Application of Full Factorial Design for Optimization of Feed Rate of Stationary Hook Hopper

Astha Kukreja, Pankaj Chopra, Akshay Aggarwal, and Pradeep Khanna

***Abstract*—the work aims at the optimization of the output feed rate of a Stationary Hook Hopper Feeder so that the best possible set of parameters affecting it can be selected to get the desired output. For this purpose the effect of various parameters on the feeder output is studied. To facilitate the study and detailed analysis, a statistical model is constructed which is used to predict and optimize the performance of the system. Efficient feed rate optimization determines the input variable settings to adjust the feed rate of the feeder according to the consumption of the parts in the next phase of production. The Stationary Hook Hopper Feeder, whose performance is to be studied, consists of a rotating circular plate and a guiding hook fixed at the centre and running up to the periphery of the plate. As the plate rotates, the parts follow the trajectory of the hook, orient themselves and then eventually are delivered through the delivery chute, tangentially to the plate. The factors influencing the feeder's performance include the speed of rotation of the disc, the population of the parts in the hopper and the size of parts to be fed. A series of experiments is performed on the three process parameters to investigate their effect on the feed rate. To study the interaction among the factors a full 23 factorial experiment approach has been adopted using the two basic principles of experimental design-replication and randomization. The process model was formulated based on Analysis of variance (ANOVA) using Minitab® statistical package. The outcome is represented graphically and in the form of empirical model which defines the performance characteristics of the Stationary Hook Hopper Feeder.**

***Index Terms*—ANOVA, design of experiments, full factorial design, stationary hook hopper feeder.**

I. Introduction

In automatic assembly lines, parts feeders are required for transferring various parts from one phase of production to the next. Parts feeders in effect convert the random mass of parts into a discrete consistent line. The parts are sequentially fed from the delivery chute at a required feed rate and desired orientation. This becomes critical in the assembly unit where consistency in the delivery of finished or unfinished parts is of upmost priority. So the performance of the feeder depends on the feed rate which should not fall below the consumption of parts by the machine or process subsequently attached to the parts feeder [3].

Parts feeders provide a cost effective alternative to manual labor, saving manufacturer's valuable time and labor costs, nonetheless bringing consistency in quality. Also where material handling by the worker could be harmful like in chemical or pharmaceutical industries, automatic feeding by such parts feeders become a necessity. Parts feeders vary in configuration and thus it becomes important to understand the performance of the feeder under the prescribed conditions to optimize the feeding mechanism.

A. Stationary Hook Hopper Feeder

The parts feeder considered for analysis is Stationary Hook Hopper Feeder; which consists of a rotating circular plate and a hook fixed at the centre, running up to the periphery of the plate, as shown in the Fig 1. The plate is provided with an internal taper to ensure that parts travel the path defined by the hook and a ledge is designed to facilitate feeding of the parts after they lose contact with the hook. As the plate rotates, the parts follow the trajectory of the hook, orient themselves and then eventually are delivered through the delivery chute. The delivery tube is tangential to the plate so that there is minimum possibility of jamming of the parts [3]. The basic rotational motion of the circular plate is provided by a DC motor connected to the plate through a belt-pullet system. An advantage of this type of feeder is its gentle feeding action, which makes it suitable for feeding delicate parts at low speeds. This feature is attributed to the curvature of the hook which is specially designed as per the parametric equation (1) so as to maintain constant speed of the parts moving along it.

$$r(\cos\theta - \mu_r \sin\theta) = K \qquad (1)$$

where r is the distance from the point on the hook to the center of the hopper, μ_r is the coefficient of dynamic friction between the part and the hook. The specimen parts under consideration for the analysis of the feed rate are metallic nuts.

B. Need for a New Approach

A series of experiments was performed [1] on the Stationary Hook Hopper Feeder to identify the effect of various parameters that influence the output response. The one factor at a time (OFAT) experiments gave satisfactory results when only one factor is changed keeping others constant. But the results obtained could not be used successfully to set the parameters for feed rate optimization.

