Research on ADC Model Based on Improved Analytic Hierarchy Process

Guoqing Huang, Pengfei Wang, and Mingxu Wang

***Abstract*—ADC model has been widely used in effectiveness evaluation, and the capability matrix C plays a vital role when building ADC model. Based on the analysis of ADC model, construct the matrix C by the improved Analytic Hierarchy Process (AHP) is proposed. The problems of traditional AHP are analyzed, which exist in the structure of capability matrix C. Then, the approximate degree used to solve the weight of judgment matrix is simulated and analyzed. Based on the analysis and the simulation, the improved AHP is proposed combined with entropy method. The results show that the improved AHP is not only to solve the weight of judgment matrix, but also to solve the difficulty when structuring capability matrix C.**

***Index Terms*—ADC model, capability matrix C, improved AHP, approximate degree, judgment matrix, weight.**

I. Introduction

ADC model is the classic model that is proposed by American Industry Weapon System Effectiveness Advisory Committee (WSEIAC). It is not only to consider the correlation between the equipment structure and the battle technical characteristics and the entire process of fight, but also to highlight the integrity of equipment. The concept of ADC model is clear and easy to understand and express, so it is widely used in the combat effectiveness evaluation of single weapon unit. However, when building the ADC model, the determination of the capability matrix C is very difficult, and the system model is to assess effectiveness by dividing the system states and calculating the state transition probability. Therefore, when the dimension of the state matrix is too large, the dimension of dependability matrix D is dramatic "expansion", which causes certain difficulties when using the ADC model.

AHP is a system analysis method proposed by the scholar of American Pittsburgh University. It is mainly to study the judgment matrix of structure model, proportional scale, consistency checking and dependability, which is a good weight calculation method. But AHP judgment matrix is constructed by experts, it ignores the difference between qualitative indexes and quantitative indexes, and has a stronger subjective judgment. And when calculating the weights, the uncertain choices of square root method and eigenvalue decomposition method can make the evaluation results different.

Based on the above analysis, first, it uses the square root method and eigenvalue decomposition method to analyze approximate degree of judgment matrix when calculating weight. And the improved AHP revised by entropy method is proposed it effectively considers both the qualitative and quantitative indexes. Then, it uses the improved AHP to construct capability matrix C of ADC model, and limits the dimension of dependability matrix D, in order to solve the problem that the dimension of the matrix D is dramatic "expansion". The results show that, based on improved AHP, it not only solves the problem of which method to calculate judgment matrix weight, but also solves the difficulty when structuring capability matrix C.

II. ADC Model and the Basic Principle of AHP

A. ADC Model

$$E = A \cdot D \cdot C \tag{1}$$

where $E=[e_1, e_2, e_3, \cdots, e_i, \cdots e_m]$ denotes the index vector of system effectiveness, e_i is defined as the effectiveness when completing the i task, $i = 1, 2, 3, \cdots, m$. $A=[a_0,\ a_1, \cdots,\ a_j, \cdots,\ a_n]$ denotes availability vector, which is the state vector at the starting of mission, a_j indicates the probability of j state at the task beginning, $j = 1, 2, 3, \cdots, n$. $D = (d_{ij})_{n \times n}$ denotes dependability matrix, which shows the probability that the system is in the i state when beginning, but during the task duration time, it is in j state. And $C=[C_1,\ C_2, \ldots,\ C_n]^{\mathrm{T}}$ denotes capability matrix, where c_i defines the probability of completing the task when the task is end.

B. The Principle of AHP

1) A hierarchical structure is determined by the evaluation objectives, judgment matrix and programs.
2) A pairwise comparison judgment matrix is constructed.
3) The relative weight of judgment matrix is derived based on the elements of single criteria.

When solving weight, it needs to consider two problems, one is the weight calculation method, and the other is the consistency checking of pairwise comparison judgment matrix [1].

The square root method or the eigenvalue decomposition method is used to calculate weights. The calculation process can be expressed as:

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Guoqing Huang is with the School of Information Engineering, Zhengzhou University, Zhengzhou, China and Department of Aeronautic Electronic Engineering, The First Aeronautical College of Air Force, Xinyang, China (e-mail: fei30923@gmail.com).

