Modeling and Simulation of Currency Exchange Rates Using Multifractional Process with Random Exponent

Sergio Bianchi, Alexandre Pantanella, and Augusto Pianese

***Abstract*—In this work, assuming as a model the Multifractional Processes with Random Exponent (MPRE), we propose a simulation algorithm able to replicate financial time series, specifically pertaining to the FX market. We show how, properly choosing the functional parameter of the MPRE, the simulated series fit with significant accuracy the actual ones. It is worthwhile to underline that the sole knowledge of the functional parameter ensures by itself that the surrogates succeed in replicating the empirical data. The results can be used in scenario analysis as well as in forecasting.**

***Index Terms*—Financial modeling, goodness of fit, multifractional processes, simulation.**

I. Introduction And Motivation

In the last quarter century a huge amount of empirical contributions questioned the effectiveness of the Brownian-based standard financial theory, unable to match most of the features displayed by actual data. Many alternative models were proposed to reproduce anomalies such as the fat tails and the high peaks in the distributions of the price variations, the absence of autocorrelation in the log returns or the highly autocorrelated volatility process.

In order to overcome the limits of the traditional models, we investigate the capability that the Multifractional Process with Random Exponent (MPRE) has to describe in a parsimonious way the empirical dynamics. The process was introduced by [1] as a generalization of the multifractional Brownian motion (mBm) [2], [3]; in its turn this extends the very well-known fractional Brownian motion (fBm) by allowing its Hurst parameter to change through time. The mBm and the MPRE – used in signal, image and texture analysis as well as in TCP traffic modeling – are generally still disregarded in finance, mainly because their nonstationarity encumbers the inference of global probabilistic properties. The main distrust of the financial community towards these models resides perhaps in the difficulty to grasp how to conciliate the no-arbitrage principle with their ability to both catch the empirically observed features of prices and give a rationale to the market mechanism. Precisely for this ability, MPRE deserves much more attention.

There are several reasons for claiming the robustness of the MPRE as a model of the financial dynamics (detailed discussions can be found in [4]–[8]); here we restrict ourselves to mention the capability the MPRE has to (a) replicate the patterns shown by financial data, and (b) provide a rationale for the trading mechanism. The formers will be shortly recalled in Section 2. Section 3 deals with the description of the model, while Section 4 introduces the simulation algorithm, whose effectiveness in capturing the main features of financial time series in discussed in Section 5. Section 6 concludes.

II. Main Stylized Facts of Financial Time Series

It is well known that, albeit widely used, the basic Gaussian model is unable to capture the main features displayed by the financial time series; in his seminal paper, Cont [9] summarizes them in the followings: (1) Absence of autocorrelation – (2) Heavy tails – (3) Gain/loss asymmetry – (4) Aggregational Gaussianity – (5) Intermittency – (6) Volatility clustering – (7) Conditional heavy tails – (8) Slow decay of autocorrelation in absolute returns – (9) Leverage effect – (10) Volatility/volume correlation – (11) Asymmetry in time scales. Although each item should deserve a detailed discussion, to quote Cont "these stylized facts are so constraining that it is not easy to exhibit even an (ad hoc) stochastic process which possesses the same set of properties and one has to go to great lengths to reproduce them with a model".

The purpose of this work resides just in addressing this issue: we show that the proper composition of two stochastic processes is able to replicate many of the features above recalled. In particular, even if the model we are going to discuss seems to be able to address all the stylized facts, we will focus for brevity on the most relevant ones, i.e. on (1), (2), (6) and (8).

III. Multifractional Processes with Random Exponent

In this Section the basics of the Multifractional Process with Random Exponent (MPRE) will be recalled. The starting point is the fractional Brownian motion (fBm), introduced 1940 by Kolmogorov and revived in 1968 by Mandelbrot and Van Ness [10]. Denoted by a+ = max(a, 0) and by W the Gaussian measure, the fBm can be represented through the Wiener integral

$$B_H(t) = \int_{\Re} \left((t-x)_+^{H-1/2} - (-x)_+^{H-1/2} \right) dW(x) \qquad (1)$$

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S. B. Author is with the Department of Economics and Law, University of Cassino, (FR), Italy, (e-mail: sbianchi@eco.unicas.it).