The reason for this can be attributed to the fact that the OFAT approach fails to depict the effect caused by the interaction of various factors on the feeder performance [2]. Interaction is defined as the failure of one factor to produce the same effect on the response at different levels of another

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factor. Therefore the need for a better statistical model was felt for optimization of feed rate and its precise prediction. In the present work this aim has been accomplished using factorial design of the experiments to be conducted. Such statistical method provides an efficient method to analyze the effect of interaction on the output response of the process that too in limited number of experimental runs.

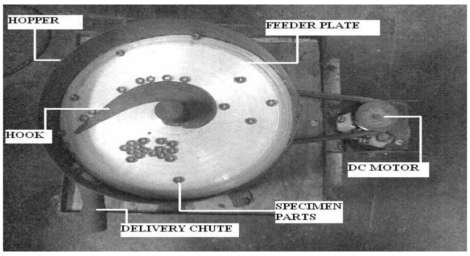


Fig 1. Stationary Hook Hopper Feeder

The reason for this can be attributed to the fact that the OFAT approach fails to depict the effect caused by the interaction of various factors on the feeder performance [2]. Interaction is defined as the failure of one factor to produce the same effect on the response at different levels of another factor. Therefore the need for a better statistical model was felt for optimization of feed rate and its precise prediction. In the present work this aim has been accomplished using factorial design of the experiments to be conducted. Such statistical method provides an efficient method to analyze the effect of interaction on the output response of the process that too in limited number of experimental runs.

C. Literature Survey – ANOVA

Analysis of Variance (ANOVA) is a statistical technique for modeling the relationship between a response variable and independent variables (factors). Each factor consists of two or more levels. The sequence of operations performed for ANOVA is as follows. After identifying factors of interest and a response variable along with their levels the order is randomized in which each set of conditions is run to obtain data. Conclusions are then drawn and results are organized.

Degree if Freedom is an important number for analyses of results and in statistical analysis, DOF is an indication of the amount of information contained in a data set. The number of degrees of freedom for any interaction is always equal to the product of the number of degrees of freedom of the main effects involved in the interaction.

DOF of a factor = number of level of factor – 1

II. Experimental Design

A. Selection of Factors

The first task before conducting the experiments is selection of potential parameters to be varied. It is difficult to modify the feeder disk and stationary hook so as to get different mechanical parameters like coefficient of friction, speed of the part along the hook, radius of the disc, etc. because of mechanical constraints. These factors, when varied, might exert some effect on the response but for purposes of the present experimentation these factors were held constant at a specific level. Uncontrollable variables like change in friction due to environmental conditions weight of parts left in the hopper, etc. are difficult to control during an experiment and thus are responsible for variability in the feeder performance if any. We extended our factorial experiment design to three design factors namely:

1) Part Size (A)
2) Part population of the parts to be fed (B)
3) Rotational velocity of the rotating disk (C)

B. 2^3 Factorial Approach

TABLE I. Process Parameters and their Levels

Factor	RPM, A	Part Pop., B	Part Size, C
Lower Limit	10	20	10 mm (M10)
Higher Limit	25	50	12 mm (M12)

TABLE II. Design Matrix and readings for 4 replicates

RUN ORDER	CODED FACTORS			FEED RATE			
	A	*B*	*C*	*R1*	*R2*	*R3*	*R4*
1	-1	-1	-1	35.156	36.383	36.355	32.571
2	-1	-1	+1	40.000	45.000	35.000	31.500
3	-1	+1	-1	93.034	122.667	106.207	119.800
4	-1	+1	+1	65.132	75.832	67.385	66.880
5	+1	-1	-1	227.077	245.660	249.577	230.583
6	+1	-1	+1	186.206	184.375	190.430	178.411
7	+1	+1	-1	401.176	424.091	439.535	473.182
8	+1	+1	+1	83.684	75.333	76.780	79.560

