Abstract—The agitator is a kind of general equipment that is widely used in industrial and agricultural production. It involves many fields such as petroleum, chemical industry, rubber and coatings. All of which aim at obtaining homogeneous materials, improving the service performance of materials or accelerating the reaction. Compared with the ordinary agitator, the planetary agitator has a wider agitating range. This article introduces the structure and working method of planetary wheel agitator. The mathematical model of the movement track of a point on the mixing shovel is established, an example model of planetary stirring is designed and verified by comparison between Adams software and MATLAB software. On the basis of this, the change law of motion trajectory and coverage rate is analyzed; In addition to this, the influence of speed ratio on speed distribution is analyzed by EDEM simulation, which provides reference for subsequent design improvement.

Index Terms—Planetary wheel, stirring equipment, motion trajectory, coverage.

I. INTRODUCTION

Mixing equipment is a kind of mechanical product with wide application and variety [1]. Chaoming Wang [2] used computational fluid dynamics to analyze the flow field of planetary mixing and single paddle mixing, and found that planetary mixing has a wider mixing range; At the same time, Adams is used to simulate and analyze the planetary mixing system with multiple planetary wheels and blades. Yujian Jiang [3] conducts a single paddle stirring simulation through Fluent. The mathematical model of planetary stirring trajectory and planetary speed ratio was established, which clarified the relationship between planetary speed ratio and planetary stirring trajectory. Compared with ordinary agitator, the stirring effect of planetary agitator is more uniform. Stirring trajectory plays an important role in planetary stirring [4]. If the fixed trajectory sweeps a small range, it means that there are many blank areas left by stirring and the stirring effect is not good [5]-[7]. Therefore, the search for suitable planetary stirring trajectory plays an important role in improving stirring performance.

II. PLANETARY WHEEL STIRRING DEVICE

Planetary stirring device is mainly composed of driving device, stirring barrel, gear box, stirring device, crushing frame, discharging device, sealing device and frame, among which the stirring device is composed of stirring arm and stirring shovel. The simplified structure is shown in Fig. 1. It can be seen from Fig. 1 that the planetary stirring device consists of a gear box, a stirring frame, a crushing frame and two discharging shovels, and each stirring frame is equipped with three stirring shovels [8].

![Image](image_url)

Fig. 1. Schematic diagram of the structure of the planetary wheel stirring device.

In the gear box is a transmission system composed of a sun gear, a planetary gear, a planetary carrier and a ring gear. The stirring frame is installed on the planet wheel, and the power is input by the sun gear. The stirring frame on the planet wheel can rotate around its own axis while revolving. The planetary gear outputs two speeds through the sun gear, so that the planetary gear drives the stirring rack to make complex trajectories. During the mixing and stirring process, there are convection, shear and diffusion movements at the same time, so as to realize the uniform distribution of the mixture stirring. In convective motion, various components of the mixture migrate in a large range under the action of external force, and their velocity directions and sizes are different, and their motion trajectories cross each other. In the shearing motion, the cohesive clusters of the mixture produce relative displacement along the sliding surface under the action of external force to break the cohesive clusters and redistribute the small-size particles. In the diffusion movement, the fine components of the mixture move randomly in a small range, and the mixture continuously gathers and redisperses.
III. MATHEMATICAL MODEL OF PLANETARY WHEEL STIRRING TRAJECTORY

Take any point on the mixing shovel to analyze its motion trajectory. Fig. 2 is a schematic diagram of the relative position of planetary gears. Suppose the radius of the index circle O of the ring gear is \( R_i \), and the radius of the index circle O' of the planetary gear is \( R_p \). Take O as the origin and OA as the x axis to establish a rectangular coordinate system. At the starting position, any point \( P(x, y) \), when circle O' rotates by \( \sigma \) angle, circle center O' rotates by \( \theta \) angle relative to circle O, namely \( \angle BOA = \theta, \angle BO'P = \sigma \).

Then there is
\[
\overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP}
\]  
(1)
\[
\overrightarrow{OQ} = ((R_i - R_p) \cos \theta, (R_i - R_p) \sin \theta)
\]  
(2)

The arc lengths are equal, so: \( R \theta = \overline{AB} = \overline{PB} = \pi \sigma \) namely:
\[
\sigma = \frac{R_i}{R_p} \theta
\]  
(3)

The directed angle between the vector \( \overrightarrow{OP} \) and the x axis is:
\[
\varphi = -(\sigma - \theta) = \frac{R_p - R_i}{R_i} \theta
\]  
(4)

