Economic Design of Reinforced HSC T-Sections in Flexure

B. Tiliouine and F. Fedghouche

***Abstract*—This paper reports on the development of a minimum cost design model and its application for obtaining economic designs for reinforced High Strength Concrete (HSC) T-sections in bending under ultimate limit state conditions. Cost objective functions, behavior constraints including material nonlinearities of steel and HSC, conditions on strain compatibility in steel and concrete as well as geometric design constraints are derived and implemented within the Generalized Reduced Gradient optimization algorithm. Particular attention is paid to problem formulation, solution behavior and economic considerations.**

***Index Terms*—Eurocode2 (EC-2), generalized reduced gradient algorithm, cost optimization, high strength concrete (HSC) T-sections, nonlinear programming, ultimate limit state (ULS).**

I. Introduction

Structural elements with T shaped -sections represent major components in various applications involving building and bridge structures. For repeated and large scale use of these components, as may be the case for precast reinforced High Strength Concrete (HSC) component production, special consideration should be devoted to their optimal design in order to make effective use of construction materials and ensure overall cost reduction of the project. By utilizing HSC [1], [2], the cross-section dimensions of the elements can be reduced. Consequently, less concrete, less formwork and less amount of steel reinforcement are needed. The net result is that the least expensive T-beam can be achieved with the smallest concrete cross-section, the least amount of reinforcement and the highest available concrete strength.

At the present time, the cost of HSC for concrete strength class C80/95 is about 1.50 higher than that of ordinary concrete of strength class C30/37. For HSC with higher classes such as C90/105, the over cost is of the order of 1.80. However, this over cost is rather negligible as compared to the economic advantages achieved thanks to the reduction in the quantities of construction materials to be used. This, in turn, will result in weight reduction and hence lighter and less costly foundations.

Another important aspect in developing a cost effective design approach is the use of a suitable optimization algorithm. Optimization techniques can be globally divided into three main categories: mathematical programming techniques [3], [4], methods based on optimality criteria [5] and heuristic search algorithms [6], [7].

In this paper, a non-linear mathematical programming technique based on the Generalized Reduced Gradient optimization algorithm is used. Numerical example are presented to illustrate the applicability of the minimum cost design model, solution behavior and economic considerations. Results are confronted to design solutions derived from conventional design office methods obtained in accordance with Eurocode 2 design code (EC2) [8] to evaluate the performance of the cost model.

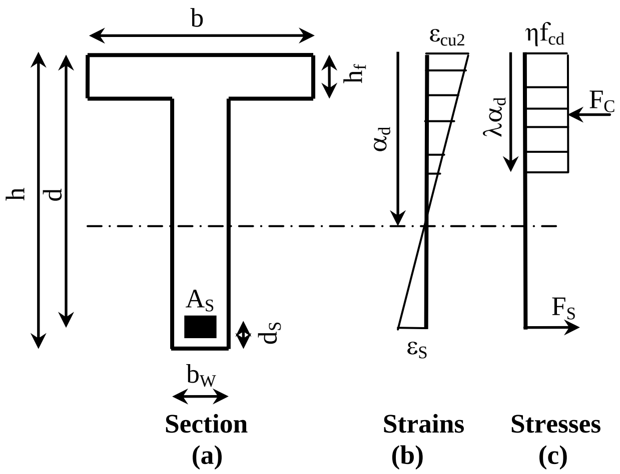

Fig. 1. (a) Typical T-Beam cross section; (b) strains and (c) stresses

II. Problem Statement

To obtain the design variables b, b_w, d, h_f, the amount of steel As and the relative depth of compressive concrete zone α (cf. Fig. 1 and Ref.[8]), given that:

Beam span: L

Ultimate bending moment capacity including self weight: M_{Ed}

Ultimate shear capacity including self weight: V_{Ed}

Characteristic compressive cylinder strength of HSC at 28 days: f_{ck}; with $50 \leq f_{ck} \leq 90$MPa