The purpose of the experimentation is to establish a statistical model to predict the output feed rate and its successful optimization using 2k factorial design. The three factors chosen for experiment are the controllable variables that have a key role to play in the process characterization. These design factors have a certain range within which they can be varied for the useful functioning of the system. The ranges of individual factors were chosen on the basis of pilot runs and process knowledge based on practical experience [1]. The upper and lower bounds of the range of each factor, which were coded as +1 and -1, are given in the Table 1. Since we have three factors to be considered, the experiment design is called a 23 full factorial design which required eight test runs, each with combinations of the three factors across two levels of each. According to the general statistical approach for experimental design four replicates were obtained to get a reliable and precise estimate of the effects. Therefore, thirty-two observations were taken in all to employ full factorial design as shown in Table 2. Throughout the experiment it was assumed that: the factor is fixed, the design was completely randomized and the usual normality assumptions of the data were satisfied.

III. Analysis

Minitab® is an excellent statistical package that assists in data analysis. Various plots like Cube plot, Interaction plot and Main Effects plot are obtained to examine effects of factors on output. Pareto plot and Normal plot of the standardized effects are obtained to compare the significance of each effect. Analysis of Variance (ANOVA) table is constructed for the significant factors affecting the output response.

A. Effect of Factors on Feed Rate

The cube plot for feed rate (Fig 2) shows the average feed rates at critical points. The critical points are those points where all the parameters have limiting values. We gather that a minimum feed rate of 35.116 parts per minute can be achieved for which we need to select minimum part population and minimum rpm for the small part size. The maximum achievable feed rate is 434.496 parts per minute at maximum rpm, maximum part population, and with small parts.

Fig 3 depicts a plot of average output for each level of the factor with the level of the second factor held constant. These plots called interaction plots are used to interpret significant interactions between the process parameters. Interaction is present when the response at a factor level depends upon the levels of other factors. Since they can magnify or diminish the main effects of the parameters, evaluating interactions is extremely important.

In the Interaction plot for feed rate, the lines in RPM versus part population plot are approximately parallel, indicating a lack of interaction between the two factors. It suggests that mutual interaction between RPM and part population has negligible effect on the feed rate. In the second plot, there exists antagonistic interaction between the rotational speed and part size as the lines of the graph cross each other. Similarly, the third plot depicts synergic interaction between part population and part size. Although the lines on the plot do not cross each other but lack of parallelism of the lines exhibit significant interaction. The greater the departure of the lines from the parallel state, the higher the degree of interaction.

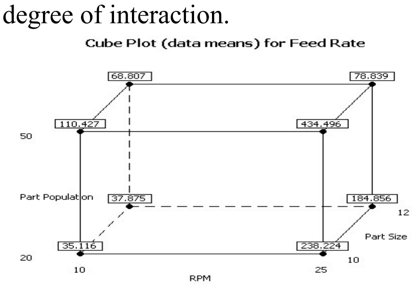


Fig 2. Cube Plot

It is also important to know how the system behaves when variation is brought upon by varying only one parameter keeping the others constant. This gives the dependence of the system over the varied parameter. A main effect occurs when the mean response changes across the levels of a factor. The main effect graphs (Fig 4) can be used to compare the relative strenght of the effects across factors. It can be asserted from the graph that the rotational speed and part population have positive effects while the part size has negative effect on the output feed rate. It can also be concluded that RPM has profound effect on the output followed by part size and part population.

B. Signifance of Various Factors

The analysis of Table 3 shows that all the effects except Rpm* Part Population are highly significant. All those effects have very small P-values. Since the P-value of the effect Rpm* Part Population is greater than the chosen value of α=0.05 for the analysis, it has a negligible effect on the output feed rate.

