Effects of Materials Uncertainties on the Variability of the Response of Vibrating Systems

M. Hammoutene, B. Tiliouine, and B. Benahmed

***Abstract*—We present the main results of a numerical investigation for the probabilistic description of the maximum response of systems under seismic excitations, taking into account random variations in the values of the damping due to the constituent materials. A probabilistic theory, using order statistics, is presented to find the most probable amplitude of the maxima of a random structural response. Among other results interpreted in terms of excitation levels and vulnerability of structures, are of high importance to engineers and their use can be extended to improve current seismic regulations.**

***Index Terms*—Monte Carlo method, response spectra, strong ground motions, uncertain damping.**

I. Introduction

Characterization of levels of displacements in modeling the dynamic response of structures is not well understood nowadays as it depends on the damping. Indeed, the improvements brought during the measurements and the conception are characterized by an increase in stiffness and a decrease in the weight of materials and links, which in general has the effect to reduce the damping and hence to increase the levels of structural response.

Therefore, when designing a structure, knowing the effects of uncertainties related to the damping is essential. This requires the development of appropriate mathematical tools to incorporate such uncertainties into the modeling of structures, as well as in the development of methods to analyze these mathematical models [1].

As a first step, we analyze, for a fixed value of the coefficient of variation (COV) of the damping, the effect of the variability of the damping on the structural response within different structural vibration frequency bandwidths. Thereafter, we proceed to an analysis of the influence of the extent of the range of variation of the damping on the spectral amplitudes. The results obtained for sure very important for engineers are obtained and their use can be extended to improve seismic regulations currently in use worldwide.

II. Variability of the Dynamic Structural Response

The order statistic probabilistic theory is used here in order to include the uncertainty of damping in the estimation of the structural response. Indeed, for an oscillator that has deterministic properties, the peak amplitude of the n^{th} ordered response $x\ (t)$ is expressed by:

$$a_{(n)} = \eta_{(n)}\ a_{\text{rms}} \tag{1}$$

where $\eta_{(n)}$, the expected amplitude of the n^{th} order peak when peaks response are arrayed in a decreasing order of magnitude, is estimated by utilizing the probability density function of peaks [2], [3]. The parameter a_{rms} representing the root mean square of the response, is defined by:

$$a_{rms} \ = \ \sqrt{m_0} \tag{2}$$

where m_0 is the Zeroth Order moment of the power density function $GD(f)$ of the displacement of the oscillator with a frequency value f_n and damping ratio ξ [4]. The expression of m_0 is given as follow:

$$m_0 \ = \ \int_0^{+\infty} GD(f)\ df \tag{3}$$

And

$$GD(f) \ = \ G(f)\left|H(f,\ f_n,\ \xi\right|^2 \tag{4}$$

$G(f)$ being the unilateral power density function of the process characterizing the ground seismic acceleration and H $(f,\ f_n\ \xi)$ the displacement transfer function of the structure.

The variance $\sigma^2_{a_{rms}}$ associated to a_{rms} and generated by the uncertainties of $\Delta\ \xi\ =\ (C_\xi.\ \ \mu_\xi)$ due to the damping ξ is obtained by an analytical development which modifies the expression of m_0 after expressing $GD(f)$ by performing a first order Taylor series expansion of the transfer function squared. All calculations will give the following result:

$$\sigma^2_{a_{rms}} \ = \ \frac{1}{4\ \overline{m}_0}\left|\int_0^{+\infty} G(f)\left[\frac{\partial\left|H(f,\ f_n,\ \xi\ \xi)\ \right|^2}{\partial\xi}\right]_\xi df\right|^2 E\left[(\Delta\xi)^2\right] \ = \ \frac{1}{4\ \overline{m}_0}\ \sigma^2_{m_0} \tag{5}$$

By replacing a_{rms} in the equation (1) by (a_{rms} ± $\sigma_{a_{rms}}$) and $\eta_{(n)}$ by $\eta_{(1)} = \eta_{max}$ (the largest peak), we obtain the amplitudes of the spectral responses associated with the damping's uncertainties of the oscillator:

$$SD\,(\,f\,,\xi) = \eta_{\max}\ \left(a_{rms}\ \pm\ \sigma_{a_{rms}}\ \right) \tag{6}$$

It should also be noted that the velocity and acceleration

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pseudo - spectra response are obtained in a conventional way by multiplying *SD* (f, ξ) by ω and ω^2 respectively.