Design strength factor: η =1.0-(f_{ck}-50)/200

Compressive zone depth factor: λ=0.8-(f_{ck}-50)/400

Strain at maximum stress for (σ_c-ε_c) power law; ε_{c2}(‰)=2.0+0.0085(f_{ck}-50)0.53

Ultimate strain of compressive concrete for (σ_c-ε_c); ε_{cu2}(‰)=2.6+35[(90-f_{ck})/100]4

Factor; n=1.4+23.4[(90-f_{ck})/100]4

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Design yield strength of steel reinforcement: $f_{yd}=f_{yk}/\gamma_s$
Partial safety factor: γ_s
Characteristic elastic limit for steel reinforcement: f_{yk}
Young's elastic modulus: E_s
Total cost per unit length of HSC T-beam: C
Unit cost of reinforcing steel: C_s
Unit cost of HSC concrete: C_c
Unit cost of formwork: C_f

III. Minimum Cost Design Model of HSC T-Beam

In developing a minimum cost design model [9], it is necessary to include in the model, design constraints. For illustrative purposes, the design constraints will be herein defined in accordance with the EC2 design code specifications. Thus and without loss of generality, the formulation of the minimum cost design of HSC T-beams under ultimate loads can be mathematically stated as follows:

Find the design variables b, b_w, d, h_f, A_s, and α that minimize the total cost C per unit length of HSC T-beam such that:

$$\frac{C}{C_c} = b_{wd} + (b-b_w)h_f + (\frac{C_s}{C_c})A_s + \frac{C_f}{C_c}[(b+2(d_s+d)]. \quad (1)$$

Subject to the following constraints:

A. Behavior Constraints:

$$M_{Ed} \le \eta f_{cd}(b\text{-}b_w)h_f(d\text{-}0{,}50h_f) + \eta\lambda_c f_{cd} b_w d^2 \alpha(1\text{-}0{,}5\lambda\alpha). \quad (2)$$

(External moment including self weight ≤ Resisting moment of the cross section)

$$\alpha = (\frac{f_{yd}}{f_{cd}})(\frac{A_s}{\eta\lambda b_w}d)\text{-}(b\text{-}b_w)\frac{h_f}{\lambda b_w}d \quad (3)$$

(internal force equilibrium)

$$\frac{A_s}{b_w d} \ge p_{\min} \quad (4)$$

(minimum steel percentage)

$$\frac{A_s}{b_w h + (b-b_w)h_f} \le p_{\max} \quad (5)$$

(Maximum steel percentage)

B. Conditions on Strain Compatibility in Steel and Concrete:

$$\varepsilon_{cu2}((1/\alpha)-1) \ge \frac{f_{yd}}{E_S} \quad (6)$$

(Optimal use of steel requires that strains in steel must be limited to plastic region at the ULS)

$$\lambda\alpha(1\text{-}0{,}5\lambda\alpha) \le \mu_{\lim} \quad (7)$$

(Compression reinforcement is not required)

C. Geometric Design Variables Constraints Including Pre-Design Rules of Current Practice:

$$h \ge L / 16 \quad (8)$$

$$d / h = 0.90 \quad (9)$$

$$0.20 \le b_w / d \le 0.40 \quad (10)$$

$$(b\text{-}b_w) / 2 \le L / 10 \quad (11)$$

$$b/h_f \le 8 \quad (12)$$

$$h_f \ge h_{min} \quad (13)$$

In the above relationships, the following definitions are used:

μ_{limit}: limit value of reduced moment
h_{min}: minimum depth of flange

In equations (2) and (3) above, it is assumed that the neutral axis position is under the beam flange which ensures that the section is behaving as the T-beam section shown in Fig. 1(a).

IV. Solution Methodology

The objective function Eq.(1) and the constraints equations, Eq.(2) through Eq.(13), together form a nonlinear optimization problem. The reasons for the nonlinearity of this optimization problem are essentially due to the expressions for the cross sectional area, bending moment capacity and other constraints equations as well as the requirement to update iteratively the self weight of the T-beam, both in the constraints functions and the objective function. Both the objective function and the constraint functions are nonlinear in terms of the design variables.