A. P. Author is with the Department of Economics and Law, University of Cassino, (FR), Italy, (e-mail:a.pantanella@eco.unicas.it).

A. P. Author is with the Department of Economics and Law, University of Cassino, (FR), Italy, (e-mail: pianese@unicas.it).

The pointwise Hölder exponent[1] $\alpha_{\text{fBm}}(t,\omega) = H$ almost surely at any point t. This value tunes the strength of dependence of the process: $H = 0.5$ recovers as a particular case the Brownian motion, whereas values larger (lower) than 0.5 indicate persistence (antipersistence), the stronger the more H deviates from 0.5.

In many situations, the constancy of the pointwise regularity can be too restrictive, so the most immediate generalisation – known as multifractional Brownian motion (mBm) and independently introduced by [2] and [3] – replaces the parameter H by a (Hölderian) function $H(t)$.

The moving average representation becomes

$$B_{H(t)}(t)=\int_{\Re}\left(\left(t-x\right)_+^{H(t)-1/2}-\left(-x\right)_+^{H(t)-1/2}\right)dW(x) \tag{2}$$

and the pointwise Hölder exponent $\alpha_{\text{mBm}}(t,\omega)$ equals almost surely $H(t)$ at any point t.

The process has covariance given by

$$\frac{g(H_t,H_s)}{2}\left\{|t|^{H_t+H_s}+|s|^{H_t+H_s}-|t-s|^{H_t+H_s}\right\} \tag{3}$$

where $g(H_t,H_s)=\{I(H_t)I(H_s)\}^{-1/2}I\{(H_t+H_s)/2\}$ and

$$I(H)=\begin{cases}\frac{\Gamma(1-2H)}{H}\sin\left\{\frac{\pi}{2}(1-2H)\right\} & H\in(0,\tfrac{1}{2})\\ \pi & H=\tfrac{1}{2}\\ \frac{\Gamma\{2(1-H)\}}{H(2H-1)}\sin\left\{\frac{\pi}{2}(2H-1)\right\} & H\in(\tfrac{1}{2},1)\end{cases}$$

Notice that the mBm assumes $H(t)$ to be a deterministic function; Papanicolaou and Sølna [11] suggest to replace it by a stationary stochastic process $\{S(t)\}_{t\in[0,1]}$ with smooth paths and decaying correlation function independent on W. The work is developed by Ayache and Taqqu [1], who define the *Multifractional Processes with Random Exponent* (MPRE).

Their construction starts from the definition of (a) the Gaussian field $\{B_H(t)\}_{(t,H)\in[0,1]\times[a,b]\subset(0,1)}$ depending on t and H, with integral representation provided by (1) and (b) the stochastic process $\{S(t)\}_{t\in[0,1]}$ with values in the arbitrary fixed compact interval $[a,b]$. Equipped with this notation, Ayache and Taqqu define the MPRE of parameter $\{S(t)\}_{t\in[0,1]}$ as the stochastic process

$$Z(t,\omega)=f_2\left(f_1(t)\right)=B_{S(t,\omega)}(t,\omega) \tag{4}$$

Any trajectory of $Z(t,\omega)$ is the composition of two functions:

- f_1: $[0,1]\to[0,1]\times[a,b]$, $t\to(t,S(t,\omega))$, that builds the random process which serves as functional parameter, and
- f_2: $[0,1]\times[a,b]\to\Re$, $(t,H)\to B_H(t,\omega)$, that rules the resulting process.