Pengfei Wang and Mingxu Wang are with the School of Information Engineering, Zhengzhou University, Zhengzhou, China (e-mail: wmxxg66@yahoo.com.cn).

1) Eigenvalue Decomposition Method (EDM)

$$A\overline{w} = \lambda_{\max}\overline{w} \tag{2}$$

where $\lambda_{\max}$ is the largest eigenvalue of A, and $\overline{w}$ is the corresponding eigenvector. w is the normalized weight vector of $\overline{w}$ .The eigenvalue decomposition method is earlier proposed and widely used for solving the index weigh in the AHP. It plays an important role to the development of the AHP in theory.

2) Square Root Method (SRM)

$$\overline{w_i} = (\prod_{j=1}^{n} a_{ij})^{\frac{1}{n}}, i = 1, 2, \cdots, n \tag{3}$$

Then, let the $\overline{w} = (\overline{w}_1, \overline{w}_2, \cdots, \overline{w}_n)^T$ be normalized to obtain the weight vector w .

It is impossible to get the exactly same judgment matrix by pairwise comparison, and it also exist some errors. In order to avoid the contradiction occurrence that A is more important than B, B is more important than C, but C is more important than A when structuring judgment matrix. So, the consistency checking is very important and necessary to judgment matrix [2].

Consistency Index (CI) can be defined as:

$$CI = (\lambda_{\max} - n)/(n-1) \tag{4}$$

where $\lambda_{\max}$ is the largest eigenvalue of A, and n is the order of A .

Consistency Proportion (CR) can be written as:

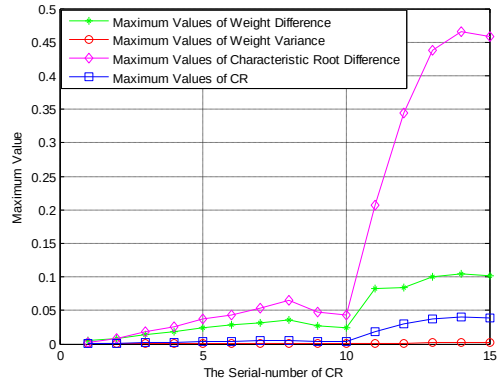
$$CR = CI / RI \tag{5}$$

where RI (random index) can reference the paper [3].When $CR < 0.1$, it can be considered that the pairwise comparison judgment matrix meets the consistency requirement. If $CR > 0.1$, the judgment matrix must be revised or re-constructed. The exceptional case is, when n=1 or 2, the judgment matrix is always consistent.

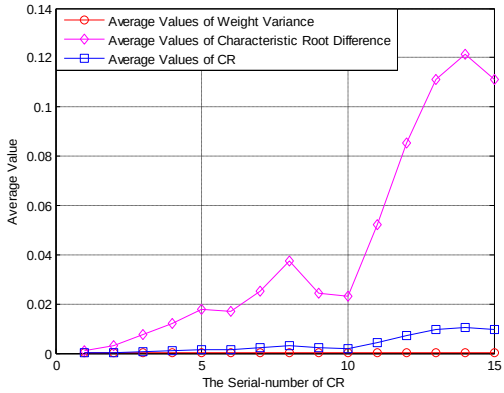
III. The Approximate Degree Simulation of SRM and EDM for Solving Weight

A. The Maximum Value and the Average Value of the Corresponding Different CR Interval

The approximate degree used the two above methods in different CR interval is simulated. The simulation parameters are set as follows: we fix the order of judgment matrix as 9, the CR interval is divided in 15 intervals, such as 0~0.01, 0.01~0.02, 0.02~0.03, 0.03~0.04,…, 0.08~0.09, 0.09~0.1, 0.1~0.2, 0.2~0.3, 0.3~0.4, 0.4~0.5 and 0.5~0.6. Meanwhile, 20,000 judgment matrixes are constructed for each CR interval. We can simulate out the maximum of the absolute value of the weight difference, the maximum and average values of the weight variance, the maximum and average values of the absolute value of the characteristic root difference, and the maximum and average values of the absolute value of CR difference used two methods in different CR interval. The simulation results are shown in Fig. 1:



(a) The maximum value.



