Lambda-Statistical Limit Inferior and Limit Superior for Sequences of Fuzzy Numbers

F. Berna Benli

Abstract— **Aytar, Mammadov and Pehlivan have introduced the concepts of statistical limit inferior and limit superior for sequences of fuzzy numbers. In this paper, we define lamda-statistical limit inferior and limit superior for sequences of fuzzy numbers. Also we will discuss the relations among lambda-statistical limit inferior and limit superior for sequences of fuzzy numbers**

Index Terms—**Fuzzy number, lambda-statistical limit inferior, lambda-statistical limit superior.**

I. Introduction

The idea of statistical convergence of a sequence was introduced by Steinhaus [1] and Fast [2]. Statistical convergence was generalized by Buck [3].

Matloka [4] was defined bounded and convergent sequences of fuzzy numbers, also showed that every convergent sequence is bounded. Later, Nanda [5] studied the spaces of bounded and convergent sequences of fuzzy numbers and shoved that these are complete metric spaces. Then, Nuray and Savas [6] have extended and also discussed the concepts of statistically convergent and statistically Cauchy sequences of fuzzy numbers. Lambda-statistically Cauchy sequences of fuzzy numbers have been introduced by Tuncer and Benli [7]. Also, Tuncer and Benli [8] have defined lambda-statistical limit and lambda-statistical cluster points of a sequence of fuzzy numbers and the concepts of lambda-statistically monotonic and lambda-statistically bounded sequences of fuzzy numbers have been given in [9].

Recently, sup and inf notions have been given only for bounded sets of fuzzy numbers ([10] and [11]). Then in [12] Aytar, Mamedov and Pehlivan have introduced the concepts of statistical limit inferior and limit superior for statistically bounded sequences of fuzzy numbers.

In this paper we will define lamda-statistical limit inferior and limit superior for sequences of fuzzy numbers, then we will prove that some results established for sequences of real numbers [13] are also valid for sequences of fuzzy numbers.

Shortly, we recall some of the basic notations in the theory of fuzzy numbers.

Let D denote the set of all closed bounded intervals $A=[\underline{A},\overline{A}]$ on the real line R. For $A,B\in D$ define

$$A\le B \Leftrightarrow \underline{A}\le \underline{B} \quad and \quad \overline{A}\le \overline{B},$$

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$$d(A,B)=\max\left(\left|\underline{A}-\underline{B}\right|,\left|\overline{A}-\overline{B}\right|\right)$$

It is easy to see that d defines a metric on D and (d,D) is a complete metric space. Also $\le$ is a partial order in D.

A fuzzy number is a fuzzy subset of real line R which is bounded, convex and normal. Let $L(R)$ denote the set of all fuzzy numbers which are upper semicontinuous and have compact support. In other words, if $X\in L(R)$, then for any $\alpha\in[0,1]$, X^α is compact, where

$$X^\alpha=\begin{cases} t: X(t)\ge\alpha & if\ \alpha\in(0,1],\\ t: X(t)>0 & if\ \alpha=0.\end{cases}$$

Define a map $\overline{d}: L(R)\times L(R)\to R$ by

$$\overline{d}(X,Y) = \sup_{\alpha\in[0,1]} d\ (X^\alpha,\ Y^\alpha).$$

It is known that ([14]) $L(R)$ is a complete metric space with the metric $\overline{d}$.

For $X, Y\in L(R)$ define $X\le Y$ if and only if $X^\alpha\le Y^\alpha$ for any $\alpha\in[0,1]$.

The fuzzy numbers X and Y are said to be incomparable if neither $X\le Y$ nor $Y\le X$. We use the notation $X \not\sim Y$ in this case.

For every $X, Y, Z\in L(R)$, we say that Z is the sum of X and Y, written $Z=X+Y$, if for every $\alpha\in[0,1]$, $\underline{Z}^\alpha=\underline{X}^\alpha+\underline{Y}^\alpha$ and $\overline{Z}^\alpha=\overline{X}^\alpha+\overline{Y}^\alpha$.