The functional parameter $\{S(t,\omega)\}$ is not necessarily stationary nor independent from the white noise W in (2); when independence is assumed, the main results known for the mBm can be extended to the MPRE.

The continuity of the MPRE ultimately relies on the continuity of the trajectories of the process that provides its random exponent. So, choosing the stochastic functional parameter properly, one can simulate in a parsimonious way even very complex phenomena, ranging from finance to physics, from information theory to physiology.

Fig. 1 displays sample paths of (*a*) an fBm with parameter H=0.6, (*b*) a shifted mBm with sine functional parameter rescaled to range in the interval [0.2, 0.8] and (*c*) an MPRE with random exponent in the interval [0.25, 0.65].

As a term of comparison, Fig. 2 shows the British pound/Canadian dollar quotes of the last five years. Even at a first glance, the series shares with the MPRE many features, such as the volatility clustering or the local trends which alternate to antipersistent phases. Notice that for the fBm the pointwise regularity is the same along the whole trajectory; it changes with the deterministic function $H(t)$ and with the random function $\{S(t,\omega)\}$ in the case of the mBm and of the MPRE, respectively.

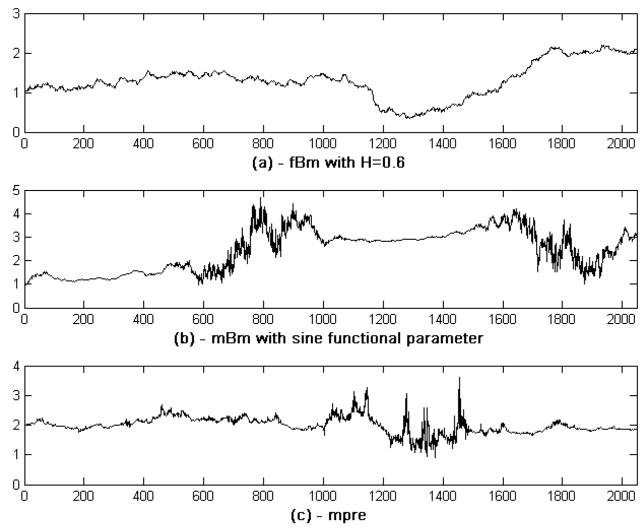

Fig. 1. Surrogated time series

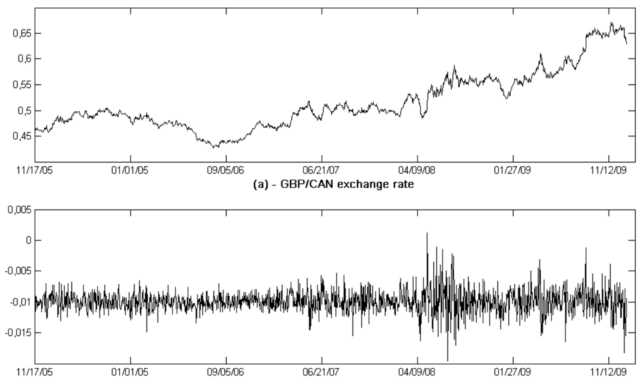

Fig. 2. GBP/CAN (01/10/2005-06/30/2010)

Just its larger flexibility makes the MPRE a process to be

[1] Remind that the Hölder exponent measures the degree of irregularity of the graph of a function. Given the function $f(x)$, if there exist a constant C and a polynomial P_n of degree $n<h$ such that $|f(x) - P_n(x - x_0)| \le C|x- x_0|^h$, the Hölder exponent $H(x_0)$ is defined as the supremum of all h's such that the above relation holds. The polynomial P_n is often associated with the Taylor expansion of f around x_0, but the relation is valid even if such expansion does not exist.

kept at due distance because of its complexity and, at the same time, a good candidate to model the financial dynamics.

In fact, as suggested in [4], [5], [12] and [13], the pointwise regularity can be assumed as a proxy of the weight that investors assign to the past prices in taking their trading decisions.

