Method for the Kinematic Analysis of the Vehicle Axle Guiding Mechanisms

Cătălin Alexandru

Abstract—**This article deals with the development of an analytical method for the kinematic analysis of the axle guiding mechanisms that are commonly used to guide the rear axles of commercial and off-road vehicles. The method can be applied / adapted for most types of axle guiding mechanisms (at least those commonly used). Unlike the most existing kinematic analysis methods, which involve the development and solving of nonlinear equation systems, with the subsequent disadvantages, the method proposed here is based on linear systems, which are easy to solve. The computation program was developed by using the programming environment Borland Delphi.**

Index Terms—**Vehicle axle, guiding mechanism, kinematics, analytical algorithm.**

I. Introduction

With increasing the vehicle travel speed, the problems raised by the need to improve the comfort and stability have led to equipping the vehicles with increasingly advanced suspension systems, capable of achieving a barrier of vibrations and noise between the rolling system and the car body, all the more much as the vehicle speed on uneven roads is not limited by the performance of the propulsion system, but by the quality of the suspension. One of the most important achievements in this field was the implementation of the guiding mechanism interposed between the wheel and the car body, which determines the position of the wheel against the car body and takes over the forces that appear at the wheel - ground contact. The guiding mechanism along with the elastic and damping components form the vehicle suspension system.

The spatial guiding of the bodies can be performed in two ways: by guiding one of its characteristic points and orienting the body in relation to this point (what is commonly used in industrial robots), or by guiding several points of the body on appropriate surfaces and curves. The latest solution is frequently used for the multi-link guiding mechanisms of the vehicle wheels, the disposing of the guiding links following the fulfillment of the specific kinematic and dynamic requirements [1]-[4]. In the relative movement to car body, the vehicle wheels can be guided independently - by means of a guiding mechanism for each wheel (independent suspension), or dependent - by a guiding mechanism of the rigid axle (dependent suspension). The first solution is frequently used for passenger cars (for the both front and rear wheels), while the second solution is mainly used for the rear axles of commercial and off-road vehicles.

The great variety of guiding mechanisms that can be imagined implies (in order to minimize the restrictions to be fulfilled in the design phase) to identify the types of mechanisms that can be used for the proposed purpose (in terms of suspension system overall setup, axle operating mode, gauge and constructive limits/constraints). The constructively compatible solutions should be evaluated from a technical and economic point of view, highlighting their advantages and disadvantages, which ultimately lead to the selection of the principle solution. This solution is then subject to optimization studies regarding the kinematic and dynamic behavior, which is a continuous concern and challenge for the research in the field, given the increasing demands regarding the suspension system performances.

The analysis of the wheel/axle guiding mechanisms can be carried out in two ways: through analytical methods, which consist of modeling the guiding mechanisms using the equation systems that describe their kinematic/dynamic behavior, those solving is performed by in-house made specialized programs (with the help of various types of programming languages); through automated formalisms integrated into high performance commercial software solutions based on the multi-body systems theory, which automatically form and solve the motion equations [5]-[11].

When working with simplified models of wheel/axle guiding mechanisms, such as the kinematic models (which do not take into account the forces acting in the suspension system, neglecting also the elasticity of the elements and the car body movement), the reduced number of degrees of mobility (DOM), which is given by the number of independent generalized coordinates (movements), allows to approach the study by analytical methods, with the advantage of customizing the computation program. In this regard, the scientific literature reveals several methods (vector, matrix or geometric methods) for the kinematic analysis of the wheel/axle guiding mechanisms [12]-[20]. Most of these methods are developed considering particular variants of guiding mechanisms, being difficult to understand how they could be generalized for a general and unitary approach. At the same time, the existing methods are based on complex mathematical algorithms, which require the use of numerical methods for solving.

This work deals with the development of an analytical method for the kinematic analysis of the axle guiding mechanisms (as mentioned above, these mechanisms are commonly used to guide the rear axles of commercial and off-road vehicles). The method is a general one, which can be applied / adapted for most types of axle guiding mechanisms (at least those commonly used), and it can be easily programmed and transposed on computer. The computation program was realized by using the programming environment Borland Delphi (based on Object Pascal).

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C. Alexandru is with the Product Design, Mechatronics and Environment Department, Transilvania University of Braşov, 500036 Braşov, Romania (e-mail: calex@unitbv.ro).