(b) The average value

Fig. 1. The maximum value and the average value of the different CR interval.

The simulation results analysis is as follows:

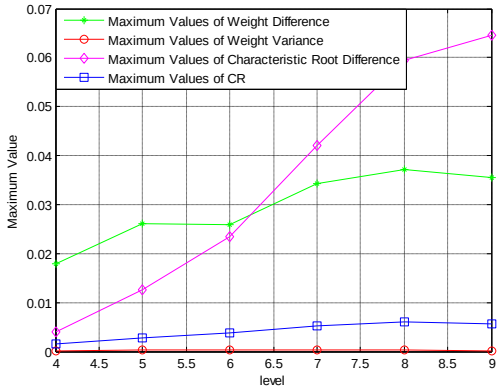
1) The approximate degree of the two methods when it meets the consistency is better than that does not meet the consistency.
2) Overall, the approximate degree of the two methods is getting better with the decrease of CR. When it meets the consistency, the approximate degree of the two methods is getting worse firstly and then the approximate degree is getting better with the increase CR.

When the approximate degree meets the consistent requirements, the obtained maximum value of weight difference is higher than 0.03, the weight variance value is less than 0.2e-3 and the average value is a little larger than 0.1e-3. When using the square root method replaces the eigenvalue decomposition method, the overall impact is small, but if the weight difference reached the maximum, the impact on index weight is larger.

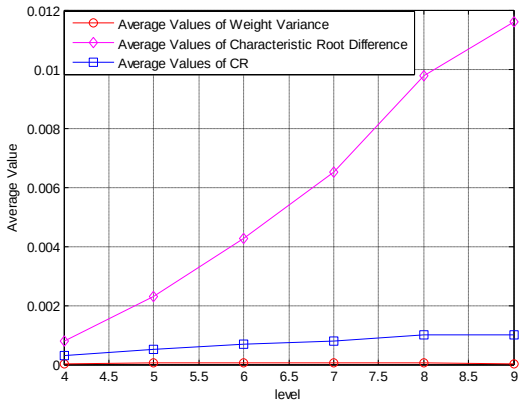
B. The Maximum Value and the Average of the Corresponding Different Orders

The approximate degree used the two above methods in different orders is simulated. The simulation parameters are set as follows: Let the judgment matrix meet the consistency requirements, then the order of judgment matrix can be gotten from 4 to 9, and 20,000 judgment matrixes are

constructed for each order. We can simulate out the maximum of the absolute value of the weight difference, the maximum and average values of the weight variance, the maximum and average values of the absolute value of the characteristic root difference, and the maximum and average values of the absolute value. The simulation results are shown in Fig. 2.



(c) The maximum value



(d) The average value

Fig. 2. The maximum value and the average value of the different order.

The simulation results analysis is as follows:

1) The approximation degree of the two methods is getting worse firstly, and then it is getting better with increasing order, if the order is 8, the approximation degree is the worst.
2) When using the SRM replaces the EDM, the overall impact is less, the average value is less than 5e-5, but if the weight difference reached the maximum, the impact on weight is larger.
3) The maximum value of consistency proportion difference is a little higher than 0.006. The consistency checking result of judgment matrix is always the same.

The consistency ratio is determined by the largest eigenvalue linear, but because of the different order, the curve shape of SR difference and CR difference is slightly different.

IV. Improved Analytic Hierarchy Process

The construction of the judgment matrix mainly depends on the experts in traditional AHP, and the objectivity is poor. Meanwhile, it also ignores the difference between qualitative indexes and the quantitative indexes, and the accuracy is poor [4]. In order to solve the above problems, the improved AHP is proposed, the steps are as follows:

1) The index weights of all levels are obtained by AHP.
2) The index weights obtained by AHP are revised by the entropy method.

