Abstract—In this paper is presented the research work done to create and optimize a new mechanical structure of industrial robot by finite element method, commanded and controlled in the spirit of Industry 4.0 by a performance controller. This mechanical structure is provided as an end-effectors, with a high speed electric motor (electrical spindle motor), provided with an end-mill tool. This assembly is intended for use in industrial applications for cleaning and machining PVC welded profiles for window frames. The material used in the manufacture is aluminum alloy.

Index Terms—Forward and inverse kinematics, frequency and static finite element analysis.

I. INTRODUCTION

The manufacture of PVC window frames, nowadays, is custom made all over the world, because the clients want a customization of the windows according to size, model, shape and color. Therefore, they must use flexible manufacturing equipment, with industrial robots, which change only the programs, to perform different operations. For cleaning the burrs after thermal welding, the most efficient system is made by the company GED through RC2000, which uses an ABB robot IRB140-6 Kg payload. What we propose, in this work, is the creation of a new robot 6DOF, more efficient than RC2000. The first step in improvement is a 10 Kg payload, which allows us to create a more efficient EE. Then, increasing the performance of the mechanical structure by optimizing it through the finite element method for a better quality of operations

The main operations are: 1) cleaning the molten plastic resulting from the thermal bonding process; 2) making water drainage holes, through PVC profiles; 3) machining the slots in PVC frames for fixing the closing / opening devices of the windows.

The use of a high speed electric motor as an EE, implies the induction of an additional spectrum of vibrations in the mechanical structure. The structure must be able to withstand this vibration that appeared in addition to those created by its own movement with the speeds imposed by the controller [1].

A lot of companies prefer to put the robots on the ceiling, due to space considerations and easier cleaning of the workspace, especially in the wood industry and wood derivatives, where dust creates work problems.

For this reason we designed and optimized this dual-use structure both for placement in a normal position on the floor or suspended on a metallic structure (Fig. 1). Several technical specifications: payload 10 kg, total weight 86 Kg, maximum length of the arm 110 mm, maximum speed 1.2 m / s.

II. THE MATHEMATICAL MODEL OF 5 DOF ROBOT

The mathematical model is required for real-time control for all six axes. The algorithms used are specific for solving the direct and inverse kinematics problems with the 4×4 transformation matrices using Denavit & Hartenemberg parameters [2].

In order to solve the problems of kinematics, we proceeded to the preparation of the kinematic diagram of the robot, presented together with the main elements in Fig. 2 [3].

The Fig. 2 presents the robot modules, the kinematic elements, the joints, the segments dimensions, the axes and
the angles of rotation. To obtain homogeneous 4×4 transformation matrices, we created the consecutive coordinate systems shown in Fig. 3. All the joints are rotation [4].

Inverse kinematics algorithm (IK), can be solved by several methods: inverse transformation, dual matrices, dual quaternion, iterative or geometric approaches [5]. For the equivalent 6R robot, the configuration is shown in Fig. 3. For serial robots, Denavit–Hartenberg (DH) parameters are used to describe the position and orientation of the EE. The transformation matrix relating the joint to joint could be given by [6].

\[ T^{-1}_{ji} = \begin{bmatrix} c_{i} & -s_{i} & 0 & a_{i} \\ s_{i}c_{i}a_{i-1} & c_{i}c_{i-1} & -s_{i}a_{i-1} & d_{i}c_{i-1} \\ s_{i}s_{i}a_{i-1} & c_{i}s_{i-1} & c_{i}a_{i-1} & d_{i}s_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

where \( s_{i} = \sin \theta_{i}, c_{i} = \cos \theta_{i}, c_{i}a_{i} = \cos \alpha_{i} \cdot s_{i}a_{i} = \sin \alpha_{i}. \)

The direct kinematics of the robot can be calculated with the relationship (2).

\[ T_{0}^{n}(\theta_{1})T_{1}^{2}(\theta_{2})T_{2}^{3}(\theta_{3})T_{3}^{4}(\theta_{4})T_{4}^{5}(\theta_{5})T_{5}^{6}(\theta_{6}) = T_{0}^{6} \]

For IK, \( T_{0}^{6} \) is known and described by (3), where \( n, o \) and \( a \) are three unit orientation vectors, and \( p \) is the position vector of

\[ T_{0}^{6} \cdot T_{0}^{n} = \prod_{j=1}^{6} T_{j}^{j+1} = \begin{bmatrix} n_{x} & o_{x} & a_{x} & p_{x} \\ n_{y} & o_{y} & a_{y} & p_{y} \\ n_{z} & o_{z} & a_{z} & p_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