IV. The Simulation Algorithm

Roughly speaking, the construction of the MPRE described in the previous Section counts two steps: the first one consists in generating a (eventually continuous) stochastic process valued in the rectangle $[0,1]\times[a,b]$; the second one consists in compounding the image of this random function with a fractional Brownian motion, depending on $t\in[0,1]$ and $H\in[a,b]$. The resulting process will have at t the pointwise Hölder exponent given by the value of the stochastic process built in step one. This construction is in principle somewhat simple but problems arise when one uses it to fit an empirical (financial) time series. To this aim, it is necessary to have a clear idea of how the pointwise Hölder exponent of the series one wants to replicate evolves through time. Therefore, the very basic step resides in estimating the pointwise Hölder exponent of the empirical time series. The knowledge of its dynamics allows to choose the stochastic process $S(t,\omega)$ and to start the construction of the MPRE. In order to accomplish this fundamental step we use the estimator introduced in [4] and improved in [13]. Since its description goes beyond the scope of this work, we will just recall that the estimator – written for the q-lagged increments of a series $\{X_{j,n}\}$ with n data sampled in discrete time on the grid $t = 1,\ldots, n$ and with unit time variance equal to K^2 – is referred to a window of length δ and reads as

$$H^k_{\delta,q,n,K}(t)=\frac{\log\left(\frac{\sqrt{\pi}}{\delta-q+1}\sum_{j=t-\delta}^{t-1}\left|X_{j+q,n}-X_{j,n}\right|^k/2^{k/2}\Gamma\left(\frac{k+1}{2}\right)K^k\right)}{k\log\left(\frac{q}{n-1}\right)} \qquad (5)$$

The moving-window estimator (5):

- is based on the assumption that the increments $X_{j+q,n} - X_{j,n}$ of the series are normally distributed within the window δ with mean zero and variance equal to $K^2\left(q/(n-1)\right)^{2H_{t/(n-1)}}$;
- is itself normally distributed with mean equal to the parameter to be estimated and variance that can be explicitly calculated when $H(t) = 0.5$. Confidence intervals for the each δ and $H(t)$ are provided by Monte Carlo simulations in [13];
- has a $(\sqrt{\delta}\log n)$ -rate of convergence, δ and n respectively being the length of the estimation window and the number of sampling points.

Just to get an idea of the goodness of fit of estimator (5), consider Fig. 3, which displays in panel (b) the estimation of the δ-pointwise Hölder exponent for the path surrogated in Figure 1, panel (b). For the series considered in the example, made of 2,048 data points, a window δ=25 was assumed (i.e., a trading month is sufficient to get rather reliable estimates on a sample of five years). While for modeling purposes it is necessary to infer the properties of the stochastic process which is supposed to have generated the set of values estimated by $H^k_{\delta,q,n,K}(t)$, for simulation purposes we will be content with replicating the series, a part from the knowledge of the generating process.

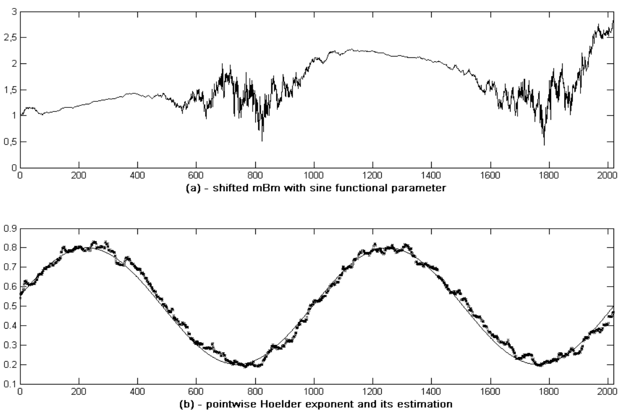

Fig. 3. Estimated δ-pointwise Hölder exponent

In fact, the analysis of the random exponent process – strongly dependent on the area of interest the model is applied to – would deserve for financial time series a separate and more extensive discussion, started in [13] with reference to the stock markets.