III. Numerical Results and Discussion

A. Statistical Characterization of the Damping

The evaluation of the damping, resulting in part to the materials used the structure's conception is a controversial subject in practice. To identify this problem, several investigators have conducted tests at different levels of response amplitudes as well as on different types of structural systems and sizes of existing buildings. The information obtained was then assembled by [5], [6]. The Table I shows the results for few buildings.

TABLE I: Values of the Critical Damping Ratio Associated to Existing Structures.

Structure	Damping ratio ξ (%)
- Building of the Gas and electricity society, San Diego	1.6 %
- Transamericain Building, San Francisco	0.9 %
- Twelve floors RC building, Skopje	1.1 %
- The Canterbury University Building of Physics/Chemistry	8.5 %
- Nine floors steel building, south - California	0.6 %
- R. M. Parsons Company international headquarter Building, California	2.5 %
- Oak Center Tower building, Oakland, Californie.	1.7 %

The results of these tests showed that the Log-normal and Gamma probability density distributions provide the best adjustments to the variations of damping, which have a coefficient of variation (COV) $C_\xi = \sigma_\xi / \mu_\xi$ whose values are contained in the interval [33% - 87%].

B. As said, to Insert Effects of Damping Uncertainty on the Dynamic Response.

The theoretical background presented above was applied to the estimation of response spectra for an average value $\mu_\xi = 5\%$ and a standard - deviation $\sigma_\xi = C_\xi\ \mu_\xi = 0.02$ of the random damping (i. e. $C_\xi = 40\%$). The accelerogram recorded during the 27th June 1966 Parkfield earthquake was used as seismic excitation.

Fig. 1(a) shows the variations of displacement response spectra (*SD*) and the pseudo velocity spectra (*PSV*) and pseudo acceleration spectra (*PSA*) obtained. For comparison purposes, the Fig. 1(b) shows the corresponding spectra obtained by using the Monte Carlo simulation technique [7].

The results show a low frequency range [<~ 3Hz] where the response of the oscillator is controlled mainly by the displacement of the support (flexible structure) because the mass does not move in the absolute axes and there is a strict equality between maximum displacements (relative for the mass, absolute for the support), regardless to the damping. Another frequency range [> ~ 8Hz], where the response of the oscillator is controlled mainly by the acceleration of the support, in which the influence of damping variations is least. The largest fluctuations are located in the intermediate frequency range [~ 3Hz - 8Hz ~]. where the values of the spectral response has presents randomly distributed extrema. These extrema are obtained regardless of the value of damping selected from the range of 1% to 20%. We notice that there is a great sensitivity of the spectral response values when damping fluctuates, in effect, small changes in damping generate relatively large variations in the response. Similar results were obtained with the Monte Carlo method (Fig. 1(b)).

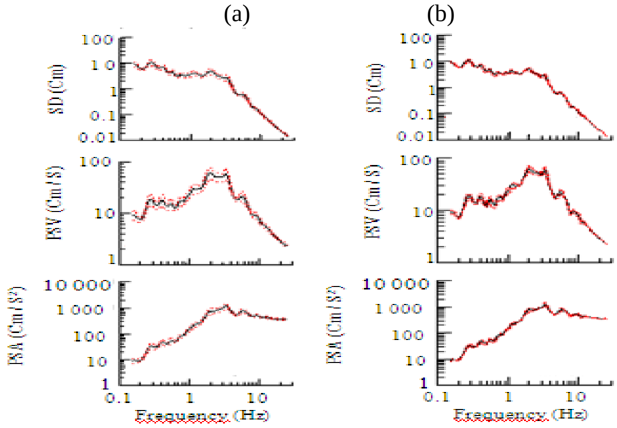

Fig. 1. Mean Spectrum (line) and Mean Spectrum ±1standard deviation (red dashed lines): (a) analytique approche (b) Monte Carlo simulation.

