Abstract—Most of the studies in Response Surface Methodology commonly involve one response or quality characteristics, whereas in most industrial applications considering all responses simultaneously is required. Multiple Response Surface (MRS) Optimization Problems often deal with responses that are conflicting. In dealing with incommensurate responses, incorporating a decision maker’s preference information into the problem has lots of advantages although a few researches in MRS literature has taken this into attention. This paper tends to take a detailed look at the most prominent approaches that has been suggested so far in MRS, also review and discuss the classifications with a special focus on the decision maker’s preference information. In today’s competitive market satisfying the customer is of high importance. The DM can be a customer and reaching a compromise with an interactive method would help the firm to succeed in having loyal customers. Results of the case study shows that applying a meta-heuristic algorithm with existing methods can improve the results. Finally future areas of research are discussed.

Index Terms—multiple response surface optimization; response surface methodology; decision maker; design of experiments.

I. INTRODUCTION

Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques that has been widely used in developing, improving, and optimizing processes [1]. In most RSM problems, the form of relationship between the response variables and the independent variables is not known. Therefore, the first step in RSM is to seek for a suitable approximation. Commonly a low-order polynomial is employed. For any curvature in the system, a polynomial of higher degrees, mostly second order is applied. Then, by the method of steepest ascent or steepest descent the most appropriate set of input for response is determined [2]. The RSM method was first introduced by Box and Wilson in 1951, although it was not considered for optimizing multiple responses variables until many years later [3].

Many industrial and manufacturing problems commonly deal with several quality characteristics to be optimized at the same time. For simultaneous consideration of multiple responses, first building appropriate response surface model for each response is required. Then we attempt to find a set of operating conditions that optimizes all responses or at least keep them in desired ranges [4]. There are 3 stages in these problems: 1) collecting data 2) building the model 3) optimization.

A multiple response surface problem is formally formulated as the following:

\[
\text{Optimize } y_1(x), \ldots, y_k(x) \quad \text{s.t. } x \in \psi \quad (1)
\]

\[\hat{y}_i(x) \text{ is the } i\text{th estimated response model and input variables are in the experimental region.}\]

In the multi objective decision making problems there are several techniques has been used solving optimization (MOO) literature (Hwang et al. 1979) [5]. In fact the timing of the decision maker’s preference information is important. In the first category prior information of the Decision Maker (DM) is needed when the problem solving has not started yet. Most of the studies in MRS problems required prior information from the DM. The second category is comprised of techniques that in which an interaction with the DM is required. And for the last category a posteriori information from the DM is required.

In this paper, we review and discuss various approaches in MRS and pay special attention to the DM role in the method. Section 2 reviews the MRS approaches. Section 3 is about Decision Maker’s preference Information in the MRS and classifies some of the most prominent articles into a table. Section 3 discusses a case study and compares the result. Finally section 4 is the conclusions.

II. MRS APPROACHES REVIEW

Many Methods have been introduced for multiple response optimization. Pignatiello (2004) categorized the existing methods in three categories [6]. Then Tajbakhsh and Noorossana (2005) classified them in four basic categories [7]. Here we focus on the most prominent approaches of the newest classification. In the next section according to the most recent studies we classify the approaches considering the decision maker’s preference information in the problem solving process.

A. Overlaying Contour Plots

Myers and Montgomery suggested that an approach for optimizing several responses is to overlay the contour plots for each response. The experimenter can visually examine the contour plot to discover the appropriate operating conditions [1]. It must be emphasized that it is mainly used...
when there are few process variables. For more than 3 design variables this approach becomes awkward.

In this approach there is no need to the DM’s information. And Contour plots play the main role.

B. Constrained Optimization Problem

A popular approach is to formulate and solve the problem as a constrained optimization problem [1]. Kim classified it as a priority based approach [8]. The priority based approach, that is similar to method bounded objective in the Multi objective decision making problems, chooses the response with the highest importance as the objective function and the rest of the functions are considered as constraints, Although it is not always much straightforward.

The idea was first suggested by Myers and Carter (1973) [9]. 2 responses are assumed and referred as a “primary response” and a “constraint response”. The goal is to find conditions on a set of design variables which maximize the primary response function subject to the constraint response function. Biles (1975) Considered multiple process responses and extended the myers and carter idea [10]. The priority based approach was studied in the later years like Delcastillo and Montgomery (1993) [11].

C. Desirability Function Approach

In this approach an estimated response such as \(\tilde{y}(x)\) is transformed to a scaled free value \(d_i\) that is called desirability. It changes from 0 to 1. The overall desirability (D) is also in the [0,1] interval is obtained by combining all desirabilities (di) [12]. Derringer and Suich (1980) extended the idea and presented a method to construct an overall desirability [13].