The idea we use in this contribution is somewhat simple but effective. Once the random exponent process has been estimated, we set as functional parameter of the MPRE the sequence of values obtained by (5). The assumption $S(t,\omega) = H^k_{\delta,q,n,K}(t)$, $t = \delta,\ldots, n$ allows to generate as many replicates as necessary, all sharing the same pointwise regularity structure.

In order to simulate the MPRE we use the modified Chan and Wood algorithm [14]. Basically, denoted by n the number of grid points and by Δ the grid width, the algorithm simulates a $B_H(t)$ at locations on the grid Δ, 2Δ, …, $n\Delta$. Setting $t_j = j\Delta$, the simulation of the mBm whose parameters are $H_1,\ldots,H_m$ on a finite grid of m points, proceeds through the following steps:

1. simulate $\{Y_{j,u}=\Delta^{-Hu}(B_{Hu}(t_j) - B_{Hu}(t_{j-1})): j=1,\ldots,n; u=1,\ldots,m$
2. for each j and u, calculate $B_{Hu}(t_j) =\Delta^{Hu}(Y_{1,u}+\ldots+ Y_{j,u})$
3. for each j, "predict" $B_{H(t_j)}(t_j)$ using some kriging based on the values $\{ B_{Hu}(t_j): j=1,\ldots,n; u=1,\ldots,m\}$. At each location $(H(t_j), t_j)$, the kriging is built through a set of neighbours $N_j = \{(\nu, k)\}$ and the mBm is predicted by

$$B_{H(t_j)}(t_j)=\sum_{(\nu,k)\in N_j}\gamma^{(j)}_{\nu,k}B_{H_\nu}(t_k)$$

where $\gamma^{(j)}_{\nu,k}=\{cov(B_{H_\nu}(t_k)\}^{-1}cov\left(B_{H_\nu}(t_k),B_{H(t_j)}(t_j)\right)$ and the covariance is given by (3).

As it will be shown in the next Section, we find evidence that the information provided by $S(t, \omega)$ is sufficient to simulate series sharing with the empirical ones most of the features included in the list of Section 1.

V. Application To Currency Exchange Rates

For brevity, the analysis we discuss here concerns only two exchange rate series: the Euro-Dollar (EUR/USD) and the Euro-Yen (EUR/JPY). According the the European standard, the quotes use the so called price notation; this means that the values are expressed in units of the base currency per unit of the target currency. A number of financial series were analysed, not only with regard to the FX market but also for the stock markets, but no relevant differences were found in terms of goodness of fit. Table I summarizes the main statistical features of the two datasets: in both the cases the kurtosis is significantly higher than 3, the value characterizing the normal distribution. Also the skewness and the tail index (both should be equal to zero for the Gaussian law) denote that the distributions are not symmetric and that the tails decrease more slowly than one would expect for the normal law.