The IK problem is to calculate the joint angles \( \theta_{i} \) through the matrix (2), which will change as:

\[ T_{i}^{j}(\theta_{i})T_{i}^{j+1}(\theta_{j})T_{i}^{j+2}(\theta_{j+1})T_{i}^{j+3}(\theta_{j+2})T_{i}^{j+4}(\theta_{j+3})T_{i}^{j+5}(\theta_{j+4})T_{i}^{j+6}(\theta_{j+5}) = T_{i}^{j} \]

After several calculations on (4), results:

\[ c_{1}p_{x} + s_{1}p_{y} = d_{4}s_{23} + a_{2}c_{2} \]

Once these mathematical relationships are determined, we obtain from (6) two values of \( \theta_{1} \):

\[ \theta_{1} = a \tan 2(p_{y}, p_{x}) \]

\[ \theta_{1} = \begin{cases} \theta_{i} - p_{i}, (\theta_{i} > 0) \\ \theta_{i} + p_{i}, (\theta_{i} \leq 0) \end{cases} \]

From both sides of (5), (6) and (7), we calculate the sum of the squares:

\[ 2a_{1}d_{4}s_{3} = p_{y}^{2} + p_{x}^{2} + p_{z}^{2} - d_{4}^{2} - a_{2}^{2} \]

The J3 angle \( \theta_{3} \) is obtained as:

\[ \theta_{3} = 2a \tan \frac{\sqrt{1-k^{2}}}{k} \]

where \( k = (p_{y} + p_{x} + p_{z} - d_{4} - a_{2})/2a_{1}d_{4} \)

To obtain \( \theta_{2} \) we will modify (2) as:

\[ T_{i}^{3}(\theta_{i})T_{i}^{4}(\theta_{j})T_{i}^{5}(\theta_{j})T_{i}^{6}(\theta_{i}) = T_{i}^{3}(\theta_{i})^{-1}T_{i}^{4}(\theta_{j})^{-1}T_{i}^{5}(\theta_{j})^{-1}T_{i}^{6}(\theta_{i}) \]

By equating (1, 4) and (2, 4) matrix elements of each side in (11), we obtain \( \theta_{2} \) as:

\[ \theta_{2} = a \tan 2(p_{x}a_{c_{3}} + k_{2}(a_{s_{3}} + d_{4}), k_{1} + k_{2}a_{c_{3}}) - \theta_{3} \]

where \( k_{1} = -p_{z}d_{4} - p_{z}s_{3} \) and \( k_{2} = p_{z}a_{3} + p_{z}a_{3} \)

By equating (1, 3) and (3, 3) matrix elements of each sides in (11), we obtain

\[ \theta_{3} = a \tan 2(a_{1}s_{1} + a_{1}c_{1}a_{s_{23}} + a_{1}c_{23}s_{1} + a_{1}c_{23}c_{1}) \]

Calculating through a similar procedure we obtain the values for \( \theta_{3} \) and \( \theta_{6} \).

**III. STATIC ANALYSIS WITH FINITE ELEMENTS OF THE ROBOT’S MECHANICAL STRUCTURE**

After performing the kinematic calculations and establishing the distances between the joints, we proceeded to the 3D drawing of each structural element, in order to achieve the 6 desired DOFs, to be functional within the

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**Fig. 3. The kinematic scheme.**

**Fig. 4. Mechanical properties of material.**
allowed kinematic limits. We used the finite element method to optimize the mechanical structure. This optimization was done gradually, eliminating every critical point. The base of the robot was fixed and at the EE the driving forces were established, in the value of 100N on each of the 3 axes in the directions established in Fig. 5.

Fig. 4 shows the type of material used in the construction of the robot, as well as its mechanical properties. [7].

These forces generate tensions in the structure, leading to uneven deformations in the various nodes.