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II. The Kinematic Functions Of The Rear Axle Guiding Mechanisms

As mentioned, in the kinematic analysis the car body is fixed (rigidly connected to ground), the kinematic behavior of the guiding mechanism being expressed by the relative movement (position and orientation) of the rear axle in relation to the car body (Fig. 1). Thus, the global reference frame OXYZ it attached to car body, where X is the longitudinal axis of the vehicle (positively oriented towards the front of the vehicle), Y - the transversal axis (oriented from right to left), and Z - the vertical axis (oriented upwards). The origin of the global reference frame in located at the front axle level (halfway between the front wheels centers). The axle reference frame $PX_PY_PZ_P$ is located at the middle of the rear axle (halfway between the rear wheels centers); in the initial position (i.e. the static rest position of the vehicle), the local axes are oriented parallel to those of the global reference frame. During the operation of the guiding mechanism (when the wheels are moving up/down), the movement of the axle relative to the car body is reported by the position and orientation of the axle reference frame $PX_PY_PZ_P$ with respect to the global reference frame OXYZ.

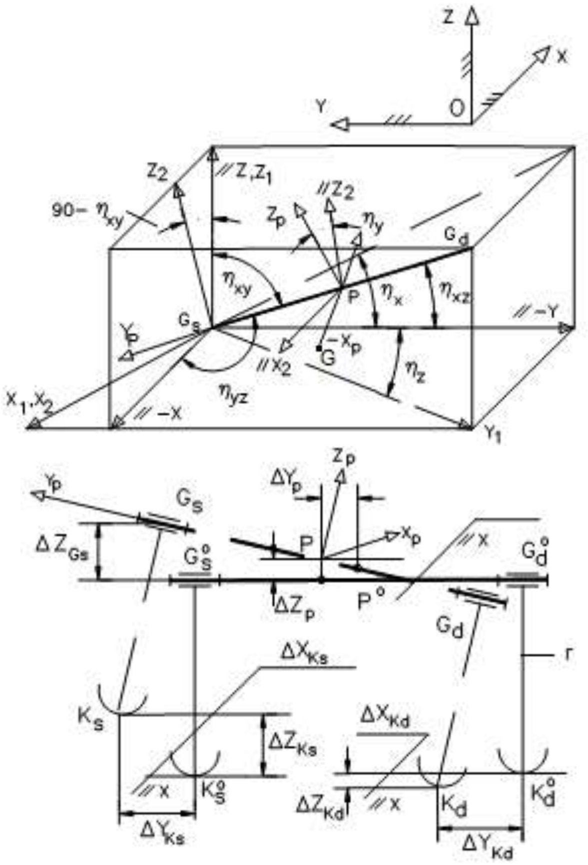

Fig. 1. The spatial position and orientation of the rear axle.

The kinematic functions of the rear axle guiding mechanisms, which are defined by considering three specific points of the axle, namely the centers of the wheels (G_s, G_d) and a point (G) located on the longitudinal axis (X_P) of the axle reference frame, have the following analytical forms (for notations, see Fig. 1):

- displacements of the axle center:

$$\Delta X_P = X_P - X_P^0, \Delta Y_P = Y_P - Y_P^0, \Delta Z_P = Z_P - Z_P^0, \tag{1}$$

- pivoting rotation of the axle:

$$\eta_Z = \operatorname{arctg} \frac{X_{Gd} - X_{Gs}}{Y_{Gd} - Y_{Gs}}, \tag{2}$$

- roll rotation of the axle:

$$\eta_X = \operatorname{arctg} \frac{Z_{Gd} - Z_{Gs}}{Y_{Gd} - Y_{Gs}}, \tag{3}$$

- rotating the axle around its own axis:

$$\eta_Y = \arcsin \frac{\left[Z_G - Z_G^0\right] - \left[Z_P - Z_P^0\right]}{\left|X_{G(P)}\right|}, \tag{4}$$

- spatial position of the rear axle axis, in OXYZ:

$$\eta_{YZ} = \arccos \frac{X_{Gd} - X_{Gs}}{\left|K_s K_d\right|} = \sphericalangle\left(G_s G_d, OX\right),$$

$$\eta_{XZ} = \arccos \frac{Y_{Gd} - Y_{Gs}}{\left|K_s K_d\right|} = \sphericalangle\left(G_s G_d, OY\right), \tag{5}$$

$$\eta_{XY} = \arccos \frac{Z_{Gd} - Z_{Gs}}{\left|K_s K_d\right|} = \sphericalangle\left(G_s G_d, OZ\right),$$