Consider a fuzzy number $\mu\in L(R)$. Let $\mu^\alpha=[\underline{\mu}^\alpha,\overline{\mu}^\alpha]$, $\alpha\in[0,1]$, be α - level sets of μ. Given a positive number $a>0$, we define the fuzzy numbers $\mu+a_1$ and $\mu-a_1$ as follows [15]

$$(\mu+a_1)^\alpha=[\underline{\mu}^\alpha,\overline{\mu}^\alpha]+[a,a]=[\underline{\mu}^\alpha+a,\ \overline{\mu}^\alpha+a],$$

$$(\mu-a_1)^\alpha=[\underline{\mu}^\alpha,\overline{\mu}^\alpha]-[a,a]=[\underline{\mu}^\alpha-a,\ \overline{\mu}^\alpha-a],$$

where

$$a_1(x)=\begin{cases}1, & x=a\\ 0, & otherwise.\end{cases}$$

Clearly $\mu+a_1$, $\mu-a_1\in L(R)$. In addition, we have $\mu-a_1<\mu<\mu+a_1$ and $\overline{d}\ (\mu,\mu+a_1)=\overline{d}\ (\mu,\mu-a_1)=a.$

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A sequence $X=(X_k)$ of fuzzy numbers is said to be bounded if the set $\{X_k : k\in N\}$ of fuzzy numbers is bounded.

Definition 1.1. The sequence $X=(X_k)$ is said to be convergent to the fuzzy number (X_0), written as $\lim_k X_k = X_0$ [6], if for every $\varepsilon>0$ there exists a positive integer $n=n_0(\varepsilon)$ such that

$$\overline{d}(X_k, X_0)<\varepsilon \text{ for every } k>n_0.$$

If K is a subset of positive integers N, then K_n denotes the set $\{k\in K : k\le n\}$. The natural density of K is given by $\delta(K)=\lim_{n\to\infty}\frac{|K_n|}{n}$, where $|K_n|$ denotes the number of elements in K_n. Clearly, finite subsets have zero natural density and $\delta(K^c)=1-\delta(K)$ where, $K^c=N-K$ is the complement of K. If $K_1\subseteq K_2$, Then $\delta(K_1)\le\delta(K_2)$ (see [16]). For a sequence of real numbers $x=(x_k)$, the notions of statistical limit superior and limit inferior are defined as follows [13];

$$st-\lim\sup x=\begin{cases}\sup B_x, & B_x\neq\phi\\ -\infty, & otherwise\end{cases}$$

$$st-\lim\inf x=\begin{cases}\inf A_x, & A_x\neq\phi\\ +\infty, & otherwise\end{cases}$$

where $A_x=\{a\in R : \delta(\{k\in N : x_k<a\})\neq 0\}$ and

$$B_x=\{b\in R:\ \delta(\{k\in N : x_k>b\})\neq 0\}.$$

Definition 1.2. The sequence of fuzzy numbers $X=(X_k)$ is statistically convergent to X_0 if the set $\{k\in N : \overline{d}(X_k, X_0)\ge\varepsilon\}$ has natural density zero for every $\varepsilon>0$. We will use the notation $st-\lim X_k = X_0$ as in [6].

The sequence $X=(X_k)$ is said to be statistically bounded from above if there exists a fuzzy number μ (called the statistical upper bound) such that

$$\delta\left(\{k\in N : X_k>\mu\}\cup\{k\in N : X_k \not\sim \mu\}\right)=0$$

The statistical lower bound can be defined similarly.

If the sequence $X=(X_k)$ is both statistically bounded from above and statistically bounded from below then it is called statistically bounded [17].

Definition 1.3. ([18]) Let $I_n=[n-\lambda_n+1, n]$, $\lambda=(\lambda_n)$ be a non-decreasing sequence of positive numbers tending to ∞, $\lambda_{n+1}\le\lambda_n+1$, $\lambda_1=1$ and $X=(X_k)$ be a sequence of fuzzy numbers. A sequence $X=(X_k)$ of fuzzy number is said to be λ-statistically convergent or $s\lambda$-convergent to fuzzy numbers X_0, written as $s\lambda-\lim X_k = X_0$ if for every $\varepsilon>0$

$$\lim_n \frac{1}{\lambda_n}\left|\{k\in I_n : \overline{d}(X_k, X_0)\ge\varepsilon\}\right|=0 \text{ or}$$
$$\delta_\lambda(\{k\in I_n : \overline{d}(X_k, X_0)\ge\varepsilon\})=0$$

The sequence $X=(X_k)$ is said to be λ-statistically bounded from above if there exists a fuzzy number M (called the λ-statistical upper bound) such that

$$\delta_\lambda\left(\{k\in I_n : X_k>M\}\cup\{k\in I_n : X_k \not\sim M\}\right)=0$$

The λ-statistical lower bound can be defined similarly.

If the sequence $X=(X_k)$ is both λ-statistically bounded from above and λ-statistically bounded from below then it is called λ-statistically bounded [9].