The direction is shown in Fig. 2. Therefore:
\[
\overrightarrow{QP} = (L \cos \frac{R_p - R_i}{R_i} \theta, L \sin \frac{R_p - R_i}{R_i} \theta)
\]  
(5)

\[
\overrightarrow{OP} = ((R_i - R_p) \cos \theta + L \cos \frac{R_p - R_i}{R_i} \theta, (R_i - R_p) \sin \theta - L \sin \frac{R_p - R_i}{R_i} \theta)
\]  
(6)

The parameter equation of a point trajectory on the mixing shovel is obtained
\[
\begin{align*}
x &= (R_i - R_p) \cos \theta + L \cos \frac{R_p - R_i}{R_i} \theta \\
y &= (R_i - R_p) \sin \theta - L \sin \frac{R_p - R_i}{R_i} \theta
\end{align*}
\]  
(7)

Namely
\[
\begin{align*}
x &= \frac{m}{2} (Z_3 - Z_2) \cos \frac{\alpha t}{1 - \frac{i_{13}^H}{i_{13}}} + L \cos \frac{(Z_3 - Z_2) \alpha t}{Z_t (1 - \frac{i_{13}^H}{i_{13}})} \\
y &= \frac{m}{2} (Z_3 - Z_2) \sin \frac{\alpha t}{1 - \frac{i_{13}^H}{i_{13}}} - L \sin \frac{(Z_3 - Z_2) \alpha t}{Z_t (1 - \frac{i_{13}^H}{i_{13}})}
\end{align*}
\]  
(8)

where, \( i_{13}^H \) is the transmission ratio of the 1st and 3rd wheels in the conversion gear train, \( Z_t, Z_2, \) and \( Z_3 \) are the tooth numbers of the sun gear, planetary gear and ring gear respectively, and \( m \) is the modulus. From the parameter equation, the trajectory curve can be drawn by Matlab software, so as to carry out the research of this paper.

IV. EXAMPLE CHECKING CALCULATION

The parameters of the designed example planetary gear model are shown in Table I. A point on the mixing shovel falls on the planetary gear index circle (\( L = R_2 \)). Its trajectory is the hypocycloid [10].

<table>
<thead>
<tr>
<th>Module ( m )</th>
<th>Sun gear teeth ( Z_1 )</th>
<th>Planetary gear teeth ( Z_2 )</th>
<th>Gear ring teeth ( Z_3 )</th>
<th>Input speed ( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>16</td>
<td>32</td>
<td>80</td>
<td>3\pi rad/s</td>
</tr>
</tbody>
</table>

The three-dimensional model of planetary gear train is established by SolidWorks and imported into Adams, and the motion track of a point on the planetary gear graduation circle is created as shown in Fig. 3. Fig. 4 is the trajectory diagram of planetary stirring made by Matlab software, and its geometric parameters are set in accordance with the 3D model. It can be seen that the mixing trajectories made by the two methods are consistent, which are all five lobes, that is, composed of five arcs. The mathematical model is verified to be correct, and the correlation analysis of planetary gears is carried out with this model.
Fig. 4. Matlab trajectory graph.

V. TRAJECTORY CHARACTERISTIC ANALYSIS

According to the characteristics of hypocycloid, when \( \frac{R_3}{R_2} \) is a rational number, the minimum period of hypocycloid is \( \theta = \frac{2\pi R_2}{gcd(R_3, R_2)} \), where the \( gcd \) (\( R_3, R_2 \)) means the greatest common divisor of \( R_3 \) and \( R_2 \). When \( \frac{R_3}{R_2} \) is irrational, the period is infinite.

When applied to planetary gears, \( \frac{R_3}{R_1} = \frac{Z_1}{Z_3} \), and the number of planetary gear teeth are all integers, that is, \( \frac{Z_1}{Z_3} \) are all rational numbers. Assuming that the simplest integer ratio of \( \frac{Z_2}{Z_1} \) is \( \frac{a}{b} \), the planetary gear needs to go around the ring gear \( a \) times, and point \( p \) can return to the starting point, and the pattern at this time has \( b \) lobes.

As shown in Table II, when the number of teeth of the ring gear is 80, the trajectory distribution under the relationship of part of the number of teeth. Fig. 5 is a trajectory diagram of serial numbers 1, 3, 5, and 6 in the table. The number of lobes of each trajectory in the figure is the value of the denominator \( b \) calculated in Table II.