**TABLE I: STRUCTURE COMPUTATION**

<table>
<thead>
<tr>
<th>Components</th>
<th>Number of nodes</th>
<th>Number of elements</th>
<th>Number of DOF</th>
<th>Number of linear tetrahedron</th>
<th>Number of coefficients</th>
<th>Number of constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fr(N)</td>
<td>-1.000e-002</td>
<td>1.000e-002</td>
<td>2.137e+010</td>
<td>2.373e+010</td>
<td>2.021796</td>
<td>2.250</td>
</tr>
<tr>
<td>Fz(N)</td>
<td>1.000e-002</td>
<td>1.000e-002</td>
<td>2.317e+009</td>
<td>2.337e+010</td>
<td>2.207e+011</td>
<td></td>
</tr>
<tr>
<td>Mz (Nmm)</td>
<td>-1.892e+001</td>
<td>1.806e+000</td>
<td>2.450e+009</td>
<td>2.420e+010</td>
<td>2.792e+011</td>
<td></td>
</tr>
<tr>
<td>My (Nmm)</td>
<td>-7.313e+001</td>
<td>7.318e+001</td>
<td>4.892e+010</td>
<td>7.390e+011</td>
<td>2.2312e+010</td>
<td></td>
</tr>
</tbody>
</table>

The octree tetrahedron mesh has the element type linear shape, with maximum size at 1 inch. The connections between elements are type contact and rigid. The structure computation is presents in Table I.

**TABLE II: TORKES AND FORCES EQUILIBRIUM**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Tz (%)</th>
<th>Tx (%)</th>
<th>Ty (%)</th>
<th>Rx (%)</th>
<th>Ry (%)</th>
<th>Rz (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.762e+000</td>
<td>0.65</td>
<td>0.18</td>
<td>0.22</td>
<td>0.10</td>
<td>0.90</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>7.699e+000</td>
<td>14.96</td>
<td>0.02</td>
<td>0.62</td>
<td>0.18</td>
<td>0.92</td>
<td>0.18</td>
</tr>
<tr>
<td>3</td>
<td>8.339e+000</td>
<td>0.05</td>
<td>10.77</td>
<td>0.11</td>
<td>0.70</td>
<td>0.44</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>1.631e+001</td>
<td>0.57</td>
<td>0.27</td>
<td>0.17</td>
<td>0.10</td>
<td>0.37</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>2.794e+001</td>
<td>0.08</td>
<td>15.71</td>
<td>0.17</td>
<td>0.25</td>
<td>0.78</td>
<td>0.18</td>
</tr>
<tr>
<td>6</td>
<td>3.881e+001</td>
<td>0.09</td>
<td>10.38</td>
<td>0.10</td>
<td>0.59</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>4.992e+001</td>
<td>0.42</td>
<td>0.04</td>
<td>1.11</td>
<td>0.16</td>
<td>0.46</td>
<td>0.01</td>
</tr>
<tr>
<td>8</td>
<td>8.793e+001</td>
<td>0.74</td>
<td>0.14</td>
<td>0.25</td>
<td>0.10</td>
<td>0.67</td>
<td>0.17</td>
</tr>
<tr>
<td>9</td>
<td>1.087e+002</td>
<td>0.82</td>
<td>0.17</td>
<td>0.65</td>
<td>0.10</td>
<td>0.67</td>
<td>0.17</td>
</tr>
<tr>
<td>10</td>
<td>1.685e+002</td>
<td>0.97</td>
<td>0.04</td>
<td>0.75</td>
<td>0.07</td>
<td>0.31</td>
<td>0.23</td>
</tr>
</tbody>
</table>

The Von Mises stress is calculated in all nodes of the network, and the strongest are highlighted in the color code shown in Fig. 6. Red color represents the bigger deformation. The highest value is found on the robot arm near the J2 and J3 joints.

**IV. FREQUENCY ANALYSIS OF STRUCTURE**

Frequency analysis is mandatory because we have two vibration generating systems: one made by the electric motor that will operate slots in PVC profiles and the second is the robot's own mechanical system. The superposition of the two sources of vibrations can lead to obtaining resonant frequencies, which can destroy the mechanical structure. After performing the frequency analysis, we discovered 10 resonance frequencies, shown in Fig. 8, which also shows the modal participation.

**Fig. 6. Amplified deformation of the structure to highlight critical points.**

**Fig. 7. Unaccepted maximum translational displacements.**

One of the most dangerous resonance frequencies is 568,657 HZ, which can cause a stress in the structure up to 3.75 x e ^ 10 N / m ^ 2. Another destructive resonance frequency is 48 HZ, which can cause a theoretical...
deformation in the structure up to 12.3 inch. (See figure 9) all measures had to be taken to prevent these frequencies from occurring, because their overlap will lead to serious damage to the mechanical structure, and errors in the positioning mechanisms, which will make the entire system impossible to control [10].