II. Lambda Statistical Limit Inferior and Limit Superior for Sequences of Fuzzy Numbers

In this section, we introduce the concepts of λ-statistically limit superior and limit inferior for λ-statistically bounded sequences of fuzzy numbers. Given a sequence $X=(X_k)$ let us define:

$$A_X^\lambda=\left\{M\in L(R): \lim_n\frac{1}{\lambda_n}\left|\{k\in I_n : X_k<M\}\right|\neq 0\right\}$$

$$\overline{A_X^\lambda}=\left\{M\in L(R): \lim_n\frac{1}{\lambda_n}\left|\{k\in I_n : X_k>M\}\right|=1\right\}$$

$$B_X^\lambda=\left\{M\in L(R): \lim_n\frac{1}{\lambda_n}\left|\{k\in I_n : X_k>M\}\right|\neq 0\right\}$$

$$\overline{B_X^\lambda}=\left\{M\in L(R): \lim_n\frac{1}{\lambda_n}\left|\{k\in I_n : X_k<M\}\right|=1\right\}.$$

The sets $\overline{A_X^\lambda}$ and $\overline{B_X^\lambda}$ are the sets of λ-statistical lower bound and λ-statistical upper bounds, respectively.

Theorem 2.1. If the sequence $X=(X_k)$ is λ-statistically bounded, then $\inf A_X^\lambda=\sup\overline{A_X^\lambda}$ and $\sup B_X^\lambda=\inf\overline{B_X^\lambda}$.

Proof. We will prove the first equality. Let $m=\inf A_X^\lambda$ and $M=\sup\overline{A_X^\lambda}$. By definition of A_X^λ and $\overline{A_X^\lambda}$ we have $m\le\tilde{m}$ for $\forall\tilde{m}\in A_X^\lambda$ and $M\ge\tilde{M}$ for $\forall\tilde{M}\in\overline{A_X^\lambda}$. For every $\tilde{m}\in A_X^\lambda$ and $\tilde{M}\in\overline{A_X^\lambda}$, we have

$$\lim_n\frac{1}{\lambda_n}\left|\{k\in I_n : X_k<\tilde{m}\}\right|\neq 0 \quad\text{and}$$

$$\lim_n\frac{1}{\lambda_n}\left|\{k\in I_n : X_k>\tilde{M}\}\right|=1.$$ This means that

$\delta_\lambda\left(\{k\in I_n : X_k<\tilde{m}\}\cap\{k\in I_n : X_k>\tilde{M}\}\right)\neq 0$. In other words, there is a number $k\in I_n$ such that $\tilde{M}<X_k<\tilde{m}$. Therefore,

$$\tilde{M} < \tilde{m} \text{ for all } \tilde{m} \in A_X^{\lambda},\ \tilde{M} \in \overline{A_X^{\lambda}} \quad (2.1)$$

From (2.1), it follows that $\tilde{M}$ is a lower bound of the set A_X^{λ} .Then by definition of infimum we have $\tilde{M} \le m = \inf A_X^{\lambda}$ This inequality holds for all $\tilde{M} \in \overline{A_X^{\lambda}}$. Then by definition of supremum we have

$$M \le m \quad (2.2)$$

Now we show that the case $M < m$ cannot be place.

To the contrary, assume that $M < m$. This means that there is a number $\alpha \in [0,1]$ such that $\underline{M^{\alpha}} < \underline{m^{\alpha}}$ or $\overline{M^{\alpha}} < \overline{m^{\alpha}}$. For

The sake of definiteness we will consider the case

$$\underline{M^{\alpha}} < \underline{m^{\alpha}} \quad (2.3)$$

and then we will show that this leads to a contradiction. Denote $b = m\left(\underline{M^{\alpha}}\right)$. It is clear that $b < \alpha$ (b can be zero). Moreover, for all $\lambda \in (b, \alpha]$ the inequality $\underline{M^{\lambda}} < \underline{m^{\lambda}}$ holds. Since the functions $M(x)$ and $m(x)$ are upper semi-continuous then there exists a point (z, β) such that $z \in \left(\underline{M^{\alpha}}, \underline{m^{\alpha}}\right)$, $\beta \in (b, \alpha)$ and

$$\underline{M^{\lambda}} < z,\ \underline{m^{\lambda}} > z \quad \text{for all } \lambda \in [\beta, \alpha] \quad (2.4)$$