- displacements of the wheel-ground contact points:

$$\begin{aligned} &\Delta X_{Ks,d} = X_{Ks,d} - X_{Ks,d}^0, \Delta Y_{Ks,d} = Y_{Ks,d} - Y_{Ks,d}^0, \\ &\Delta Z_{Ks,d} = Z_{Ks,d} - Z_{Ks,d}^0; \end{aligned} \tag{6}$$

where:

$$\begin{aligned} X_{Ks,d} &= X_{Gs,d} + r \cdot \sin \eta_z \cdot \cos \eta_{XY}, \\ Y_{Ks,d} &= Y_{Gs,d} + r \cdot \cos \eta_z \cdot \cos \eta_{XY}, \\ Z_{Ks,d} &= Z_{Gs,d} - r \cdot \sin \eta_{XY}; \end{aligned} \tag{7}$$

- wheel track variation:

$$\Delta E = \left(Y_{Ks} + \left|Y_{Kd}\right|\right) - \left(Y_{Ks}^0 + \left|Y_{Kd}^0\right|\right), \tag{8}$$

The values with "zero" exponent corresponding to the initial position of the axle guiding mechanism (these values are known input data for the kinematic analysis).

The instantaneous revolute axis of the rear axle (figured in Fig. 2) is collinear with the vector ω and passes through a point R whose position vector $d=PR$, in the axle reference frame, is obtained from the Euler's velocity distribution:

$$\overline{V}_R = \overline{V}_P + \overline{\omega} \times \overline{d}. \tag{9}$$

Multiplying this expression by ω,

$$\begin{aligned} &\overline{\omega} \times (\overline{V}_P - \overline{V}_R) = \overline{\omega} \times (\overline{d} \times \overline{\omega}), \\ &\overline{\omega} \times \overline{V}_P - \overline{\omega} \times \overline{V}_R = \omega^2 \cdot \overline{d} - (\overline{\omega} \cdot \overline{d}) \cdot \overline{\omega}, \end{aligned} \tag{10}$$

since $\overline{\omega}\times\overline{V_R}=\overline{\omega}\times\overline{V_\omega}=0,\ \overline{\omega}\cdot\overline{d}=0(\overline{\omega}\perp\overline{d})$, it will result:

$$\overline{d}=\frac{\overline{\omega}\times\overline{V}_P}{\omega^2}. \tag{11}$$

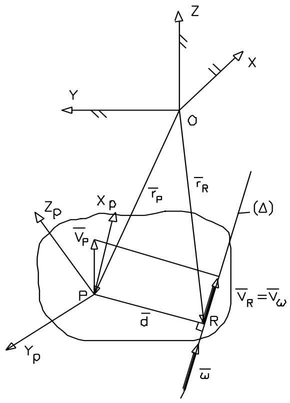

Fig. 2. The instantaneous revolute axis of the rear axle.

The position vector of the point R, relative to the origin O of the global reference frame XYZ, is defined by the equation:

$$\bar{r}_R=\bar{r}_P+\overline{d}=\bar{r}_P+\frac{\overline{\omega}\times\overline{V_P}}{\omega^2}, \tag{12}$$

Thus resulting its global coordinates:

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix}=\begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix}+\frac{1}{\omega^2}\begin{bmatrix} \omega_y\cdot Z_P-\omega_z\cdot Y_P \\ \omega_z\cdot X_P-\omega_x\cdot Z_P \\ \omega_x\cdot Y_P-\omega_y\cdot X_P \end{bmatrix}. \tag{13}$$

The direction of the instantaneous axis is defined by the director cosines:

$$\alpha_R=\arccos\frac{\omega_x}{\omega},\beta_R=\arccos\frac{\omega_y}{\omega},\gamma_R=\arccos\frac{\omega_z}{\omega}, \tag{14}$$

The canonical equations of the axis having the form:

$$\frac{X-X_R}{\omega_x}=\frac{Y-Y_R}{\omega_y}=\frac{Z-Z_R}{\omega_z}. \tag{15}$$

Finally, the global coordinates of the axle oscillation center (I), which is defined as intersection between the instantaneous axis and the vertical-transversal plane that contains the axle axis, are obtained, as follows:

$$\begin{gathered} X_I=X_P^0,Y_I=Y_R+\frac{\omega_y}{\omega_x}\cdot\left[X_P^0-X_R\right], \\ Z_I=Z_R+\frac{\omega_z}{\omega_x}\cdot\left[X_P^0-X_R\right]. \end{gathered} \tag{16}$$

III. The Kinematic Analysis Algorithm

The guiding links of the axle suspension mechanisms are connected to the adjacent parts (axle and car body) by compliant joints (bushings). For the kinematic analysis, where the force generating elements (as the bushings actually are) are not taken into account, the bushings are modeled as spherical joints, thus neglecting the linear (axial and radial) deformations [21]-[24].