Fig. 9. Danger stress and deformation.

Von Mises stress [nominal values]:

\[ 1.31 \times 10^{-6} \text{ N}_m^2 \]

On Boundary

Fig. 10. Allowed structure deformation.

Translational displacement vector:

\[ \begin{align*}
0.00429 \\
0.00535 \\
0.00225 \\
0.00147 \\
0.00117 \\
0.00102 \\
0.00085 \\
0.000634 \\
0.0004 \\
0.0003 \\
0.000284 \\
0.00025 \\
0.000217 \\
0.000184 \\
0.000133 \\
0.000117 \\
0.035e+05 \\
5.16e+05 \\
0.34e+05 \\
1.67e+05 \\
0.357
\end{align*} \]

On Boundary

Fig. 11. Translational displacements of the structure are within accepted limits.

V. ANALYSIS OF RESULTS AND PROPOSING SOLUTIONS FOR IMPROVEMENT

As can be seen in the simulation of Fig. 6-7, deformations of the structure are very high. If stress values are within the breaking limit, 0.044 inch (1.1 mm) displacement values can’t be accepted (Fig. 7). This is the reason why we have proceeded to successively improve the rigidity of the mechanical structure [9].

Several successive modifications were made at the places shown in Fig. 6, in particular on the following elements: 1 - shoulder body element; 2 - the arm attachment of the shoulder body; 3 - the shape and size of the lower part of the arm, where the structure is most affected by the forces applied; 4 - the shape and thickness of the arm; 5 - Rotary rod of 4-th DOF; 6 - the forearm fasteners; 7 - the shape and thickness of the forearm.

Fig. 12. Errors of mathematical model tolerated.

This is highlighted in red and has the value of \( 1.31 \times 10^6 \text{ N}_m^2 \). This value is below the breaking strength of the material, which will ensure durability over time. The maximum deformation of the metal structure is found on the mechanical hand module and the value is 0.00429 inches (0.1 mm). By successive calculations, we reached these values of deformation and tension in the mechanical structure. These values confirm a solid structure, capable of supporting the additional demands created by the vibrations of the cutting spindle at the end of the last degree of mobility.

This deformation is shown in Fig. 11, with the red color, and the deformation values are shown in the color table on the left side of the figure. This positioning accuracy allowed by the mechanical structure is in line with the electronic positioning system on the trajectory, consisting of AC servomotors and absolute optical encoder, with 2500 pulses per revolution.

Fig. 12 shows the places where the largest errors of the mathematical model are found. Estimated local error (relative strain energy variation [J]) images are used to visualize computation error maps, which represent scalar field quantities defined as the distribution of energy error norm estimates for a given computation. The program evaluates the validity of the computation and provides a
global statement about this validity. It also displays a predicted energy error norm map which gives qualitative insight about the error distribution on the part, visualized in fringe pattern mode, along with a color palette. This map provides qualitative information about the way in which estimated computation errors are relatively distributed on the part. These errors are very small, as shown in the table in the figure, which leads us to the conclusion that the choice of the mathematical modeling method through the analysis of the finite elements has led us to very good results [11].

VI. CONCLUSION AND FUTURE WORK

In this scientific work are presented the ways of designing a new structure of industrial robots with 6 degrees of freedom by mathematical modeling and finite element analysis. The mathematical kinematic algorithms IK and FD presented in the matrix form are easy to implement in the robot controller, who allows a better correlation of the movements in real time, allows us a better determination of the position, speeds and accelerations in the robot's joints and a better positioning accuracy of the EE. Due to the simplicity of the movement algorithms, we can process more information of movement and the diagnosis in real time. We can determine the value of current, forces and moments in the joints. Their knowledge helps us to optimize the 3D trajectory movement and to protect the robot system components.

The finite element analysis allowed us to optimize the component parts and thus ensured a maximum deformation of 0.00429 inches, which represents a very good accuracy for the payload of 10 Kg. What remains to be done is the practical realization of the structure, the implementation of mathematical algorithms in the controller and the practical laboratory tests. The next step is to reduce trajectory positioning errors for speeds and high accelerations.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Niculae Mihai wrote the paper and designed and optimized the mechanical structure of the robot. Paul Badescu has developed the command and control systems for robot movement and performing motion tests on space trajectories. All authors had approved the final version.

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REFERENCES


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