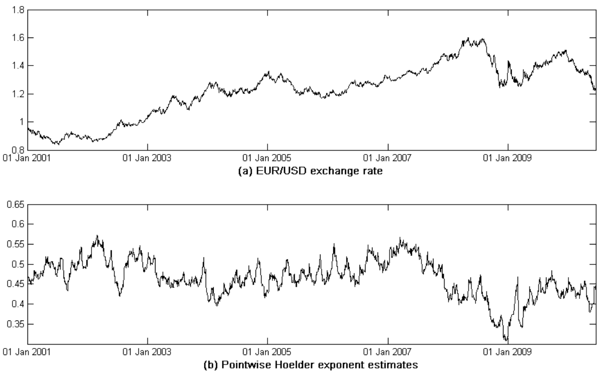

Fig. 4. EUR/USD Exchange Rate

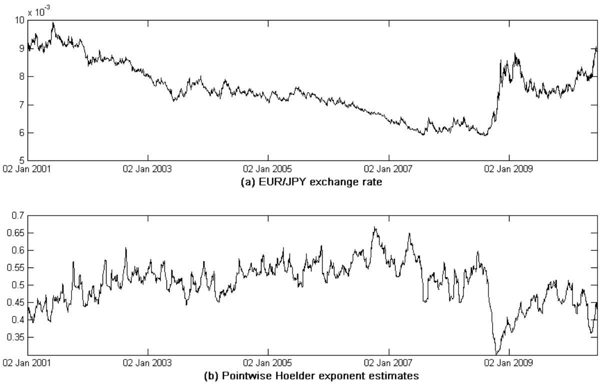

Fig. 5. EUR/JPY Exchange Rate.

TABLE I: Analysed Exchange Rate Series

	Mean	St.Dev.	Kurtosis	Skewness	Tail-index
EUR/USD	1.18×10^{-4}	0.0082	6.6818	0.1661	0.1535
EUR/JPY	-7.17×10^{-7}	0.0062	9.3274	0.5156	0.1424

The very first step consisted in estimating the random exponent process. Fig. 4 and 5 display – in panel (a) – the exchange rate series and – in panel (b) – the pointwise Hölder exponent estimation. Notice that in both the cases, the random exponent – although with large deviations – fluctuates around 0.5 until 2008, year in which it drops downwards. Given the meaning of the pointwise exponent, the path designed by $H(t)$ summarizes the state of the market; it captures both the trends typical of `bull' or `bear' markets and the mean-reversion, symptomatic of the frenetic buy-and-sell activity occuring during the financial crises. It is worthwhile to underline the consistency of the estimates: the exchange series show an overall strong negative correlation[2] equal to –0,7959, while the exponents series display a significant positive correlation (0.5789) and this indicates that the information flow – that ultimately engenders the variations of $H(t)$ – acts similarly for the two series. Regardless of the financial interpretation, here we are interested in replicating the features of Table I (high-peaked, fat-tailed distributions) as well as the behavior of the sample autocorrelation (remind that the price variations are not autocorrelated whereas the absolute or squared variations do show slowly decaying autocorrelations). Applying the algorithm described in the previous Section we generated 1,000 samples of MPRE for each time series, all sharing the same pointwise Hölder function.

Fig. 6 displays the normal probability plots [15] for both the datasets. The data (actual and surrogated ones) are plotted against the theoretical normal distribution represented by the straight line. Obviously, departures from the line indicate departures from normality. In detail, the Gaussian model fails in capturing the fat tails, which are on the opposite fitted by the surrogated series, that in addition match almost “perfectly” the distributions of the empirical data.

TABLE II: Surrogated Time Series

	Mean	St.Dev.	Kurtosis	Skewness	Tail-index
EUR/USD	-5.15×10^{-7}	0.0063	5.7725	-0.0600	0.0377
EUR/JPY	-7.64×10^{-6}	0.0083	8.0751	-0.0233	0.1077

These findings are summarized in Table II, which displays the distributional properties of the increments of the surrogated series. The standard deviation is slightly lower than the actual one and the same occurs for the kurtosis. This effect can be explained in terms of the smoothing introduced by the sliding window of length δ the estimator works with. Improvements can be achieved by perturbation of the estimated Hölder exponents, but the issue is going to be faced in a future work.

The symmetry of the paths of the multifractional Brownian motion, simulated by $B_{H(t_j)}(t_j)$ (step 3 of Section 4), does not allow to replicate the positive (negative) skewness of the empirical data and some refinements - that we will not dwell upon here - should be considered in order to correct for this effect.

