An Optimization Model for Vehicle Routing in Urban Cold-Chain Logistics

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Abstract—In the present study, we construct an improved vehicle routing problem with simultaneous delivery and pick-up model to reduce unnecessary waste and cost in urban cold-chain logistics distribution. This model considers real-time outside temperature during distribution and optimizes the total cost of transportation, cargo damage and refrigeration. The model is solved using a genetic algorithm to determine the distribution path with the optimal cost. Finally, the validity of the model and the effectiveness of the algorithm are numerically tested.

Index Terms—Urban cold-chain logistics, vehicle routing problem, genetic algorithm

I. INTRODUCTION

In the modern world, people become increasingly concerned about the quality of life in various respects. The consumption of high-quality fresh products is increasing in both urban and rural areas, and food safety is drawing considerable attention. Therefore, urban cold-chain logistics, as a key factor that ensures cargo quality to reduce unnecessary loss during transportation, plays an increasingly important role in daily life. Urban cold-chain logistics is undergoing rapid growth and unprecedented development.

However, due to the limited level of urban cold-chain logistics, many problems occur in the distribution process under traditional conditions [1]. These issues greatly affect circulation speed, and transportation efficiency and cause enormous waste in the distribution process.

In this paper, a new model for the vehicle routing problem with simultaneous delivery and pick-up (VRPSDP) is constructed to improve the efficiency of cold-chain logistics transportation and reduce cargo damage and other logistics costs. This model takes real-time outside temperature into account, to fulfill the food quality required by customers while reducing the distribution cost.

The vehicle routing problem was introduced by Dantzig and Ramser in 1959 [2]. They discussed a vehicle path problem where a single distribution center provides a distribution service to customer nodes with certain demands. The path optimization problem aims to complete distribution tasks under the constraints of a certain number of vehicles with the minimal distribution cost. On this basis, many different directions and variations of the VRP have been developed for academic research and practical application [3]-[15].

To extend the VRP to bring it closer to reality, Hokey [4] proposed the VRPSDP, which has become an important form of the VRP. In the VRPSDP model, a client node contains pick-up and delivery requirements. In 2014, Zhang and Chen [5] studied the distribution of a variety of frozen foods with different temperature requirements that are loaded together, considering time window and loading weight constraints, especially the loading volume related to the units of different frozen foods. Hsu and Chen [6] studied the optimization of vehicle fleet size and distribution scheduling for the cold delivery of various food products in different temperature zones.

Hu et al. [7] constructed a VRPSDP model with different freezer zone compartments for various temperature requirements, taking into account the characteristics of the multi-temperature co-distribution transport process and the time window constraints of client nodes. In 2017, Liang and Zou [8] established a cold-chain logistics vehicle path optimization model that minimizes the total distribution cost including cargo damages. To weight the temperature-related cost of cargo damage and refrigeration, the model uses the temperature setting of vehicles as a decision variable, and a suitable temperature within the cargo’s required temperature range is selected as the vehicle’s distribution temperature; these measures lead to the optimal total distribution cost.

This study discusses the vehicle routing problem in urban cold-chain logistics. The aim is to minimize the distribution cost, including transportation, damage and refrigeration costs. We pay special attention to real-time outside temperature in the distribution process and consider both delivery and pick-up requirements. An improved model for distribution path optimization is developed and solved using a genetic algorithm [9].

The remaining part of this paper is organized as follows. Section II describes studied problem. Section III shows the model formulation, and Section IV presents the solution method, which adopts a genetic algorithm for the improved model. Section V explains the comprehensive computational experiment conducted, to demonstrate the validity of the proposed model and the effectiveness of the algorithm. Section VI contains the conclusion and future works.
II. PROBLEM DESCRIPTION

This study considers an urban cold-chain with single distribution center and multiple vehicles delivering cargo to multiple client nodes simultaneously, as shown in Fig. 1. The client nodes have both delivery and pick-up requirements.

Based on the characteristics of cold-chain logistics distribution, the distribution cost of the vehicles can be divided into three parts, namely, transportation cost, cargo damage cost, and refrigeration cost. The transportation cost depends on the transport cost per kilometer of each vehicle and its mileage. The cost of cargo damage includes two parts: (1) that caused by the accumulation of time during the distribution process, and (2) that caused during the loading and unloading when the vehicle door is opened and closed. The cost of refrigeration consists of two components: (1) that due to the difference between the outside and inside temperatures of the vehicle during delivery and (2) the cost of additional refrigeration consumed during loading and unloading at a client node when the door is opened [10].

