of search trajectories to be successfully found. It also appears that there may be an optimal range for D_ε. Both seem to depend heavily on the differences in performance of the carrier-passenger search pair with and without the passenger. This analysis includes experiments with other optimization solvers. Another direction of research is investigating how to solve optimization problems for a larger range of D_ε and D_P values. Both D_ε and D_P have an effect on the optimal plans found. Thus, further analysis will be performed to discover how the magnitude of D_ε and D_P affect optimal search plans.

In addition, future research should be done to find performance baselines. This will be important if this methodology is used in a production application.

Future research will also focus on optimizing more complex problems. The scenario and search pairs in this paper are relatively trivial. Future research would focus on searchers that could deploy multiple searchers, adding constraints to when a searcher returns to its transport, and searchers that could redeploy from the same searcher.

These enhancements could make Pathfinder immensely useful for ASW. For example, a P-8 ASW aircraft [25] could be given optimal search plans that include where it will deploy several sonar buoys. In areas like the Greenland, Iceland, and United Kingdom Gap, this could be useful in international border enforcement. For example, P-8s could be teamed with submarines, ships, sonar arrays, and other aircraft to prevent submarines from traversing this strategic area. This would be an important path forward for this line of research.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Both authors contributed to the optimization model. Both authors have approved the final version of the manuscript.

Acknowledgment

The authors would like to acknowledge the Institute for Defense Analyses and the researchers who provided invaluable knowledge in the field of Search Theory.

References

[1] B. Koopman, *Search and Screening*, 1958.
[2] L. Stone, J. Royset, and A. Washburn, *Optimal Search for Moving Targets*, Springer, 2016.
[3] K. Q. Qi, "Analytical optimization on GNSS buoy array for underwater positioning," *Acta Oceanol. Sin.* pp. 137-143, 2019.
[4] X.-Y. Chen, "Genetic algorithm and its application in optimal sensor layout," *Sensors and Transducers*, pp. 55-60, 2015.
[5] R. Grasso, M. Cococcioni, B. Mourre, J. Osler, and J. Chiggiato, "A decision support system for optimal deployment of sonobuoy networks based on sea current forecasts and multi-objective evolutionary optimization," *Expert Systems with Applications*, pp. 3886-3899, 2013.
[6] Y. Ma, Z. Y. Mao, W. J. Ding, R. Luo, Y. J. Xiao, and X. Y. Meng, "A quick deployment method for sonar buoydetection under the overview situation ofunderwater cluster targets," *IEEE Access*, 2019.
[7] J. Furlong and J. Ryder. (2018, June 13). Hunting Submarines from the Air. [Online]. Available: https://physicsworld.com/a/hunting-submarines-from-the-air/
[8] K. Noonan and T. Rogoway. (2019, Octover 22). Confessions of A Submarine Hunting S-3 Viking Crewman During the Twilight of the Cold War. [Online]. Available: https://www.thedrive.com/the-war-zone/30525/confessions-of-a-submarine-hunting-s-3-viking-crewman-during-the-twilight-of-the-cold-war
[9] R. A. Holler, "The evolution of the sonobuoy from world war ii to the cold war," *U.S. Navy Journal of Underwater Acoustics*, 2013.
[10] H. Simon, *Models of Bounded Tationality*, Cambridge MA: MIT Press, 1982.
[11] J. Frost and L. Stone. (2001, September). Review of Search Theory: Advances and Applications to Search and Rescue Decision Support. Retrieved from Defense Technical Information Center. [Online]. Available: https://apps.dtic.mil/dtic/tr/fulltext/u2/a397065.pdf
[12] K. Iida, "Inverse Nth power detection law for washburn's lateral range curve," *Journal of the Operations Research Society of Japan*, pp. 90-101, 1993.
[13] H. Ding and D. Castanon, *Moving Object Search with Multiple Agents*, 2018.
[14] J. S. Przemieniecki, "Mathematical methods in defense analysis," *Mathematical Methods in Defense Analysis*, pp. 278-279, 2000.
[15] U. Wilensky. (1999). Netlogo. [Online]. Available: https://ccl.northwestern.edu/netlogo/
[16] B. Murtagh and M. Saunders, "Large-scale linearly constrained optimization," *Mathematical Programming*, vol. 14, pp. 41–72, 1978.
[17] AMPL Optimization inc. (2021). *MINOS for AMPL*. [Online]. Available: AMPL: https://ampl.com/products/solvers/solvers-we-sell/minos/
[18] Legend *Class National Security Cutter*. (2021). [Online]. Available: https://www.naval-technology.com/projects/legendclassnsc/
[19] *ScanEagle*. (2021). [Online]. Available: https://www.insitu.com/products/scaneagle
[20] C. Collins, *Defense Media Network, Coast Guard Unmanned Aircraft Program Update: Under Secretary for Management*, 2018.
[21] J. Pike, *HH-60J Jayhawk Specification*. [Online]. Available: GlobalSecurity.org.
[22] 29 Defiant. (2021). [Online]. Available: https://www.metalsharkboats.com/29defiant/
[23] J. Frost, *Principles of Search Theory*, 2000.
[24] USCG, *USGS Addendum to the US National Search and Rescue Supplement (NSS).* Washington DC: USCG, 2013.
[25] *P-8: An Intelligence, Surveillance and Reconnaissance Solution*. (2021). [Online]. Available: https://www.boeing.com/defense/maritime-surveillance/p-8-poseidon/index.page

Jarrod Grewe received his BS in mathematics from College of the Ozarks in 2009 and his MS in applied mathematics at University of Missouri-Columbia in 2011. He is currently a Ph.D candidate at George Mason University in Fairfax Virginia USA. He is currently a Senior Database Administrator at the Institute for Defense Analyses.

His research interests are in the areas of search theory, agent based modeling, and nonlinear optimization.

Igor Griva is an associate professor of Mathematical Sciences at George Mason University. His research focuses on the theory and methods of nonlinear optimization and their application to problems in science and engineering.