Let us define the fuzzy numbers γ_1 and γ_2 as

$$\gamma_1(x) = \begin{cases} 0, & x < \underline{X^0} \\ \beta, & x \in \left[\underline{X^0}, z\right) \\ 1, & x = z \\ 0, & x > z \end{cases} \qquad \gamma_2(x) = \begin{cases} 0, & x < z \\ \beta, & x \in \left[z, \overline{X^0}\right) \\ 1, & x = \overline{X^0} \\ 0, & x > \overline{X^0} \end{cases}$$

where the numbers

$\underline{X^0} = s\lambda - \liminf \underline{X_k^0} - 1$ and

$\overline{X^0} = s\lambda - \limsup \overline{X_k^0} + 1$ arefinite. It is not difficult to observe that

$$M \not\sim \gamma_1,\ m \not\sim \gamma_2 \quad (2.5)$$

This follows from

$$\underline{M^{\beta}} \ge s\lambda - \liminf \underline{X_k^{\beta}} \ge s\lambda - \liminf \underline{X_k^0} > \underline{X^0} = \underline{\gamma_1^{\beta}},$$

$$\underline{M^{\alpha}} < z = \underline{\gamma_1^{\alpha}} \text{ and } \underline{m^b} \le \underline{M^{\alpha}} < z = \underline{\gamma_2^b},\quad \underline{m^{\beta}} > z = \underline{\gamma_2^{\beta}}.$$

Now let us consider the sets

$$C_1 = \left\{k \in I_n : \underline{X_k^{\lambda}} \le z,\ \text{for some } \lambda \in (\beta, \alpha]\right\}$$

$$C_2 = \left\{k \in I_n : \underline{X_k^{\lambda}} > z,\ \text{for some } \lambda \in (\beta, \alpha]\right\}.$$

Clearly $C_1 \cup C_2 = I_n$ and therefore,

$$\delta_{\lambda}(C_1) + \delta_{\lambda}(C_2) \ge 1 \quad (2.6)$$

First we assume that $\delta_{\lambda}(C_1) > 0$. Taking in to account the structure of the fuzzy number γ_2 and the real number $\overline{X^0}$, we can obtain that $X_k < \gamma_2$ for all $\forall k \in C_1 \setminus K_1$, where $K_1 = \left\{k \in I_n : \overline{X_k^{\lambda}} > \overline{X^0},\ some\ \lambda \in [0,1]\right\}$. It is clear that $\delta_{\lambda}(K_1) = 0$, hence we have $\delta_{\lambda}(C_1 \setminus K_1) = \delta_{\lambda}(C_1)$. Thus, $\lim_n \frac{1}{\lambda_n}\left|\{k \in I_n : X_k < \gamma_2\}\right| \ge \delta_{\lambda}(C_1) > 0$. This means that $\gamma_2 \in A_X^{\lambda}$ and therefore, by definition of $\inf A_X^{\lambda}$ we obtain that $\gamma_2 \ge m = \inf A_X^{\lambda}$. This contradicts (2.5) (that is $m \sim \gamma_2$).

Hence, we have shown that $\delta_{\lambda}(C_1) = 0$.

Now from (2.6), it follows that $\delta_{\lambda}(C_2) = 1$. Taking into account the structure of the fuzzy number γ_1 and the real number $\underline{X^0}$, we obtain that $X_k > \gamma_1$ for all $\forall k \in C_2 \setminus (C_1 \cup K_2)$,

where $K_2 = \left\{k \in I_n : \underline{X_k^{\lambda}} < \underline{X^0},\ some\ \lambda \in [0, \beta]\right\}$.

It is obvious that $\delta_{\lambda}(K_2) = 0$, hence we conclude $\delta_{\lambda}(C_2 \setminus (C_1 \cup K_2)) = 1$. This implies

$$\lim_n \frac{1}{\lambda_n}\left|\{k \in I_n : X_k > \gamma_1\}\right| \ge \delta_{\lambda}(C_2 \setminus (C_1 \cup K_2)) = 1,$$

Which means that $\gamma_1 \in \overline{A_X^{\lambda}}$. Thus $\gamma_1 \le M = \sup \overline{A_X^{\lambda}}$. This contradicts (2.5) (that is, $M \not\sim \gamma_1$).

This completes the proof.

Definition 2.1. If $X = (X_k)$ is a λ-statistically bounded sequence of fuzzy numbers, then the λ-statistical limit inferior of $X = (X_k)$ is given by $s\lambda - \liminf X = \inf A_X^{\lambda}$.

Also, the λ-statistical limit superior of $X = (X_k)$ is given by $s\lambda - \limsup X = \sup B_X^{\lambda}$.