In order to solve this nonlinear optimization problem, the Generalized Reduced Gradient method [4] is used as it is widely recognized as an efficient method for solving a relatively wide class of nonlinear optimization problems. A brief summary of the solution methodology is presented in the Appendix. Further details can also be found in [10], [11].

V. Numerical Results and Discussion

Typical design examples are now given, illustrating the performance of the minimum cost design model. Particular attention is paid to solution behavior and economic considerations. The objectives of this application test are:

1) to evaluate the performance of the minimum cost design model

2) to examine the characteristics of the solution in order to identify the binding and the nonbinding constraints,

3) to provide minimum cost design solutions that can be used as a basis for comparisons in future investigations.

The optimal solutions are compared to the standard design solutions obtained in accordance with EC-2 design code. The results in terms of the corresponding gains are presented in tabular form. Increase in cost saving due to the requirement to update the cross section dimensions with new self weight of the optimized beams, is also investigated.

It should be noted that the solution vector of the above problem cannot be considered as the final solution of the minimum cost design problem. As a matter of fact, because of the requirement to update the geometric dimensions of the section with the new self weight of the optimized beam, the degree of nonlinearity of the resulting optimization problem enhances further. The final optimal solution is thus obtained in two phases:

Phase 1 is concerned with the determination of the optimal solution using the initial loading parameters (i.e. with initial self weight corresponding to the starting solution).

Phase 2 is concerned with the requirement to update the self weight of the beam (both in the constraints functions and the objective function) with the geometric dimensions of the optimized section obtained in phase 1. The modified forces due to the new self weight are computed, the new dimensions of the beam are optimized and the process continued until convergence is achieved. In the present example, the optimal solution vector is reached after 3 cycles of iteration only.

A. Comparison between the HSC Optimal and Standard Design Solutions

The solution vectors of design variables including the geometric dimensions of the T-beam cross section and the area of tension reinforcement as obtained from the application of standard design procedure and the proposed approach are shown in Table I.

TABLE I: COMPARISON BETWEEN HSC CLASSICAL DESIGN SOLUTION AND OPTIMAL COST DESIGN SOLUTION EXCLUDING OR INCLUDING SELF WEIGHT EFFECTS (L=29M; M_{ED} =8MNM; V_{ED}=3MN; HSC GRADE C80/95; GRADE OF STEEL S500; C_S/C_C=25 FOR HSC; C_F/C_C=0; ρ_{MIN} =0.25%; ρ_{MAX} =4%).

	HSC Classical solution	HSC Optimal solution	HSC Optimal solution including self weight effects
b(m)	1.00	0.80	0.80
b_w(m)	0.40	0.32	0.30
h(m)	1.70	1.80	1.80
d(m)	1.53	1.62	1.62
h_f(m)	0.12	0.10	0.10
A_S(m^2)	126x10^{-4}	122 x10^{-4}	114x10^{-4}
α	0.159	0.236	0.214
C / C_c	0.999773	0.876514	0.858287

From the above results, it is clearly seen that the relative depth of the compressive concrete zone associated with the optimal solution is 48% larger than that given by the classical solution, thus leading to a much better use of the concrete. It is also seen from the values of the relative costs C/C_c associated with the classical and optimal solutions, that a significant cost saving of the order of 14% can be obtained by using the proposed formulation. Note also that this cost saving can be increased up to 16% when comparing the standard design solution to the HSC optimal design solution including self weight effects.

B. Behavior of Minimum Cost Design Solution

A study of the inequality constraints indicated that the design constraints of the beam were all non binding except for the behavior constraints associated with ultimate bending moment capacity Eq.(2) ; the geometric design constraints Eq.(8); Eq.(10); Eq.(12); and Eq.(13). The values of the geometric design variables b_w (web width), h_f (flange depth) and h (total depth) are all on the specified lower limit values.