A comprehensive structural systematization of the axle guiding mechanisms based on spherical joint assumption was presented in [21], according to the number of degrees of mobility (DOM=1 or DOM=2, by case), which can be determined by the Gruebler's count [25], [26]:

$$DOM = 6\cdot n - \Sigma r \tag{17}$$

where n is the number of moving parts (the axle and the guiding mechanism arms/links), and Σr - the sum of geometric restrictions (introduced by joints).

In order to develop the kinematic analysis algorithm, it will be considered one of the most used types of axle guiding mechanism, which is schematically represented in Fig. 1 in both Chebyshev (a) or Watt (b) configurations [27]. The mechanism, which contains three guiding links/arms of which two (the lower arms - 1_s, 1_d) are double hinged to car body, is codified 2SR-1SS, by considering the types of joints on the adjacent bodies (axle and car body), where S - spherical joint, R - revolute joints (the pair of two spherical joints on car body, $M_{0s'}$ - $M_{0s''}$ or $M_{0d'}$ - $M_{0d''}$, determines in fact o revolute joint). Thus, the mechanism has one active degree of mobility, corresponding to the axle vertical travel.

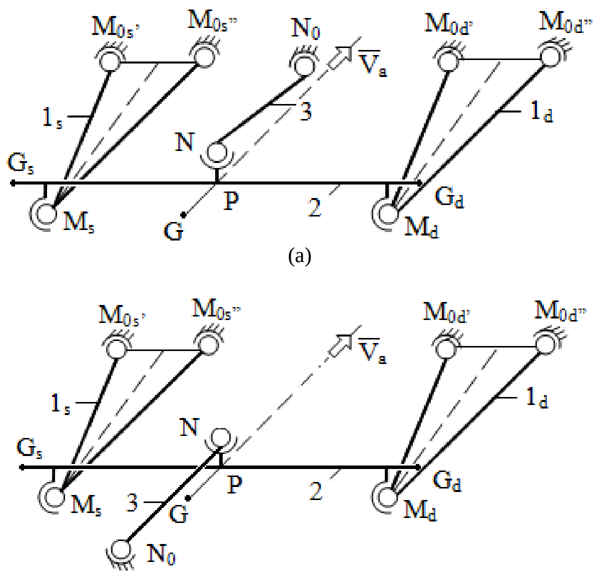

Fig. 3. The axle guiding mechanism of type 2SR-1SS.

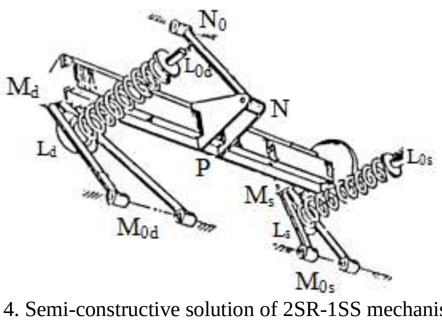

Fig. 4. Semi-constructive solution of 2SR-1SS mechanism.

A semi-constructive solution of the 2SR-1SS axle guiding mechanism in Watt configuration is shown in Fig. 4. Except for the notations that can be found in Fig. 3, the ends of the shock absorbers ($L_{s,d}$ - to the lower guiding arms, and $L_{0s,d}$ - to the car body) are also marked in Fig. 4 (although the damping elements are not considered in the kinematic analysis). The motivation for considering these points will be seen from the kinematic analysis algorithm.