Desirability function for the larger-the-better case is as following:

\[
d_i = \begin{cases} 
0 & \tilde{y}_i(x) \leq y_i^{\min} \\
\frac{\tilde{y}_i(x) - y_i^{\min}}{y_i^{\max} - y_i^{\min}} & y_i^{\min} \leq \tilde{y}_i(x) \leq y_i^{\max} \\
1 & \tilde{y}_i(x) \geq y_i^{\max}
\end{cases} \quad (2)
\]

In the above equation The min and max indexes on the y denote the lower and upper limits accepted for \(\tilde{y}_i(x)\) respectively.

The most important advantage of this approach is that the Decision maker’s preference information be easily applied in the model. In addition it is easy to use and popular among the authors.

D. Loss Function Approach

Pignatiello (1993) first suggested a squared error loss function as follows [14]:

\[
L(y(x)) = (y(x) - \Phi)’C(y(x) - \Phi) \quad (3)
\]

Where

\(y(x)\) = response vector
\(\Phi\) = target vector
\(C\) = cost matrix

The cost matrix determines the relative importance of the response variables.

Extended studies about this approach can be found in Vinning (1998), Tsui (1999), Riberio et al. (2000) [15, 16].

E. Process Capability Approach

Process capability index is used to evaluate whether a process is able to meet current specification limits [18]. Hsiang and Taguchi presented the index \(C_{pm}\) as follows [19]:

\[
C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \quad (4)
\]

USL and LSL are specification limits. The mean, variance and target are other elements of the above equation.

In addition Chan et al., in an independent work proposed this index later. This index can be applied in multi response optimization. Plante considered the maximization of process capability as a criterion for multi response optimization [20]. For a detailed study about this approach the reader can refer to [18],[20].

F. Distance Function Approach

This approach was proposed by Khuri and Conlon [21]. The distance function is [21]:

\[
distance[\hat{y}(x), T] = [\hat{y}(x) - T]'\sum_{i(x)}^{-1}[\hat{y}(x) - T] \quad (5)
\]

\(T\) represents the target value. \(\hat{y}(x)\) is the predicted response and \(\sum_{i(x)}\) is the variance-covariance matrix of the predicted responses. The optimal settings are achieved if the distance function gets minimized.

Figure 1: MRS optimization approaches classification

All the discussed approaches are depicted in fig.1. need to be highly robust in order to avoid getting stuck at a local
optimal solution. The benefit of GA is just able to obtain the
global optimal solution rather fairly. Also GA does not
require the specific mathematical analysis of optimization
problems [22].

Pignatiello et al (2004) for the first time used both
Desirability approach and Genetic Algorithm in multi
applied the genetic algorithm with desirability function
framework for a multi response simulation optimization
problem [2].

Neural network(NN) is is a class of adaptive systems
consisting of a number of simple processing elements,called
neurons.Neurons are interconnected to each other in a feed
forward way. A significant contribution of NN is the ability
to learn to perform operations, not only for inputs like the
training data, but also for new data that may be incomplete
or noisy [22].

Noorossana et al (2005) applied an Artificial Neural
Network Approach to Multiple Response Optimization.
They have chosen the data to be qualitative and has applied
fuzzy theory for them.results of the paper shows great
stability [23]. Bashiri et al.(2009) used neural network based
on a desirability function .they used a feed forward back
propagation with two hidden layers.the number of neurons
in the hidden layers are determined using MSE criterion for
testing data and training [24].

III. DECISION MAKER’S PREFERENCE PARAMETERS IN THE
MRS OPTIMIZATION

In this section the most recent works in various MRS
approaches are classified into 4 categories : no information
from the DM,prior information from the DM,interaction
with DM,posterior information from the DM. The Preference
parameters from the DM can be specification limits,
including LSL and USL. Also target value or the shape of
the function. Another option is assigning weights to
different responses.

A. No Information from the DM

Despite the effectiveness of a DM information preference,
several studies in MRS are solved with no information from
the DM. Romano et al (2004) deals with a robust parameter
design problem. they present a general framework for the
multivariate problem when data are collected from a
combined array [25].No information is obtained from a DM.
Ko et al (2005) proposed a new loss function-based method
for multi response optimization [26],although the proposed
loss function method shows more reasonable result .There is
no information in the problem by a DM. Noorossana and
Ardakani (2008) considered a robust parameter design
problem with 2 objectives. An Lp metric method was
applied because it does not need much information from the
function and simulation approach with a Genetic algorithm.
The same as other works in this group no information is
required from a DM [2].

B. Decision Maker’s Prior Information

Some research carried out so far in MRS can be
categorized here. All the DM’s preference information are
gathered before solving the problem. Sadjadi and khaledi
(2008) studied some multi objective decision making
approach for non linear multiple response surface problems
[28].a Lexicographic method is used and before problem
solving the DM’s preference information are asked.