In order to further illustrate the variability of optimal solutions with unit cost ratios C_s/C_c , the optimal solutions has also been computed for various ratios C_s/C_c = 13; 25; 36; 70; 100; 130; 160; 200.

The overall cost reduction achieved on the T-beam for a given unit cost ratio C_s/C_c, can be measured from the corresponding relative gain (in percent) defined as follows:

$$\text{Gain in percent (\%)} = \frac{C_{\text{classical}} - C_{\text{optimal}}}{C_{\text{classical}}} \times 100 \qquad (14)$$

The relative gains can be determined for the various values of the unit cost ratios. The corresponding results are reported in Table II and illustrated graphically in Fig. 2. for HSC class *C80/95*.

TABLE II: VARIATION OF RELATIVE GAIN IN PERCENT (%) VERSUS UNIT COST RATIO C_S/C_C OF CONSTRUCTION MATERIALS.

C_s/C_c	13	25	36	70	100	130	160	200
Gain in percent(%)	16	14	13	10	10	12	14	18

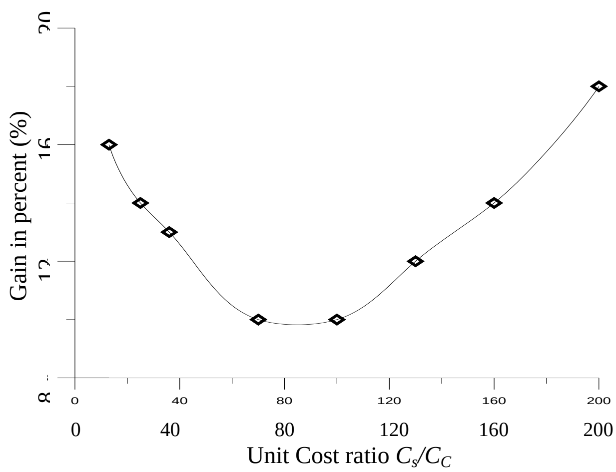

Fig. 2. Variation of gain percentage versus unit cost ratio C_s/C_c

It can be observed from Fig. 2, that the relative gain decreases rapidly for increasing values of the unit cost ratio, stabilizes around an average value approximately equal to 10% for values of 70≤Cs/Cc≤100 and then increases significantly beyond this average value.

Furthermore, the performance and sensitivity of present HSC minimum cost design model to various classes of HSC and material stress ratio prescribed in EC-2 have been examined. The results are reported in tabular form in Table III for Cs/Cc=25. It is clearly seen that the gain percentages are insensitive to changes in HSC classes and material stress ratios. Significant cost savings up to 14% (16% when including self weight effects which may important for long beam spans) can be achieved for this design example.

TABLE III: PERFORMANCE OF HSC MINIMUM COST DESIGN MODEL VERSUS HSC CLASS AND MATERIAL STRESS RATIO.

Class of HSC	*C55/67*	*C60/75*	*C70/85*	*C80/95*	*C90/105*
f_{yd} / f_{cd}	14	13	11	9	8
Gain (%)	13	14	14	14	14

VI. CONCLUSIONS

A minimum cost design model is presented for the optimal design of reinforced HSC T-beams in bending under ULS conditions considering EC2 design stress-strain relationships. Cost objective functions including cost of concrete, steel and form work, behavior constraints including nonlinearities of steel and HSC, conditions on strain compatibility in steel and concrete as well as geometric design constraints are derived and implemented within the Generalized Reduced Gradient optimization algorithm. Particular attention was paid to problem formulation, solution behavior and economic considerations.

It is shown, among others that optimal solutions achieved using the present model can lead to substantial savings in the amount of construction materials to be used.

Further practical requirements involving other design codes and manufacture constraints as well as more sophisticated cost objective functions and cross section geometry can be implemented within the present cost optimization model without major alterations.

In addition, the proposed approach is practically simple, reliable and computationally effective compared to standard design procedures used in current engineering practice.