In particular for the EUR/JPY exchange rate, the surrogated series capture the tail index too. A possible explanation of the poor fitting of the tail index for the EUR/USD series resides perhaps in the behavior of $H(t)$. Looking at panels (b) of Fig. 4 and 5, it is easy to realize that the EUR/USD series is characterized by smaller variations with respect to the EUR/JPY case. In addition this

[2] To preserve coherence with panels (a) of Figures 4 and 5 the correlations were calculated maintaining the price notation for both the currencies. Generally, the correlation is calculated assuming one of the two currencies in price notation and the other in volume notation. In this case the overall correlation between the two series would be equal to 0.7758.

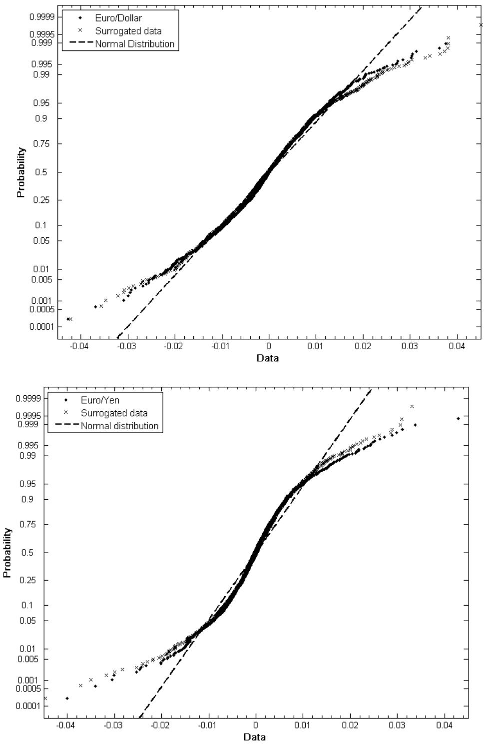

Fig. 6. Probability plot: actual and surrogated
(a) EUR/USD, (b) EUR/JPY

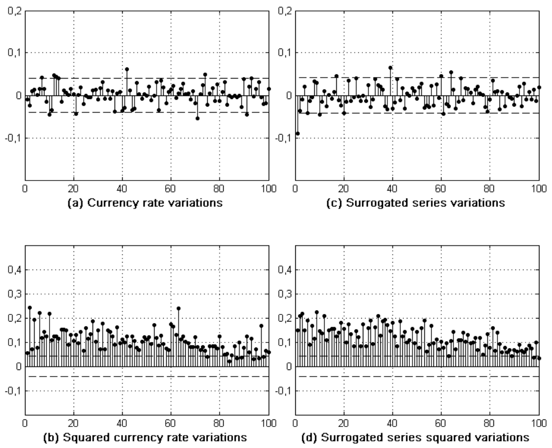

Fig. 7. EUR/USD. Sample autocorrelation function

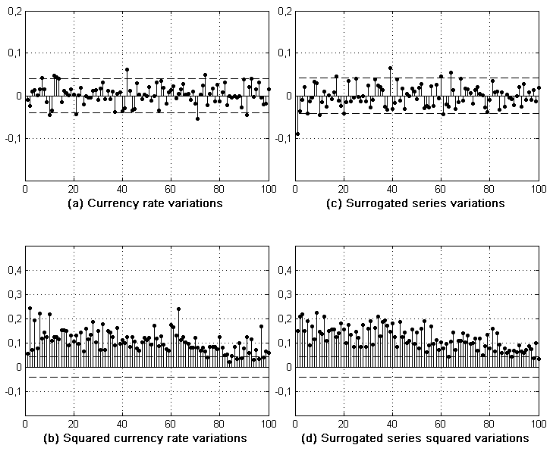

Fig. 8. EUR/JPY. Sample autocorrelation functions

series is also affected by abrupt changes in the dynamics of the pointwise exponent that impact on the extremes of the distributions.