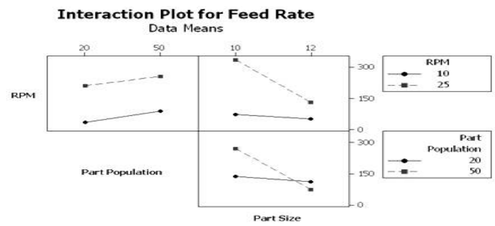


Fig 3. Interaction Plot

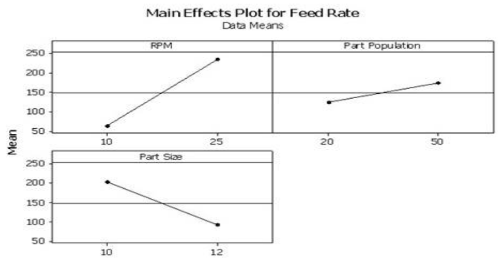
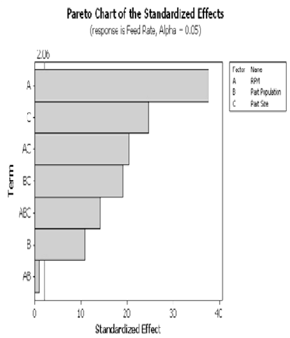


Fig 4. Main Effect Plot



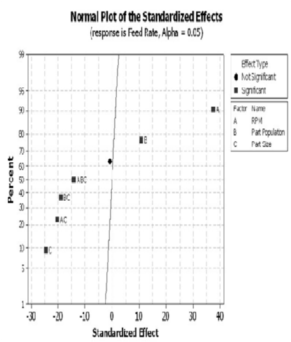


Fig 5. Pareto Chart of the Standardized effects

Fig 6: Normal Plot of the Standardized effects

The Pareto Chart of the Effects (Fig 5) and the Normal Plot of Standardized Effects (Fig 6) also assist to determine the magnitude and the importance of an effect. Pareto chart displays the absolute value of the effects and draws a reference line on the chart at t-value limit, where t is the (1 - α/2) quantile of a t-distribution with degrees of freedom equal to the degrees of freedom (24) for the error term. Any effect that extends within this reference line is statistically insignificant.

The charts indicate that the effect of AB i.e. RPM x Part Population is statistically insignificant. The effect of A has the highest standardized effect on the feed rate followed by C, AC, BC, ABC and B. Hence, the term AB should not be considered for the empirical relation. The insignificance of factor AB can also be reasserted from the normal plot, in which, the points that do not fall near the fitted line are important. The factors having negligible effect on the output response tend to be smaller and are centered around zero.

C. Significance of the Model

We have obtained two empirical relations based on five parameters namely A, B, C, BC, and AC. The first relation incorporates coded values of all the factors ranging from -1 to +1 and in the other equation, the corresponding actual values of the factors are to be inserted to solve for the unknown variable. The latter is more intuitive but it has a limitation that it is not as accurate as coded model because of round-off errors. Are statistically significant

This equation can be used to find out the values of the three factors to be set in order to achieve desired output feed rate. In an exemplary situation, a feed rate of 60 ± 2 parts per minute for the parts of size 12 is to be targeted and corresponding optimum values of the remaining two factors needs to be found. The optimization procedure picks several starting points from which search for the optimal factor settings is begun. There are two types of solutions for the search: is equal to the probability level (p). The null hypothesis can be rejected for values of the test statistic that are larger than this critical value. The F_o value of the model is 448.8447, which is very large as compared to the critical value of 2.423 and the model terms with p-values less than α=0.0500.

D. Development of Reduced Model

The parameter AB has been stated to be statistically insignificant i.e. the product of rotational speed and part population has negligible effect on the output feed rate and a reduced model was created wherein the factors AB and ABC are ignored. The removal of 3-way interaction factor ABC is essential due to the hierarchical nature of the model.

The final outcome as given by Minitab software after incorporating these changes is given below.

The empirical relation in terms of coded units:

Feed Rate =
148.58
+ 85.52 * RPM
+ 24.56 * Part Population
– 55.99 * Part Size
– 46.27 * RPM * Part Size
– 43.33 * Part Population * Part Size

The empirical relation in terms of uncoded units:

Feed Rate =
– 1792.28
+ 79.2667 * RPM
+ 33.4152 * Part Population
+ 153.090 * Part Size
– 6.16941 * RPM * Part Size
– 2.88889 * Part Population * Part Size

For a particular F-distribution and a particular probability level (α), the critical value of the F-distribution is the point along the x-axis above which the total area under the curve

Local solution: For each starting point, there is a local solution. These solutions are the combination of factor settings found beginning from a particular starting point.