VII. APPENDIX

A general constrained nonlinear programming problem [4], [10], [11] can be stated as follows:

Minimize $f(x), x \in F \subseteq S \subseteq R^n$

Subject to

$$h_i(x) = 0 \quad i = 1, 2, ..., p$$

$$g_i(x) \le 0 \quad j = p+1, ..., q$$

$$a_k \le x_k \le b_k \quad k = 1, ..., n$$

where x=[$x_{1,...,}x_n$] is a vector of n variables, $f(x)$ is the objective function, $h_i(x)$ ($i=1,...,p$) is the ith equality constraint, and $g_j(x)$ ($j=1,...,q$; $q<n$) is the jth inequality constraint. S is the whole search space and F is the feasible search space. The a_k and a_k denote the lower and upper bounds of the variables x_k($k=1,...,n$), respectively. It is assumed that all problem functions $f(x)$, $h_i(x)$, and $g_j(x)$ are twice continuously differentiable. In most of the nonlinear programming problems $f(x)$, $h(x)$, and $g(x)$ are nonconvex and the problems have multiple locally optimal solutions. In the only case where the $f(x)$ is convex, every $h_i(x)$ is linear and every $g_j(x)$ is convex, constrained local minimum is also constrained global minimum.

The GRG algorithm transforms inequality constraints into equality constraints by introducing slack variables. Hence all the constraints in the above nonlinear programming problem are of equality form and can be represented as follows :

$$h_i(x) = 0 \quad i = 1, 2, ..., q \tag{A1}$$

where x contains both original variables and slacks. Variables are divided into Dependent, x_D, and Independent, x_I, variables (or basic and nonbasic, respectively) :

$$x = \begin{bmatrix} x_D \\ ... \\ x_I \end{bmatrix} \tag{A2}$$

The names of basic and nonbasic variables are from linear programming. Similarly, the gradient of the objective function bounds and the Jacobian matrix may be partitioned as follow:

$$a = \begin{bmatrix} a_D \\ ... \\ a_I \end{bmatrix}, \quad b = \begin{bmatrix} b_D \\ ... \\ b_I \end{bmatrix}, \quad \nabla f(x) = \begin{bmatrix} \nabla_D f(x) \\ .... \\ \nabla_I f(x) \end{bmatrix},$$

$$J(x) = \begin{bmatrix} \nabla_D h_1(x) & ... & \nabla_I h_1(x) \\ \nabla_D h_2(x) & ... & \nabla_I h_2(x) \\ ... & ... & .... \\ \nabla_D h_q(x) & ... & \nabla_I h_q(x) \end{bmatrix} \tag{A3}$$

Let x^0 be an initial feasible solution, which satisfies equality constraints and bound constraints .Note that basic variables must be selected so that $J_D(x^0)$ is nonsingular.

The reduced gradient vector is determined as follows:

$$g_I = \nabla_I f(x^0) - \nabla_D f(x^0)(J_D(x^0))^{-1} J_I(x^0) \tag{A4}$$

The search directions for the independent and the dependent variables are given by

$$d_I = \begin{cases} 0 & if\ x_i^0 = a_i,\ g_i > 0, \\ 0 & if\ x_i^0 = b_i,\ g_i < 0, \\ -g_i & otherwise \end{cases} \tag{A5}$$

$$d_D = -(J_D(x^0))^{-1} J_I(x^0)\, d_I$$

A line search is performed to find the step length α as the solution to the following problem:

Minimize

$$f(x^0 + \alpha \mathrm{d}) \tag{A6}$$

Subject to

$$0 \le \alpha\alpha \le \alpha_{\max}$$

where

$$a_{\max} = \sup \left\{ \frac{\alpha}{\mathrm{a}} \le x^0 \le x^0 + \alpha d \le b \right\} \tag{A7}$$

The optimal solution α^* to the problem gives the next solution:

$$x^1 = x^0 + \alpha^* d \tag{A8}$$

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