Step 1: Let the judgment matrix $A=(a_{ij})_{n\times n}$ constructed by the AHP be normalized, the normalized matrix is:

$$\bar{A}=(\bar{a}_{ij})_{n\times n}, i=1,2,3,\cdots,n \tag{6}$$

$$\bar{a}_{ij}=a_{ij}/\sum_{i=1}^{n}a_{ij} \tag{7}$$

Step 2: Determine the information entropy and the difference coefficient of the indexes [5].

$$e_j=1/\ln n\sum_{i=1}^{n}\bar{a}_{ij}\ln(1/\bar{a}_{ij}), i=1,2,3,\cdots,n \tag{8}$$

$$d_j=1-e_j \tag{9}$$

where e_j represents the information entropy value, and d_j represents the difference coefficient.

Step 3: Calculate the entropy weight of each index.

$$\mu_j=d_j/\sum_{j=1}^{n}d_j, i=1,2,3,\cdots,n \tag{10}$$

Step 4: Revise the index weights used entropy weight.

$$\bar{w}=\mu_i w_i/\sum_{i=1}^{n}\mu_i w_i, i=1,2,3,\cdots,n \tag{11}$$

V. ADC Model Based on Improved AHP and Its Key Issues Analysis

A. ADC Model Based on Improved AHP

In the ADC model, the structure of the capability matrix C is the key, but the structure is difficult, it brings some limitations to the use of the ADC model. AHP can make the structure simplify, it decomposes a complex problem into simple factors. According to the affiliation and form a hierarchical hierarchy in final, these factors will be restructured hierarchically. First, the relative importance of the various factors in the level is determined; Second, the judgment of policy makers to determine the total ranking of the relative importance of the various factors is synthesized, which will greatly improve the effectiveness, reliability and feasibility of the decision-making. Based on the above analysis, AHP is introduced into building ADC model to build the capability matrix C [6]. The Capability matrix C is constructed as follows:

1) Set up efficiency evaluation index system;
2) Build standard of evaluation
3) Calculate the index weights by using the improved AHP in the 4th part;
4) Calculate the capability values and construct the capability matrix C.

If the effectiveness evaluation index system is a three-tier

system, first, calculate the capability value of each third-level index, which is determined by all the experts' rating standards and the number of experts; then obtain the capability values of the second-level indexes and first-level indexes value, the capability values of the second-level indexes is determined by the product of the capability values of the third-level indexes and the third-level indexes' weight, using the same way to obtain the first-level indexes' capability values. At last, the top-level index' capability value also can be gotten, in the other words, the capability matrix C is gotten.

B. Key Issues

In the AHP, the dimension of judgment matrix has been limited when structuring judgment matrix. In the other words, the every layer's elements are not more than 9. In the paper, the improved AHP is used to structure capability matrix C, the order n of capability matrix C will be limited to less than 9, along with it the dependability matrix D will be finalized. The dimension of capability matrix C can be used to determine or revise the availability matrix A and the dependability matrix D. It also solves the problem that the dimension of dependability matrix D is dramatic "expansion".

To use the SRM or EDM to solve the index weight and test the consistency in using the AHP, it gives a good answer that the SRM can be used in the situation of relatively lower accuracy requirements. But when analyzing cost-efficient or calculating the effectiveness based on the effectiveness, the EDM should be used to solve the index weights. The SRM can be used to test the consistency instead of the EDM in an inconvenient situation.

VI. Conclusions

An ADC model based on improved AHP has been presented. The model chooses the improved AHP to construct the capability matrix C. Through the simulations, it has been shown that the SRM is reasonable instead of EDM. Using the ideas of entropy method to improve the AHP, the problems of ADC model have been solved, it is not only to solve the confirmation of capability matrix C but also to limit the dimension's dramatic "expansion" of dependability matrix.

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Guoqing Huang was born in Tanghe, Henan Province, in 1963. He is a graduate and now he is an associate professor in the department of Aeronautic Electronic Engineering of the First Aeronautic Institute of Air Force and also the instructor of graduate students in zhengzhou university. His research concerns the signal and information processing.

Pengfei Wang was born in Zhengzhou, Henan province, in 1988. He is now a graduated student. His research concerns signal and information processing.

Mingxu Wang was born in Tanghe, Henan Province, in 1986. He is now a graduated student. His research concerns signal and information processing.