Global solution: There is only one global solution, which is the best of all the local solutions. The global solution is the "best" combination of factor settings for achieving the desired responses.

For each of the local solution, predicted value of the response is calculated. The desirability of each of the predicted values asses its closeness to the target value on a scale of 0 to 1. A reduced gradient algorithm with multiple starting points is employed to maximize the desirability in order to determine the numerical optimal or the global solution (Table 6).

The test runs show that a reliable and useful statistical model based on ANOVA has been thus developed. The following information about the model is also obtained:

S = 37.9014 PRESS = 56576.5
R-Sq = 92.81% R-Sq(pred) = 89.11%
R-Sq(adj) = 91.43%

R square measures the proportion of total variability explained by the model. The value of R-square is 92.81%. A potential problem with this statistic is that it always increases as factors are added to the model even if these factors are not significant. So the adjusted R-squared is calculated as 91.43%, which is a statistic that is adjusted for the "size" of the model. From PRESS (Prediction Error Sum of Squares) the prediction R-squared statistic is computed to be 89.11%. This indicates that the model is expected to explain about 89% of the variability in new data and is in reasonable agreement with the value of R-sq (adjusted).

IV. CONCLUSION

A reliable statistical model based on full factorial experiment design has been developed which can be used for the optimization of output feed rate of the stationary hook hopper feeder. The model is significant to explain 89% of variability in new data. Such a model not only assists to estimate the magnitude and direction of the effects of change in factors but also predicts the effects of their mutual interactions.

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TABLE III. ESTIMATED EFFECTS AND COEFFICIENTS FOR FEED RATE (CODED UNITS)

Term	Effect	Coef	SE Coef	T	P
Constant		148.58	2.272	65.41	0.000
RPM	171.05	85.52	2.272	37.65	0.000
Part Population	49.12	24.56	2.272	10.81	0.000
Part Size	-111.97	-55.99	2.272	-24.65	0.000
RPM*Part Population	-4.00	-2.00	2.272	-0.88	0.388
RPM*Part Size	-92.54	-46.27	2.272	-20.37	0.000
Part Population*Part Size	-86.67	-43.33	2.272	-19.08	0.000
RPM*Part Population*Part Size	-64.48	-32.24	2.272	-14.19	0.000

TABLE IV. ESTIMATED EFFECTS AND COEFFICIENTS FOR FEED RATE (CODED UNITS)

Term	Effect	Coef	SE Coef	T	P
Constant		148.58	6.700	22.18	0.000
RPM	171.05	85.52	6.700	12.76	0.000
Part Population	49.12	24.56	6.700	3.67	0.001
Part Size	-111.97	-55.99	6.700	-8.36	0.000
RPM*Part Size	-92.54	-46.27	6.700	-6.91	0.000
Part Population*Part Size	-86.67	-43.33	6.700	-6.47	0.000

TABLE V. ANALYSIS OF VARIANCE FOR FEED RATE (CODED UNITS)

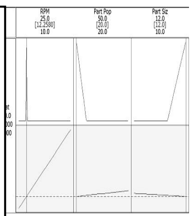
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	353665	353665	117888	82.07	0.000
2-Way Interactions	2	128600	128600	64300	44.76	0.000
Residual Error	26	37349	37349	1437		
Lack of Fit	2	33386	33386	16693	101.10	0.000
Pure Error	24	3963	3963	165		
Total	31	519614				

TABLE VI. ANALYSIS OF VARIANCE FOR FEED RATE (CODED UNITS)

Global Solution

RPM = 12.2580
Part Populatn = 20
Part Size = 12

Predicted Responses
Feed Rate = 60, Desirability = 1.000000



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