The following assumptions are made for the proposed VRPSDP in urban cold-chain logistics.  
1) This paper studies problem with a single kind of cargo for distribution;  
2) Vehicle capacity is limited.  
3) Each client node is only serviced by one vehicle.  
4) Each vehicle can service multiple client nodes.  
5) Distribution is limited to intracity service so that the process produces minimal cargo damage cost.  
6) Cargo delivery to and pick-up from client nodes can be mixed.  
7) The vehicles depart from the distribution center and must return to it after completing their delivery tasks.  
8) Each client node may have simultaneous delivery and pick-up requirements.  
9) The locations of the distribution center and client nodes and the demands of the client nodes (including delivery and pick-up volumes) are given.  
10) The distances between the client nodes and the distribution center are known, and the service time of each client node is given.  
11) Cargo damage is uniformly distributed in the entire distribution process.

Fig. 1. Example of vehicle routing problem.

III. MODEL FORMULATION

A. Notations

In this paper, a mathematical model of cold-chain logistics that considers real-time outside temperature and simultaneous delivery and pick-up is established. The model obtains optimal vehicle paths, that is, those that minimize the total cost of transportation, cargo damage and refrigeration. The main parameters are follows: 

\[ n: \text{set of client nodes.} \]
\[ |n|: \text{number of elements in } n. \]
\[ K: \text{set of vehicles.} \]
\[ a: \text{transport cost per kilometer.} \]
\[ d_{ij}: \text{distance from client node } i \text{ to } j. \]
\[ c: \text{value of unit cargo.} \]
\[ \beta_1: \text{cargo damage rate in unit time in the process of transportation.} \]
\[ \beta_2: \text{cargo damage rate during loading and unloading.} \]
\[ t_{ij}: \text{travel time of vehicle from client node } i \text{ to } j. \]
\[ T_j: \text{service time of vehicle at client node } j. \]
\[ q_j: \text{delivery volume of client node } j. \]
\[ p_j: \text{pick-up volume of client node } j. \]
\[ Q: \text{maximum load of vehicle.} \]
\[ \alpha: \text{degree of depreciation of vehicle.} \]
\[ W_1: \text{heat transmission coefficient of carrier.} \]
\[ S_1: \text{surface area of vehicle.} \]
\[ \delta: \text{unit price of refrigerant.} \]
\[ t_k: \text{time at which vehicle } k \text{ completes distribution.} \]
\[ t_{ko}: \text{time at which vehicle } k \text{ begins distribution.} \]
\[ W_2: \text{heat transmission coefficient of air.} \]
\[ S_2: \text{area where carrier door is open.} \]

Decision variables

\[ x_{ijk} = \begin{cases} 1, & \text{vehicle } k \text{ drive from client node } i \text{ to } j, \\ 0, & \text{otherwise.} \end{cases} \]
\[ y_{ik}: \text{load volume of vehicle } k \text{ at client node } i \text{ after distribution.} \]

B. Transportation Cost

The transportation cost of distribution is proportional to the distance traveled by the vehicle. It is expressed as follows:

\[ C_1 = \sum \sum \sum a_{ij} x_{ijk}. \quad (1) \]

C. Cargo Damage Cost

In cold-chain logistics, the cost of cargo damage in the process of distribution is the loss of cargo quality caused by the accumulation of time and the changes in temperature. This study mainly considers two kinds of cargo damage. One is damage due to the increase in distribution time and real-time external temperature fluctuations in the process of distribution. The other is damage that occurs during service at the customer point, caused by the opening and closing of the carrier door, which cause hot air to enter the carrier. During this process, the carrier temperature rises, and cargo is damaged. The cargo damage cost is calculated as follows:

\[ C_2 = c \left[ \beta_1 \sum \sum \sum t_{ij} y_{ik} + \beta_2 \sum \sum T_j y_{jk} \right]. \quad (2) \]
D. Refrigeration Cost

The external temperature tends to vary considerably during the day and is not constant. Assume that the temperature function of one day change in temperature with time is \( H(t) \), the difference between in temperature inside and outside the carrier is \( \Delta H(t) \), the delivery temperature of the cargo is \( H_0 \). Then, \( \Delta H(t) = H(t) - H_0 \).