In particular for the EUR/JPY exchange rate, the surrogated series capture the tail index too. A possible explanation of the poor fitting of the tail index for the EUR/USD series resides perhaps in the behavior of $H(t)$. Looking at panels (b) of Figures 4 and 5, it is easy to realize that the EUR/USD series is characterized by smaller variations with respect to the EUR/JPY case. In addition this series is also affected by abrupt changes in the dynamics of the pointwise exponent that impact on the extremes of the distributions.

Even more interesting is the behavior of the surrogated series with regard to their autocorrelation. Figures 7 and 8 reproduce the sample autocorrelation functions of actual (panels (a)-(b)) and surrogated (panels (c)-(d)) data. Notice that the behavior is virtually undistinguishable for both the simple variations – for which the autocorrelation is pretty much null (panels (a)-(c)) – and the squared variations – for which the autocorrelation is significantly positive (panels (b)-(d)).

VI. Concluding Remarks And Future Developments

In this work we have analysed the correspondence of the MPRE-based simulations with some of the most fundamental stylized facts well-known in finance. We found clear evidence that the surrogated series succeed in fitting the distributional properties and the autocorrelation behavior of the empirical data. By construction, the model can be proved to be able to replicate also other patterns of actual financial time series, but their detailed discussion goes beyond the scope of this work.

TABLE III: Checked and Potential Matches of MPRE

Stylized fact	Surrogated
Absence of autocorrelation	Y
Heavy Tails	Y
Gain/Loss asymmetry	P
Aggregational Gaussianity	P
Intermittency	P
Volatility clustering	Y
Conditional heavy tails	P
Slow decay of autocorrelation in absolute returns	Y
Leverage effect	P
Volatility/volume correlation	P
Asymmetry in time scale	P

We will be content with pointing out in Table III the features surely matched by the MPRE (Y) and those which are potentially compatible with the model but still missing of proper analyses (P).

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S. Bianchi was born in Cassino on 25th September 1967. He was graduated in Economics from University of Cassino (Italy) in 1991 and earned his Ph.D. in Actuarial and Financial Science from University of Rome "La Sapienza" (Italy) in 1997.

He taught Mathematics and Financial Mathematics at the Gregorian Pontifical University, University of Rome "La Sapienza" and University of Sassari and he is now full professor of Mathematics and Mathematical Finance at the University of Cassino, where he also held the office of Head of Department from 2005 to 2009. From 2009 he is Rector's Delegate for Research and Benchmarking. He is author of about sixty research papers concerning the modeling of financial time series by (multi)fractional processes and self-similar stochastic processes. His current research interests concern market's efficiency and behavioral finance in the light of the arbitrage principle.

Prof. Bianchi is a member of the AMASES (Association of Applied Mathematics for Economic and Social Sciences).

A. Pantanella was born in Aigle (Switzerland) on 2nd December 1975. He graduated in Mechanical Engineering from University of Cassino (Italy) in 2003 and earned his Ph.D. in Mathematics for Economics and Finance from University of Rome "La Sapienza" (Italy) in 2009.

He taught Mathematics and Computer Science at the Faculty of Economics, University of Cassino and he is now adjunct professor of Mathematics at the University of Cassino. He is author of about fifteen research papers concerning financial modeling and applications of multifractional stochastic processes to the finance. His research interests concern financial time series, stochastic modeling, simulation algorithms and wavelet analysis.

A. Pianese was born in Isola del Liri (Italy) on 2nd July 1962. He graduated in Mathematics from University of Camerino (Italy) in 1986.

He taught Mathematics and Computational methods in Applied Mathematics at the Faculty of Economics, University of Cassino and he is now Aggregate Professor of Mathematical Finance and Computer Science. From 2004 he is the Director of the Computer Lab for Advanced Scientific Applications "L.I.S.A.". He is author of several papers focused on financial and electricity market modeling. Currently, his research concern stochastic models for financial time series with a particular look at (multi)fractional and (multi)fractal processes.

Prof. Pianese is a member of the AMASES (Association of Applied Mathematics for Economic and Social Sciences).