In most of the kinematic analysis methods from literature (whether they are vector, matrix or geometric), the vertical coordinates of the wheels centers (G_s, G_d) or of the theoretical contact points between wheels and ground (K_s, K_d) are considered as independent kinematic parameters. Obviously, this is the case closest to the reality, as it is simulated the vertical travel of the wheel when passing over the bumps (road irregularities). However (and the advantage of this choice will be see hereinafter), for the method presented in this work the length of the shock absorber (i.e. the distance between its ends) will be considered as independent parameter. For the axle guiding mechanism with DOM=1, the kinematic restriction is defined by the function:

$$l_s = |L_s L_{0s}| = f(t), \tag{18}$$

while for the bi-mobile mechanisms (DOM=2) there are two kinematic restrictions:

$$l_s = |L_s L_{0s}| = f_1(t),\ l_d = |L_d L_{0d}| = f_2(t), \tag{19}$$

where (in a generalized form):

$$|LL_0| = \sqrt{(X_L - X_{L0})^2 + (Y_L - Y_{L0})^2 + (Z_L - Z_{L0})^2}. \tag{20}$$

Most of the existing kinematic analysis algorithms of the vehicle wheel/axle guiding mechanisms (no matter what method they rely on) generate nonlinear equation systems, for whose solving numerical methods are needed. In this way, not only the solving procedure is more complicated, but certain computational errors are introduced, and it is also necessary to establish an initial solution as accurately as possible. As will be seen, the method proposed here eliminates these drawbacks (disadvantages), because it allows the transformation of nonlinear systems into some linear ones, which are easy to solve.

The principle of the method is based on determining the global coordinates of a point of interest in the guiding mechanism according to the known coordinates of the other three points. For example, if M_1, M_2 and M_3 are three known locations, the global coordinates of the interest point M, which is located at known distances from the above three points, will be determined from the equation system:

$$\begin{aligned}
&(X_M - X_{M1})^2 + (Y_M - Y_{M1})^2 + (Z_M - Z_{M1})^2 = |MM_1|^2 = d_1^2,\\
&(X_M - X_{M2})^2 + (Y_M - Y_{M2})^2 + (Z_M - Z_{M2})^2 = |MM_2|^2 = d_2^2,\\
&(X_M - X_{M3})^2 + (Y_M - Y_{M3})^2 + (Z_M - Z_{M3})^2 = |MM_3|^2 = d_3^2.
\end{aligned} \tag{21}$$

The solution of the nonlinear system (with three unknowns - X_M, Y_M, Z_M) was obtained by transforming it into a linear system. For beginning, the first equation of the system was subtracted from the other two. Thus, the unknown squares X_M^2, Y_M^2, Z_M^2 are removed, resulting two linear equations in X_M and Y_M as functions of Z_M:

$$\begin{aligned}
&(X_{M1} - X_{M2})\cdot X_M + (Y_{M1} - Y_{M2})\cdot Y_M + (Z_{M1} - Z_{M2})\cdot Z_M =\\
&= \frac{1}{2}\left[\left(d_2^2 - X_{M2}^2 - Y_{M2}^2 - Z_{M2}^2\right) - \left(d_1^2 - X_{M1}^2 - Y_{M1}^2 - Z_{M1}^2\right)\right],\\
&(X_{M1} - X_{M3})\cdot X_M + (Y_{M1} - Y_{M3})\cdot Y_M + (Z_{M1} - Z_{M3})\cdot Z_M =\\
&= \frac{1}{2}\left[\left(d_3^2 - X_{M3}^2 - Y_{M3}^2 - Z_{M3}^2\right) - \left(d_1^2 - X_{M1}^2 - Y_{M1}^2 - Z_{M1}^2\right)\right],
\end{aligned} \tag{22}$$

which can be rewritten in the following form:

$$X_M = A_1 \cdot Z_M + B_1, Y_M = A_2 \cdot ZM + B_2. \tag{23}$$

Then, in the first equation of the nonlinear system (21), the unknowns X_M and Y_M are replaced by the solutions (23), obtaining a quadratic equation in Z_M,

$$A_3 \cdot z_W^2 + 2 \cdot B_3 \cdot z_W + C_3 = 0, \tag{24}$$

which has the well known solution:

$$Z_M = \frac{-B_3 \pm \sqrt{B_3^2 - A_3 \cdot C_3}}{A_3}. \tag{25}$$

In equations (23-25), the coefficients $A_{1,2,3}$, $B_{1,2,3}$ and C_3 depend on the global coordinates of the known reporting points ($M_{1,2,3}$) and the corresponding distances ($d_{1,2,3}$):