Refrigeration cost mainly includes two aspects. Heat transfer is caused by the real-time difference between the temperatures inside and outside the carrier during distribution. Moreover, the heat exchange is caused by air circulation during loading and unloading at client nodes. The damage cost can be calculated based on the refrigerant consumption. The refrigeration cost during distribution is as follows:

\[
\eta_1 = (1 + \alpha)W_1 S_1 \delta \sum_{\text{t}} \sum_{j} \sum_{k \in K} x_{ijk} \int_{t_{ko}}^{t_k} \Delta H(t) \, dt.
\]  
(3)

The refrigeration cost during service time at client nodes is as follows:

\[
\eta_2 = \omega W_2 S_2 \delta \sum_{\text{t}} \sum_{j} \sum_{k \in K} x_{ijk} T_j \Delta H(t),
\]  
(4)

where \( \omega \) is related to the frequency of the opening and closing of the door, as specified in Table I.

| TABLE I: COEFFICIENT RELATED TO DOOR-OPENING FREQUENCY |
|----------------|----------------|
| Level | Frequency of door opening | Coefficient (\( \omega \)) |
| 1     | Not open                  | 0.25                          |
| 2     | 6 times or less           | 0.50                          |
| 3     | 7–12 times                | 0.75                          |
| 4     | More than 12 times        | 1.00                          |

Then, the refrigeration cost is as follows:

\[
C_3 = \eta_1 + \eta_2.
\]  
(5)

E. Model Formulation

This paper aims to minimize the distribution cost, in consideration of the transportation cost, cargo damage cost and refrigeration cost involved in the distribution process. We focus on the real-time outside temperature in the distribution process. The improved VRPSDP model is formulated as follows:

\[
\min F(x_{ijk}) = C_1 + C_2 + C_3
\]  
(6)

s.t: \[
\sum_{\text{t}} \sum_{j} \sum_{k \in K} x_{ijk} = n
\]  
(7)

\[
\sum_{j} \sum_{k \in K} x_{ijk} = 1, \quad \forall j \in n; \ i \neq j
\]  
(8)

\[
\sum_{j} \sum_{k \in K} x_{ijk} = 1, \quad \forall i \in n; \ i \neq j
\]  
(9)

\[
\sum_{\text{t}} x_{ijk} = 1, \quad \forall j \in n
\]  
(10)

\[
\sum_{\text{t}} x_{ijk} = 1, \quad \forall i \in n
\]  
(11)

\[
y_{jk} \leq Q, \quad \forall i \in n; \ k \in K
\]  
(12)

\[
y_{ok} \leq Q, \quad \forall k \in K
\]  
(13)

The objective function (6) is the minimization of the total distribution cost. Equations (7) – (15) are constraints. Constraint (7) ensures that all client nodes are served. Constraints (8) and (9) guarantee that only one delivery vehicle can arrive at and depart from each client node, respectively. Constraints (10) and (11) indicate that the cargo in any delivery vehicle does not exceed the vehicle capacity during delivery and at the distribution center, respectively. Finally, according to constraint (14), a vehicle’s current load volume is equal to its load volume at the last client node minus the delivery volume plus pick-up volume.

IV. THE GENETIC ALGORITHM FOR THE MODEL

The improved VRPSDP model is an NP-hard problem, that is, no algorithm will exactly obtain the optimal solution. Therefore, we propose to use the genetic algorithm to solve the model.

The genetic algorithm is an iterative search algorithm where a population of individuals of solutions is evolved from generation to generation toward the optimal solution [11]. We need to carefully design the chromosomes of the individuals so that each individual corresponds to a feasible solution. The crossover and mutation operations also have to be designed well. Fitness calculation is conducted in each generation, if the preset termination conditions are met, then the population is a solution to the genetic algorithm. Otherwise, the fitness calculation is repeated after selection, crossover and mutation until the termination conditions are satisfied.

The flow of the designed genetic algorithm in this study is shown below.

Step 1: initialize a population of individuals with a chromosome length of \( |\text{ji}| \) and a population size of \( L \).

Step 2: insert the distribution center \( 0 \) into the chromosomes and calculate the individual chromosome fitness, and label the value of fitness.

Step 3: select individuals with high fitness using the roulette wheel selection strategy thereby generating a new population.

Step 4: select chromosomes for crossover operations with crossover probability. Two parental chromosomes cross genes to form a new chromosome, which then enters the new population. Chromosomes that do not undergo crossover operations enter the new population directly.