![Fig. 5. Part of the trajectory distribution map.](image)

<table>
<thead>
<tr>
<th>Serial number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_2 )</td>
<td>32</td>
<td>31</td>
<td>30</td>
<td>25</td>
<td>24</td>
<td>20</td>
<td>19</td>
<td>16</td>
</tr>
<tr>
<td>( Z_3 )</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>30</td>
<td>32</td>
<td>40</td>
<td>42</td>
<td>48</td>
</tr>
<tr>
<td>( a/b )</td>
<td>2/5</td>
<td>31/80</td>
<td>3/8</td>
<td>5/16</td>
<td>3/10</td>
<td>1/4</td>
<td>19/80</td>
<td>1/5</td>
</tr>
<tr>
<td>( L )</td>
<td>1232</td>
<td>19677</td>
<td>1968</td>
<td>3981</td>
<td>2469</td>
<td>840</td>
<td>15623</td>
<td>895</td>
</tr>
<tr>
<td>( \eta_s )</td>
<td>0.43</td>
<td>7.03</td>
<td>0.64</td>
<td>1.29</td>
<td>0.8</td>
<td>0.27</td>
<td>5.08</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Take the maximum working width of the mixing shovel section as \( d \), and the arc length of the curve moved by \( P_0 \) in one cycle as \( g \), the value of the product \( gd \) can represent the coverage area of a single blade in the basin.

\[
S_d = gd
\]  

The diameter of the mixing barrel is \( D \), and the cross-sectional area \( S_D \) of the mixing barrel is:

\[
S_D = \pi D^2/4
\]  

Then the ratio \( \eta_s \) between \( S_d \) and \( S_D \) is the track coverage rate, which can be expressed as the stirring range of the planetary paddle:

\[
\eta_s = S_d/S_D
\]  

According to the arc length formula

\[
L=\int_{0}^{\theta} \sqrt{(x')^2+(y')^2} \, d\theta
\]  

When \( d=20\text{mm}, D=280\text{mm} \), the coverage rate under the relationship of partial tooth number is shown in Table III.

![Fig. 6. Change chart of middle area.](image)
Planetary mixing performs periodic planetary motion in the mixing barrel. The actual mixing trajectory is fixed, and its coverage also increases with the number of revolutions and petals, but there are still more blank areas. As shown in Fig. 6, when the number of lobes are all 80, the trajectory distribution diagram of different revolutions. It can be seen that with the change of the stirring trajectory, the blank area in the center of the mixing barrel also changes. When the number of petals is constant, the blank area in the middle also increases with the decrease in the number of revolutions.

VI. Simulation Analysis

The stirring frame makes planetary motion in the mixing barrel, and its speed ratio directly affects the movement of the mixture in the barrel, and then affects the mixing time. From the calculation formula of transmission ratio, the transmission ratio of rotation and revolution of the stirring frame can be obtained as follows:

\[ i_{13}^H = \frac{\omega_3^H}{\omega_3} = \frac{\omega_3 - \omega_H}{\omega_H} = -\frac{z_3}{z_1} \]  \hspace{1cm} (13)

\[ i_{12}^H = \frac{\omega_2^H}{\omega_2} = \frac{\omega_2 - \omega_H}{\omega_H} = -\frac{z_2}{z_1} \]  \hspace{1cm} (14)

where the rotation speed of the gear ring is \( \omega_3 = 0 \), the transmission ratio of rotation and revolution of the stirring frame can be obtained as follows:

\[ i_{2H} = 1 - \frac{z_3}{z_2} \]  \hspace{1cm} (15)

Based on the EDEM software, single-propeller simulation is performed under four schemes with a stirring trajectory of 80 petals and a number of revolutions of 33, 27, 23, and 17 respectively. Set the total input particles of 10,000, and observe the lateral velocity streamline diagram during one revolution as shown in Fig. 7(a~d), and their speed ratios are \( i_{2H_a} = 1.42 \), \( i_{2H_b} = 1.96 \), \( i_{2H_c} = -2.48 \), \( i_{2H_d} = -3.7 \). In this simulation, an agitator model equivalent to the planetary wheel verification example was established. It can be seen from the figure that the velocity streamline near the stirring frame is greatly disturbed, and corresponding to the middle blank area of the stirring track, the blue streamline area decreases with the increase of the absolute value of the rotation speed ratio, and the particle velocity of the mixture at the edge of the stirring barrel increases.

VII. Conclusion

The stirring trajectory of planetary gear train is periodic and fixed and the ratio of the number of teeth between planetary gear and ring gear is equal to the ratio of the number of common revolutions of planetary gear to the number of lobes of complete track in one cycle. The stirring coverage rate also increases with the increase of the number of revolutions and the number of lobes, and when the number of lobes is constant, the intermediate blank area increases with the number of revolutions. The absolute value of the speed ratio increases, and the disturbance zone of the velocity streamline increases, which makes the mixing movement more intense and can effectively shorten the mixing time.
CONFLICT OF INTEREST
The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS
The first author established the mathematical model of the planetary stirring trajectory and performed the example verification. In addition, she used MATLAB software to analyze the trajectory characteristics; the second author used EDEM software to carry out the simulation analysis of planetary stirring; the third author analyzed the research status of planetary stirring device is presented. Finally, the first author completed the writing of the paper, and all authors approved the final version.

REFERENCES

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