$$\begin{aligned}
&A_1 = -\frac{1}{D}\cdot\begin{vmatrix} Z_{M2}-Z_{M1} & Y_{M2}-Y_{M1} \\ Z_{M3}-Z_{M1} & Y_{M3}-Y_{M1}\end{vmatrix}, A_2 = -\frac{1}{D}\cdot\begin{vmatrix} X_{M2}-X_{M1} & Z_{M2}-Z_{M1} \\ X_{M3}-X_{M1} & Z_{M3}-Z_{M1}\end{vmatrix},\\
&B_1 = -\frac{1}{D}\cdot\begin{vmatrix} E_2-E_1 & Y_{M2}-Y_{M1} \\ E_3-E_1 & Y_{M3}-Y_{M1}\end{vmatrix}, B_2 = -\frac{1}{D}\cdot\begin{vmatrix} X_{M2}-X_{M1} & E_2-E_1 \\ X_{M3}-X_{M1} & E_3-E_1\end{vmatrix},\\
&D = -\frac{1}{D}\cdot\begin{vmatrix} X_{M2}-X_{M1} & Y_{M2}-Y_{M1} \\ X_{M3}-X_{M1} & Y_{M3}-Y_{M1}\end{vmatrix}, E_i = \frac{1}{2}\cdot\left(d_i^2 - X_{Mi}^2 - Y_{Mi}^2 - Z_{Mi}^2\right),\\
&A_3 = A_1^2 + A_2^2 + 1, B_3 = A_1\cdot B_1 + A_2\cdot B_2 - X_{M1}\cdot A_1 - Y_{M1}\cdot A_2 - Z_{M1},\\
&C_3 = B_1^2 + B_2^2 - 2\cdot(X_{M1}\cdot B_1 + Y_{M2}\cdot B_2) - 2\cdot E_1.
\end{aligned} \tag{26}$$

In equation (25), the appropriate solution (between the two possible mathematical solutions) is chosen according to the guiding mechanism set-up in the initial/static position of the vehicle. Then, the global coordinates X_M and Y_M are obtained from equation (23). Thus, the position of the interest point M is determined for the entire vertical travel (up - down) of the suspension.

The kinematic model of the rear axle guiding mechanism is defined by the following input data (in correlation with the notations in Fig. 3 and 4):

- the global coordinates of the points of interest on car body, in *OXYZ*: $X_{M0s'}$, $Y_{M0s'}$, $Z_{M0s'}$, $X_{M0s''}$, $Y_{M0s''}$, $Z_{M0s''}$, $X_{M0d'}$, $Y_{M0d'}$, $Z_{M0d'}$, $X_{M0d''}$, $Y_{M0d''}$, $Z_{M0d''}$, X_{N0}, Y_{N0}, Z_{N0}, X_{L0s}, Y_{L0s}, Z_{L0s}, X_{L0d}, Y_{L0d}, Z_{L0d};
- the local coordinates of the points of interest on axle, in $PX_PY_PZ_P$: $X_{Ms(P)}$, $Y_{Ms(P)}$, $Z_{Ms(P)}$, $X_{Md(P)}$, $Y_{Md(P)}$, $Z_{Md(P)}$, $X_{N(P)}$, $Y_{N(P)}$, $Z_{N(P)}$, $X_{Ls(P)}$, $Y_{Ls(P)}$, $Z_{L(P)s}$, $X_{Ld(P)}$, $Y_{Ld(P)}$, $Z_{Ld(P)}$, $X_{Gs(P)}$, $Y_{Gs(P)}$, $Z_{Gs(P)}$, $X_{Gd(P)}$, $Y_{Gd(P)}$, $Z_{Gd(P)}$, $X_{G(P)}$, $Y_{G(P)}$, $Z_{G(P)}$;
- the lengths of the guiding arms: l_{1s}, l_{1d}, l_3;
- the initial position of the axle/wheels: X_P^0, Y_P^0, Z_P^0, $X_{Gs}^0, Y_{Gs}^0, Z_{Gs}^0, X_{Gd}^0, Y_{Gd}^0, Z_{Gd}^0, X_G^0, Y_G^0, Z_G^0$.

The basic output/unknown parameters (which will be determined by the kinematic analysis) are the global coordinates of the three specific points (G_s, G_d, G) that define the spatial position and orientation of the rear axle, in OXYZ: X_{Gs}, Y_{Gs}, Z_{Gs}, X_{Gd}, Y_{Gd}, Z_{Gd}, X_G, Y_G, Z_G.