Step 5: perform a chromosome mutation operation with mutation probability. Change the position of chromosome genes according to the corresponding probability to produce a new population. Chromosomes that do not undergo mutation directly enter the new population.

Step 6: generate a new generation of populations. If the termination conditions are met, then the algorithm terminates. Otherwise, revert to step 2.
Each of these steps is described below.

A. Encoding of Chromosomes

The improved VRPSDP model is designed with a natural number coding approach to its chromosome coding. A permutation of natural numbers from 1 to |n| generates the base of a chromosome for an individual. Then we insert several 0s to complete the chromosome. For example, 345216987 is a full permutation of natural numbers, and the number 0 is inserted into it; that is, the distribution center is inserted into the distribution sequence, thus constituting three distribution routes 345, 216 and 987. A sample of the chromosome for the genetic algorithm is shown in Fig. 2.

![Example of chromosome](image)

3 4 5 2 1 6 9 8 7 0
0 3 4 5 0 2 1 6 0 9 8 7 0

Fig. 2 Example of chromosome

B. Initialization

The initial population is an |n|-L 2-dimensional matrix, where |n| is the number of client nodes and L is the preset population size. The initial population of L individuals is generated randomly. Each individual stands for one solution that satisfies the problem. The population size affects the convergence speed and solution diversity.

C. Fitness Function

Genetic algorithms use fitness to assess the merit of individuals during the search process. This is an important basis for the algorithm operation. In this study, the inverse of the objective function is used as the fitness function. The fitness function is as follows:

\[
\text{Fit}(x_{ijk}) = \frac{1}{F(x_{ijk})},
\]

D. Selection Operation

In the present study, the selection of parental individuals is based on the roulette rule. Individuals with high fitness are more likely to be selected as parents to produce a new population. The basic idea of the selection of genetic operators is that the probability of an individual to be selected is proportional to its fitness [12]. A certain percentage of the best solution in each iteration is retained to keep the best individuals of the current population in the next generation. This may improve the convergence of the genetic algorithm.

E. Crossover Operation

In this study, partial matching crossover is used to generate the crossover operator, but as some information is lost in the crossover process, the gene positions of the chromosomes need to be changed according to the corresponding probability, and a variation operator is generated [13]. Individual chromosomes are selected for reversal with a certain mutation probability to produce two mutated node genes, which are then subjected to the reversal operation to form a new population.

To illustrate, an example is shown in Fig. 3. The gene values at three randomly selected gene positions in parent1 are first copied to the same gene positions in the child. Then parent2 genes with the same gene values are removed. Finally, the gene values at the remaining gene positions in parent2 are sequentially copied to the corresponding position in the child [14]. If a gene position already has a gene value, then that position is skipped, and the operation continues at next position.

F. Mutation Operation

Some important genetic information may be lost in the crossover operation, and a moderate amount of mutation (a certain probability of changing the gene positions of the chromosomes) must be introduced. In this paper, a chromosomal individual is selected with a certain probability of mutation, and two mutated gene nodes are created on the gene string of the individual. An example of the mutation operation is shown in Fig. 4, where the boldface chromosomes are the genes being mutated.

![Example of mutation operation](image)

![Example of crossover operation](image)

V. EXPERIMENTAL ANALYSIS

In order to verify the validity of the model and the effectiveness of the algorithm, we tested it through numerical experiments [15]. In the numerical example, the coordinates of the distribution center and the 100 individual client nodes, basic data about the delivery and pick-up volumes and the service times at the client nodes are generated randomly. Part of the data are shown in Table II.

<table>
<thead>
<tr>
<th>TABLE II: DATA OF CLIENT NODES</th>
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<tbody>
<tr>
<td>Node No.</td>
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<td>----------</td>
</tr>
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<td>0</td>
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<td>1</td>
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<td>...</td>
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<td>100</td>
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</table>

Based on the preceding analysis of statistical data, the distribution center has sufficient vehicles. The vehicles are of the same type, and the main model parameters are specified in Table III.

<table>
<thead>
<tr>
<th>TABLE III: MAIN MODEL PARAMETERS</th>
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<tr>
<td>a</td>
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<tr>
<td>0.25</td>
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</table>
We conducted many experiments to improve the parameter settings of the genetic algorithm, and the experimental results suggest that the parameter settings in Table IV are the best choice. In the selection operation, we retain 10% of the best solutions for the next iteration.