By Theorem 2. 1. we get $s\lambda - \liminf X = \sup \overline{A_X^{\lambda}}$ and $s\lambda - \limsup X = \inf \overline{B_X^{\lambda}}$. A simple example will help to illustrate the concepts just defined.

Theorem 2.2. Let $X = (X_k)$ be a λ-statistically bounded sequence of fuzzy numbers. If $m = s\lambda - \liminf X$ then

$$\lim_n \frac{1}{\lambda_n}\left|\{k \in I_n : X_k < m - \varepsilon_1\}\right| = 0 \quad (2.7)$$

and

$$\lim_n \frac{1}{\lambda_n}\left|\{k \in I_n : X_k < m + \varepsilon_1\} \cup \{k \in I_n : X_k \not\sim m + \varepsilon_1\}\right| \ne 0 \quad (2.8)$$

for every $\varepsilon > 0$.

Proof. To the contrary, we asuume that there exists $\varepsilon > 0$ such that $\lim_n \frac{1}{\lambda_n}\left|\{k \in I_n : X_k < m - \varepsilon_1\}\right| \ne 0$. This means that $m-\varepsilon_1 \in A_X^{\lambda}$. Then we get $m \le m-\varepsilon_1$ which is a contradiction.

Now let us show that inequality (2.8) holds. Assume that it is not true, that is, there exists $\varepsilon > 0$ such that

$$\left.\begin{array}{l} \lim\limits_{n} \dfrac{1}{\lambda_n}\left|\left\{k \in I_n : X_k < m+\varepsilon_1\right\}\right| = 0 \\ \lim\limits_{n} \dfrac{1}{\lambda_n}\left|\left\{k \in I_n : X_k \not\sim m+\varepsilon_1\right\}\right| = 0 \end{array}\right\} \qquad (2.9)$$

For each $\forall k \in N$, only the following three cases are possible:

$$X_k < m+\varepsilon_1, \quad X_k \ge m+\varepsilon_1, \quad X_k \not\sim m+\varepsilon_1 . \text{ Then}$$

$$\{k \in I_n : X_k < m+\varepsilon_1\} \cup \{k \in I_n : X_k \ge m+\varepsilon_1\} \cup \left\{k \in I_n : X_k \not\sim m+\varepsilon_1\right\} = I_n$$

Thus, from (2.9), we have

$\lim\limits_{n} \dfrac{1}{\lambda_n}\left|\left\{k \in I_n : X_k \ge m+\varepsilon_1\right\}\right| = 1$. This means that $m+\varepsilon_1 \in \overline{A_X^{\lambda}}$.

Hence we can write $m+\varepsilon_1 \le \sup \overline{A_X^{\lambda}} = \inf A_X^{\lambda} = m$.

This is a contradiction. So we get,

$$\lim_{n} \frac{1}{\lambda_n}\left|\left\{k \in I_n : X_k < m+\varepsilon_1\right\} \cup \left\{k \in I_n : X_k \not\sim m+\varepsilon_1\right\}\right| \ne 0$$

Theorem 2.3. Let $X = (X_k)$ be a λ-statistically bounded sequence of fuzzy numbers. If $M = s\lambda - \limsup X$ then

$$\left.\begin{array}{l} \lim\limits_{n} \dfrac{1}{\lambda_n}\left|\left\{k \in I_n : X_k > M+\varepsilon_1\right\}\right| = 0 \\ \lim\limits_{n} \dfrac{1}{\lambda_n}\left|\left\{k \in I_n : X_k > M-\varepsilon_1\right\} \cup \left\{k \in I_n : X_k \not\sim M-\varepsilon_1\right\}\right| \ne 0 \end{array}\right\} \qquad (2.10)$$

for every $\varepsilon > 0$.

This can be proved similarly as **Theorem 2.2.** In addition, the converse of **Theorem 2.3.** does not hold in general. Now we have the following assertion.

Theorem 2.4. For any λ-statistically bounded sequence of fuzzy numbers $X = (X_k)$,

$$s\lambda - \liminf X \le s\lambda - \limsup X .$$

Proof. We have $s\lambda - \liminf X = \sup \overline{A_X^{\lambda}}$ and $s\lambda - \limsup X = \sup B_X^{\lambda}$. By definition of the sets $\overline{A_X^{\lambda}}$ and B_X^{λ} , we see that $\overline{A_X^{\lambda}} \subseteq B_X^{\lambda}$. Hence we get $\sup \overline{A_X^{\lambda}} \le \sup B_X^{\lambda}$ which completes the proof.

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