In the concept of the proposed method, for the axle guiding mechanisms with DOM=1 (as is the guiding mechanism represented in the Fig. 3 and 4), the determination of these coordinates implies the successive repetition of the numerical algorithm defined by eq. (21-25), in the following sequence (steps):

1) determining the global coordinates of the point L_s according to the known points L_{0s}, $M_{0s'}$, $M_{0s''}$; in other words, in eq. (21-25), $M \to L_s$, $M_1 \to L_{0s}$, $M_2 \to M_{0s'}$, $M_3 \to M_{0s''}$;
2) determining the global coordinates of the point M_s according to the known points L_s (its global coordinates were determined in the 1st step), $M_{0s'}$, $M_{0s''}$; in eq. (21-25), $M \to M_s$, $M_1 \to L_s$, $M_2 \to M_{0s'}$, $M_3 \to M_{0s''}$;
3) determining the global coordinates of the point M_d according to the known points M_s (determined in the 2nd step), $M_{0d'}$, $M_{0d''}$; in eq. (21-25), $M \to M_d$, $M_1 \to M_s$, $M_2 \to M_{0d'}$, $M_3 \to M_{0d''}$;
4) determining the global coordinates of the point N according to the known points M_s (determined in the 2nd step), M_d (determined in the 3rd step), N_0; in eq. (21-25), $M \to N$, $M_1 \to M_s$, $M_2 \to M_d$, $M_3 \to N_0$;
5) determining the global coordinates of the point G_s / G_d / G according to the known points M_s (determined in the 2nd step), M_d (determined in the 3rd step), N (determined in the 4th step); in eq. (21-25), $M \to G_s$ / G_d / G, $M_1 \to M_s$, $M_2 \to M_d$, $M_3 \to N$.

As mentioned, it was considered that the independent kinematic parameter is the length of the left shock absorber (L_sL_{0s}), the kinematic restriction function being one defined by eq. (18). Once the global coordinates of the three specific points (G_s, G_d, G) have been determined, the parameters / functions that describe the kinematic behavior of the axle guiding mechanism can be computed by using the equations (1-16). At the same time, the correlation between the length of the shock absorber and the vertical displacement/position of the wheel center (Z_{Gs}), which is more realistic when simulating the wheel passing over road bumps/irregularities, can be established.

For the axle guiding mechanisms with DOM=2, with two kinematic restriction functions, as figured in eq. (19), the computation algorithm is similar to the one presented above, except that the coordinates of the equivalent point M_d are determined depending on the points L_d, $M_{0d'}$, $M_{0d''}$, where L_d was previously obtained based on the known points L_{0d}, $M_{0d'}$, $M_{0d''}$ (practically, the first two steps of the algorithm apply to the right side of the axle guiding mechanism in a similar way to the left side).

IV. Results and Conclusions

The above presented kinematic analysis method has been algorithmized and transposed on computer by using the well-known programming environment Borland Delphi (based on Object Pascal), which is currently developed and maintained by Embarcadero Technologies. The numerical simulations for this paper have been carried out by considering an axle guiding mechanism of type 2SR-1SS in Chebyshev configuration (as shown in Fig. 3.a). Through the kinematic analysis of the mechanism, different results that describe the behavior of the axle guiding mechanism have been obtained. Of these results, Table I shows the displacements of the axle center along the global reference frame axes, as well as the three rotations that determine the spatial orientation of the axle/guiding mechanism. The results are also obtained/presented in graphical forms, as shown in Figures 5 and 6.

TABLE I: Results of the Kinematic Analysis

ΔZ_{Gs}	ΔX_P	ΔY_P	ΔZ_P	η_Z	η_X	η_Y
80.0	-1.70	-3.15	4.10	0.04	-6.14	0.60
60.0	-0.97	-2.26	2.33	0.02	-4.66	0.35
40.0	-0.44	-1.41	1.05	0.01	-3.14	0.16
20.0	-0.11	-0.64	0.27	0.00	-1.59	0.04
0.0	0.00	0.00	0.00	0.00	0.00	0.00
-20.0	-0.12	0.47	0.27	-0.00	1.64	0.04
-40.0	-0.49	0.71	1.11	-0.01	3.32	0.17
-60.0	-1.14	0.65	2.56	-0.02	5.05	0.39
-80.0	-2.11	0.23	4.65	-0.05	6.85	0.71