<table>
<thead>
<tr>
<th>TABLE IV: PARAMETER SETTING</th>
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<tbody>
<tr>
<td>Population size</td>
</tr>
<tr>
<td>50</td>
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</table>

The proposed model is compared with the original VRPSDP model. Let \( F_0(X) \) and \( F(X) \) be the values of the objective functions of the original VRPSDP model and the improved VRPSDP model, respectively. Several experiments are performed for comparison. First, we solve the original VRPSDP model to obtain \( X^* \) (the optimal vehicle route of solution) and \( F_0(X^*) \) (the optimal value). Then, we substitute \( X \) into the improved model to obtain \( F(X') \), which is the value of the objective function of the improved model. Next, we solve the improved model to obtain the optimal vehicle route of solution \( X' \) and the optimal value \( F(X') \).

Table V shows the difference between \( F(X') \) and \( F(X^*) \). We can see that the improved model has a significant reduction in the average operating cost for the same operating route. Compare with to the original model, the improved VRPSDP model has an average cost reduction of nearly 1.6%. Therefore, the distribution cost can be reduced by considering the real-time external temperature of urban cold-chain logistics distribution. Therefore, the improved VRPSDP model is superior to the original one. Fig. 5 shows an examples of distribution routes, where the red star indicates the distribution center and the blue points are the client nodes.

<table>
<thead>
<tr>
<th>TABLE V: COMPARISON OF ORIGINAL AND IMPROVED VRPSDP</th>
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<tbody>
<tr>
<td>Case No.</td>
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<tr>
<td>9</td>
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<tr>
<td>10</td>
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<tr>
<td>Average</td>
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</tbody>
</table>

In addition, we conducted a number of experiments to demonstrate the effectiveness of the proposed genetic algorithm. For example, we increased the number of cities from 15 to 100 to demonstrate that the algorithm can solve small-scale problems as well as it can solve large-scale ones. In Table VI, the data for each case are the averages of 10 experiments with an initial population of 50 and 500 iterations. As can be seen in the table, as the number of cities increases, the computing time increases moderately. Hence, the algorithm effectively solved large-scale problems. However, the number of iterations needed to obtain the results is close to the maximum iteration of 500, so we need to consider increasing the number of iterations.

<table>
<thead>
<tr>
<th>TABLE VI: COMPARISON OF INCREASE OF CITIES</th>
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<tr>
<td>Number of cities</td>
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<td>25</td>
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<td>30</td>
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<tr>
<td>50</td>
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<tr>
<td>100</td>
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</table>

For further improvement of the values of the objective function of the new model in large-scale problems, we repeated the experiment with an initial population of 50 and a fixed number of cities of 100. The results shown in Table VII. From the table we can clearly see that as the number of iterations increases, the number of iterations converges increasingly steadily for both the original model and the improved model. However, given a trade-off between the computational time and the objective functions, we select 100 iterations as the final output.

<table>
<thead>
<tr>
<th>TABLE VII: COMPARISON OF INCREASE OF ITERATIONS</th>
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<tbody>
<tr>
<td>Number of iterations</td>
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<tr>
<td>500</td>
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<tr>
<td>1000</td>
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<tr>
<td>1500</td>
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</table>

VI. CONCLUSION

This paper studied the VRPSDP model in urban cold-chain logistics. Based on the characteristics of urban cold-chain logistics systems, we analyzed the costs involved in the process of cold-chain logistics distribution. We paid special attention to the real-time outside temperature and established an improved VRPSDP model to minimize the comprehensive transportation costs. Since the model is an NP-hard problem, we designed a genetic algorithm to solve the improved model. Numerical experiments demonstrated that the improved
model is superior to the original model, and reduces the distribution costs and improves the distribution efficiency. Numerical results also show that the proposed genetic algorithm can effectively obtain the operation path with optimal cost for cold-chain logistics transport vehicles.

However, there are some issues that need to be addressed in this paper. We did not examine the weight of each cost in the cold-chain logistics distribution process, and all costs were given the same weight. In addition, there is room for improvement regarding the convergence curve of the genetic algorithm. We can consider combining the exact algorithm to find the initial solution, instead of starting with a heuristic algorithm to obtain the initial solution, and then using the heuristic algorithm to solve it based on the initial solution obtained by the exact algorithm.

CONFLICT OF INTEREST
The authors declare that they have no conflicts of interest.

AUTHOR CONTRIBUTIONS
The authors contributed equally. The authors read and approved the final manuscript.

REFERENCES

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