The values obtained for the pseudo - acceleration spectrum (*PSA*) fluctuate between a minimum value equal to 0.13%, which characterizes rigid structures, and a maximum one equal to 15.39%, which is to associate with a structure in the intermediate frequency band. The average fluctuations value of the *PSA* is 7.70%.

C. Sensitivity of the Response to Different Changes of the Damping Variability.

In this section, we study the sensitivity of the seismic response to changes in the values of C_ξ. The test results on real buildings showed that C_ξ varies between [33% - 87%], which lead us to adopt, for this part of the study, the values C_ξ : 0.4 et 0.8 that represent the extreme values of the interval and 0.6 to associate with the median values of the interval.

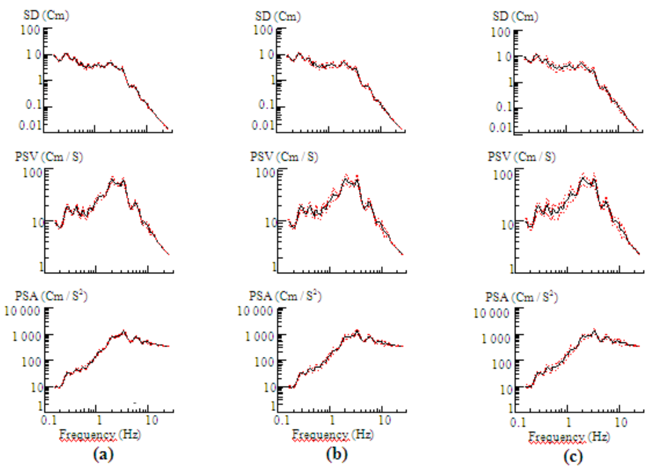

Fig. 2. Mean Spectrum (line) et Mean Spectrum ±1 standard deviation (red dashed lines):
a) C_ξ =40 %, b) C_ξ =60 % and c) C_ξ =80 %.

Fig. 2 shows the fluctuations of the amplitudes obtained for each response spectra (*SD*, *PSV* and *PSA*) and for each of the three selected values of the coefficient C_ξ. By looking at this figure, we can ,make the same observations as before regarding fluctuations spectral amplitudes within each frequency bandwidth regardless of the values that C_ξ.might take

Regarding the variations of the spectral amplitudes in regard of the values of C_ξ, Figure 5 clearly shows for each considered spectra that the variability amplitude increases with the increasing values of C_ξ.

IV. Conclusion

In this work we investigated the effect of uncertainties related to the damping of the structural seismic response. The damping of the structure, due in part to the material and links, is modeled by a random variable with a given variability and the seismic responses of structures, expressed in terms of spectra, are estimated by a probabilistic theory, using order statistics.

The results show two frequency ranges [<~ 3Hz] and [> ~ 8Hz] associated with rigid and flexible structures, respectively, where the not noticeable influence of damping results in small fluctuations of the responses around their respective averages. The largest fluctuations are obtained for the intermediate frequency range [~ 3Hz - 8Hz ~]. for which the damping effect is more significant.

It has also been shown that when the variability of the damping increases, characterized by increasing values of COV C_S, the amplitudes of the spectral response also grow. This foreseen outcome reflects the fact that the amplitude levels of the response are inversely proportional to the damping values.

This study can be extended when taking into account the effects of systematic uncertainties induced by the engineer in the values used for the dynamic parameters (mass, stiffness, natural frequency of vibration). The study can also be extended to study the influence of variations of the damping on the dynamic response of the structure by considering the aspects of the different seismic regulations in force around the world.

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