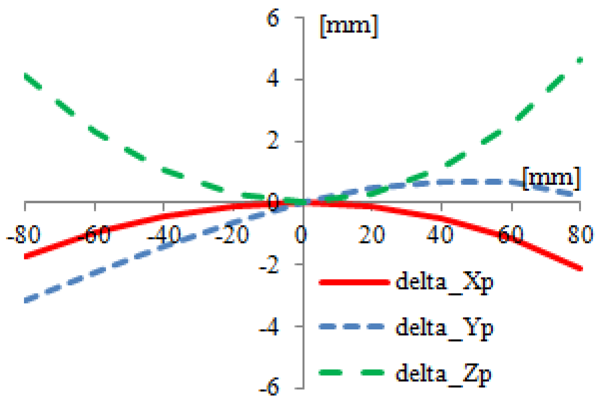

Fig. 5. The variations of the axle center coordinates.

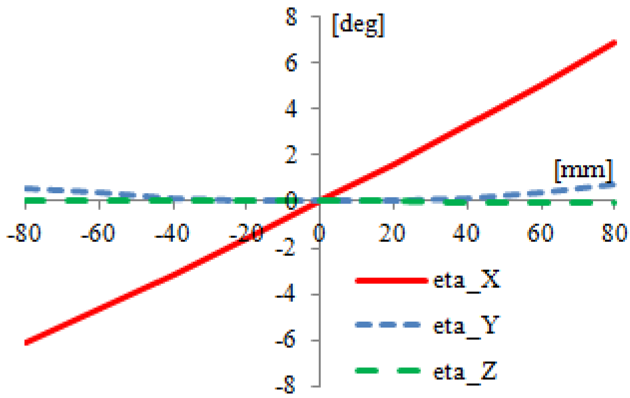

Fig. 6. The variations of the axle orientation angles.

From the ones presented above, the following conclusions can be formulated:

- the proposed method uses specific geometrical parameters of the axle guiding mechanisms, which can be obtained/extracted directly from the execution and assembly drawings;
- the method is very simple to apply, and it does not require the use of numerical methods to solve the equation system

by which the kinematic behavior of the guiding mechanism is defined/modeled;

- as a consequence of the simplicity of the method, the related computation program is easily to be algorithmized and transposed on computer;
- although the length of the shock absorber is used as an independent kinematic parameter, which does not match the real case when the suspension is operated by wheels, the correlation with the vertical displacement of the wheel can however be achieved;
- the method can be applied for most types of axle guiding mechanisms, and it can be adapted for the wheel guiding mechanisms with independent suspension, which are frequently used for passenger cars (the transposition / adaptation of the method for these types of guiding mechanisms is intended to be addressed in a future work).

Conflict of Interest

The author declares no conflict of interest.

Author Contributions

As the sole author, Cătălin Alexandru conducted the whole research and wrote the paper.

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Cătălin Alexandru was born in Cetăţeni, Romania. He received his bachelor in 1991, the PhD in 1998, the PhD supervisor in mechanical engineering at Transilvania University of Brașov, Romania, in 2009.

He is currently a professor at Transilvania University of Brașov, Romania, in the Product Design, Mechatronics and Environment Department. He published over 200 scientific articles, part of them in prestigious ISI-rated journals (such as Mechanism and Machine Theory, Applied Mathematical Modelling, Journal of Renewable and Sustainable Energy, The Scientific World Journal, Proceedings of the Institution of Mechanical Engineers - Part C: Journal of Mechanical Engineering Science, Proceedings of the Institution of Mechanical Engineers - Part D: Journal of Automobile Engineering). He owns 3 patents. He participated at numerous international conferences in Romania and abroad (England, Scotland, Portugal, Spain, France, Germany, Austria, Italy, and others). Research interests in mechanical & mechatronic systems, automotive engineering, renewable energy systems, virtual prototyping.

Prof. Alexandru is a member of International University Association for Science and Technology of Romania (UASTRO), and the International Federation for the Promotion of Mechanism and Machine Science (IFToMM), editor for the Bulletin of the *Transilvania* University of Brasov: Series I - Engineering Sciences, and reviewer for prestigious ISI-rated journals (Advances in Mechanical Engineering, Applied Mathematical Modelling, Applied Mathematics and Computation, IEEE Transactions on Industrial Informatics, IEEE Transactions on Power Electronics, Journal of Mechanical Science and Technology, Journal of Renewable and Sustainable Energy, Mechanical Sciences, Mechanisms and Machine Theory, Robotics and Autonomous Systems, Solar Energy, Renewable Energy, and others).