C. Interaction with the DM

In dealing with conflicting responses, incorporating a
decision maker’s preference information into the problem
has lots of advantages although a few research in MRS
literature has taken this into attention. According to Jeong
and Kim (2009) The DM’s preference information is
denoted through preference parameters. The shape,bound
and target of a desirability function or the cost matrix in loss
function method are examples of preference parameters[29].
Some studies that have tried to use interactive methods to
solve MRS problem are as follows : Montgomery and
Bettencourt (1977) , Mollaghasemi and Evans (1994) , Park
and Kim (2005) [30],[31],[32].

In an interaction makes DM,feels more comfortable with
the process. Therefore applying an interactive method
appropriately leads to a good compromise between the
problem solver and a decision maker. Among the research in
this field some works are more significant.

Jeong and Kim (2005) notice that as STEM is effective in
extracting a decision maker’s preference information for a
satisfactory compromise, they proposed a modified STEM
called D-STEM [33].By some examples it is concluded that
the new method is more effective.

Kazemzadeh et al (2008) demonstrate various types of
DMs relating to MRS problems. The types are as follows: customer, experimenter, producer and manager. They also
propose a general framework for Multiresponse optimization problems based on goal programming [34].

Jeong and Kim (2009) proposed an interactive version of
the desirability function approach .Their method provides
various channels through that the DM is able to articulate
his/her preference information [29].

D. Decision Maker’s Posterior Information

Lee et al. proposed a posteriori preference information
approach to dual response surface optimization in 2010 [36].

In table 1 the above discussion is summarized. Recent
works in MRS optimization are considered with special
focus on the DM’s role in the problem solving.

Table 2 is derived from works in the literature that have
used the most popular MRS approaches. The problems are
studied to see whether the DM’s preference information is
used to obtain a compromise result or not. Obviously more
focus has been on using desirability function for an
interactive approach to solve MRS problem.

<table>
<thead>
<tr>
<th>Types of DM information</th>
<th>Most popular MRS approaches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constrained Optimization</td>
<td>Desirability Function</td>
</tr>
<tr>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Prior</td>
<td>✓</td>
</tr>
<tr>
<td>Interactive</td>
<td></td>
</tr>
<tr>
<td>Posterior</td>
<td>✓</td>
</tr>
</tbody>
</table>
TABLE1.RECENT WORKS CLASSIFICATION CONSIDERING THE DM

<table>
<thead>
<tr>
<th>Recent works in MRS</th>
<th>Types of DM Information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>None</td>
</tr>
<tr>
<td>Roman o 2004</td>
<td>✓</td>
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<tr>
<td>Ko 2005</td>
<td>✓</td>
</tr>
<tr>
<td>Park 2005</td>
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<td>Jeong 2005</td>
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<td>Pasandi deh 2006</td>
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<td>Sadjadi 2008</td>
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<td>Noorosana 2008</td>
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<td>Kazem zadeh 2008</td>
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<td>Jeong 2009</td>
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<td>Lee 2010</td>
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IV. CASE STUDY

Khoo and Chen (2001) enhanced the response surface methodology with Genetic algorithm [35]. They Solved a case study as follows: there are 2 responses called mean pull strength and the minimum strength that are required to be monitored. The goal is to maximize both the mean pull strength and the minimum strength simultaneously. The response surface equations established with design of experiments are as follows:

\[
y_1 = 73.89 + 12.91x_1 + 7.11x_2 + 2.56x_3 - 1.96x_1^2 - 1.01x_2^2 + 0.22x_3^2 + 0.36x_1x_2 - 0.068x_1x_3 - 0.52x_2x_3 \tag{4}
\]

\[
y_2 = 45.06 + 14.11x_1 + 6.56x_2 + 2.17x_3 - 1.96x_1^2 - 1.02x_2^2 + 0.14x_3^2 \tag{5}
\]

where output variables are mean pull strength and minimum strength respectively. Both objective functions are going to be maximized. Input variables are bonding temperature, bonding force and bonding time respectively.

- \( x_1 \) varies from 500 to 580.
- \( x_2 \) varies from 11 to 13.
- \( x_3 \) varies from 210 to 250.

Results are produced by Matlab. The interactive method that has the desirability function framework performs much better than methods with no information from the DM.

V. CONCLUSION

In this article existing approaches in MRS are reviewed and discussed, considering a decision maker’s preference information. In dealing with conflicting responses, incorporating a decision maker’s preference information into the problem has lots of advantages mainly because of the fact that in today’s high competitive market satisfying the customer is of high importance. However, a few research in MRS literature has taken this into attention.

DM can be a customer and reaching a compromise with having loyal customers. Obtaining posteriori information from the DM is also an area with potential scope to be studied about. Results of the case study shows that applying an interactive method with existing MRS an interactive method would help the firm to succeed in approaches lead to better results. Future studies can focus on applying metaheuristic algorithms in the optimization step.

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